Multi-Spin Spacecraft And Gyrostats As Dynamical Systems With Multiscroll Chaotic Attractors

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***Abstract*—Attitude dynamics of the multi-spin spacecraft (MSSC) and multirotor unbalanced gyrostats (MUG) is considered in aspects of the possibility of the strange multiscroll chaotic attractors' (MCA) realization. Corresponding connections between the dynamical systems with MCA (Lorenz, Sprott, Wang, Qi, Li, Chen, Lü, Liu, Čelikovský, Burke, Shaw, Arneodo, Coullet, etc.) and the mathematical models of the MSSC/MUG' attitude dynamics are established/defined at the presence of only linear structures of external torques (primarily linear dissipation or excitation) and control signals in the internal rotors' engines of the MSSC/MUG.**

***Keywords*— multiscroll chaotic attractors, multi-spin spacecraft, unbalanced gyrostats, internal rotors, control systems and torques**

I. INTRODUCTION

Investigation of various aspects of the spacecraft (SC) attitude dynamics remains one of the urgent tasks of the spaceflight dynamics, which is considered in different formulations. An important feature of the SC dynamics is the irregular attitude motion at the realization of the regimes connected with chaotic attractors in the phase space of angular (attitude) motion around the SC mass center. In this connection it is quite interesting to find links of the attitude SC dynamics with the classical cases of strange chaotic attractors in the well known dynamical systems (Lorenz, Sprott, Wang, Qi, Li, Chen, Lü, Liu, Čelikovský, Burke, Shaw, Arneodo, Coullet, etc.) [1-29].

As the further development of the spacecraft construction we can indicate a multi-rotor scheme, which corresponds to a multi-spin spacecraft (MSSC) and unbalanced gyrostats [3]. Motivated by the recent wonderful results [1] we will try in this work to make a connection of the MSSC motion mathematical models with the dynamical systems containing strange multiscroll chaotic attractors (MCA) [1-29].

The MSSC [3] is constructed as the multi-rotor system with conjugated pairs of rotors placed on the all inertia principle axes of the main body (fig.1). General properties of the MSSC motion are connected with the internal multi-rotor system (the multi-rotor kernel). This «spider»-type multi-rotor system was described in [3, 4] where the attitude dynamics, spatial (attitude) reorientations of the MSSC and also roll-walking motions of multi-rotor robots are considered. The multi-rotor kernel allows to perform the attitude gyroscopic stabilization of the MSSC with the help of a compound spinup of the rotors. One of the important features of the MSSC is numerous independent internal degrees of freedom corresponding to the rotors' rotations. It is the powerful instrument for the spacecraft's attitude control and/or the angular reorientation with the help of an internal redistribution of the system angular momentum between the rotors and the main body.

The paper has the following structure. The section 1 is the introduction; the section 2 contains the main mathematical models of the attitude dynamics of multibody gyrostats and MSSC, which are used in the section 3, where the passible reduction of the dynamical models to the system with multiscroll chaotic attractors is considered; and in the section 4 we present some examples of the modelling of attitude motion based on the MCA-models.

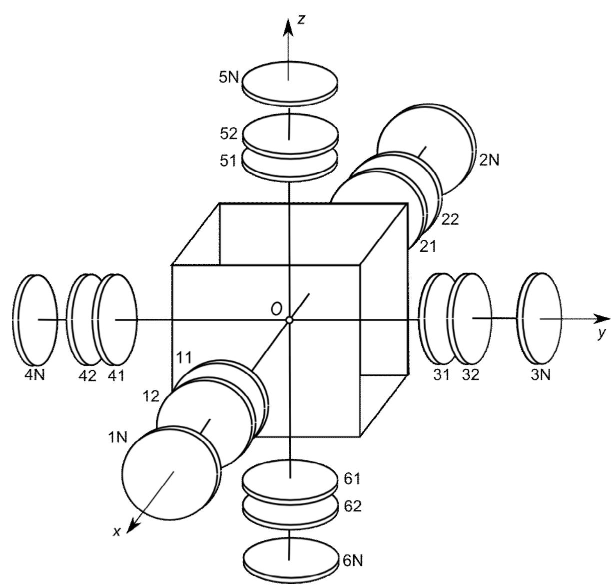

Fig.1. The MSSC mechanical internal structure

II. MATHEMATICAL MODEL OF THE MOTION

Let's assume the symmetry of rotors disposition (Fig.1) relatively point *O* and identity of their mass-inertia parameters.

Following to the work [3] we can construct the main equations of the MSSC attitude dynamics. The angular momentum of the MSSC-system can be written in projections

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onto the frame *Oxyz* axes connected with the central main body

$$\mathbf{K}=\mathbf{K}_m+\mathbf{K}_r \quad (1)$$

where $\mathbf{K}_m$ is the angular momentum of the main body with the fixed ("frozen") rotors; $\mathbf{K}_r$ is the relative angular momentum of the rotors.

The angular motion equations of the system can be written with the help of the law of the angular momentum's variation in the moving coordinates frame *Oxyz*

$$\frac{d\mathbf{K}}{dt}+\boldsymbol{\omega}\times\mathbf{K}=\mathbf{M}^e \quad (2)$$

where $\mathbf{M}^e$ is the external torque.

We can write the dynamical equations for the multi-rotor system with 6*N* rotors (Fig.1). This system contains *N* layers with rotors on the six general directions coinciding with the principle axes of the main body. The angular momentum components can be written

$$\mathbf{K}_m=\begin{bmatrix}Ap\\Bq\\Cr\end{bmatrix};\quad \mathbf{K}_r=\sum_{l=1}^{N}I_l\begin{bmatrix}\sigma_{1l}+\sigma_{2l}\\ \sigma_{3l}+\sigma_{4l}\\ \sigma_{5l}+\sigma_{6l}\end{bmatrix};$$
$$A=\tilde{A}+4\bar{J}+2\bar{I},\quad B=\tilde{B}+4\bar{J}+2\bar{I}, \quad (3)$$
$$C=\tilde{C}+4\bar{J}+2\bar{I};\ \bar{J}=\sum_{l=1}^{N}J_l;\ \bar{I}=\sum_{l=1}^{N}I_l;$$

Here $\boldsymbol{\omega}=[p,q,r]^{\mathrm{T}}$ – the vector of the absolute angular velocity of the main body; $\tilde{A}, \tilde{B}, \tilde{C}$ are the general moments of inertia of the main body; σ_{kl} is the relative angular velocity of the *kl*-th rotor (relatively the main body); I_l and J_l are the longitudinal and the equatorial inertia moments of the *l*-layer-rotor relatively the point *O*.

Then the equation (2) can be written in the following scalar form

$$\begin{cases}A\dot{p}+\sum_{l=1}^{N}I_l\left(\dot{\sigma}_{1l}+\dot{\sigma}_{2l}\right)+\left(C-B\right)qr+\\ \quad+\left[q\sum_{l=1}^{N}I_l\left(\sigma_{5l}+\sigma_{6l}\right)-r\sum_{l=1}^{N}I_l\left(\sigma_{3l}+\sigma_{4l}\right)\right]=M_x^e;\\ B\dot{q}+\sum_{l=1}^{N}I_l\left(\dot{\sigma}_{3l}+\dot{\sigma}_{4l}\right)+\left(A-C\right)pr+\\ \quad+\left[r\sum_{l=1}^{N}I_l\left(\sigma_{1l}+\sigma_{2l}\right)-p\sum_{l=1}^{N}I_l\left(\sigma_{5l}+\sigma_{6l}\right)\right]=M_y^e;\\ C\dot{r}+\sum_{l=1}^{N}I_l\left(\dot{\sigma}_{5l}+\dot{\sigma}_{6l}\right)+\left(A-C\right)qp+\\ \quad+\left[p\sum_{l=1}^{N}I_l\left(\sigma_{3l}+\sigma_{4l}\right)-q\sum_{l=1}^{N}I_l\left(\sigma_{1l}+\sigma_{2l}\right)\right]=M_z^e;\end{cases} \quad (4)$$

The relative motion equations of the rotors are $\left(l=1..N\right)$:

$$\begin{cases}I_l\left(\dot{p}+\dot{\sigma}_{1l}\right)=M_{1l}^i+M_{1lx}^e;\ I_l\left(\dot{p}+\dot{\sigma}_{2l}\right)=M_{2l}^i+M_{2lx}^e\\ I_l\left(\dot{q}+\dot{\sigma}_{3l}\right)=M_{3l}^i+M_{3ly}^e;\ I_l\left(\dot{q}+\dot{\sigma}_{4l}\right)=M_{4l}^i+M_{4ly}^e\\ I_l\left(\dot{r}+\dot{\sigma}_{5l}\right)=M_{5l}^i+M_{5lz}^e;\ I_l\left(\dot{r}+\dot{\sigma}_{6l}\right)=M_{6l}^i+M_{6lz}^e\end{cases} \quad (5)$$

where $M_{jlx}^e, M_{jly}^e, M_{jlz}^e$ are external torques acting on the *jl*-th rotor, and M_{jl}^i is the torque from internal forces acting between the main body and the *jl*-th rotor (the internal engines torques).

The equation system (4) with *N* systems (5) completely describe the angular motion of the multi-rotor system with 6*N* rotors and the attitude dynamics of the multi-spin spacecraft (Fig.1).

The main dynamical equations (4) can be rewritten in the unbalanced-gyrostat-form [3]

$$\begin{cases}\hat{A}\dot{p}+\dot{D}_{12}+\left(\hat{C}-\hat{B}\right)qr+\left[qD_{56}-rD_{34}\right]=M_x^e\\ \hat{B}\dot{q}+\dot{D}_{34}+\left(\hat{A}-\hat{C}\right)rp+\left[rD_{12}-pD_{56}\right]=M_y^e\\ \hat{C}\dot{r}+\dot{D}_{56}+\left(\hat{B}-\hat{A}\right)pq+\left[pD_{34}-qD_{12}\right]=M_z^e\end{cases} \quad (6)$$

$$\dot{D}_{12}=M_{12}^i+M_{12}^e;\quad \dot{D}_{34}=M_{34}^i+M_{34}^e;\quad \dot{D}_{56}=M_{56}^i+M_{56}^e, \quad (7)$$

where $\hat{A}=A-2\sum_{j=1}^{N}I_j;\ \hat{B}=B-2\sum_{j=1}^{N}I_j;\ \hat{C}=C-2\sum_{j=1}^{N}I_j$,

D_{ij} are the summarized angular momentums of the rotors:

$$D_{12}=\sum_{j=1}^{N}\left[\Delta_{1j}+\Delta_{2j}\right],\quad D_{34}=\sum_{j=1}^{N}\left[\Delta_{3j}+\Delta_{4j}\right],$$
$$D_{56}=\sum_{j=1}^{N}\left[\Delta_{5j}+\Delta_{6j}\right];$$
$$\Delta_{1j}=I_j\left(p+\sigma_{1j}\right);\quad \Delta_{2j}=I_j\left(p+\sigma_{2j}\right); \quad (8)$$
$$\Delta_{3j}=I_j\left(q+\sigma_{3j}\right);\quad \Delta_{4j}=I_j\left(q+\sigma_{4j}\right);$$
$$\Delta_{5j}=I_j\left(r+\sigma_{5j}\right);\quad \Delta_{6j}=I_j\left(r+\sigma_{6j}\right);$$

and the summarized rotors' torques are

$$M_{12}^i=\sum_{l=1}^{N}\left(M_{1l}^i+M_{2l}^i\right),\quad M_{12}^e=\sum_{l=1}^{N}\left(M_{1lx}^e+M_{2lx}^e\right)$$
$$M_{34}^i=\sum_{l=1}^{N}\left(M_{3l}^i+M_{4l}^i\right),\quad M_{34}^e=\sum_{l=1}^{N}\left(M_{3ly}^e+M_{4ly}^e\right) \quad (9)$$
$$M_{56}^i=\sum_{l=1}^{N}\left(M_{5l}^i+M_{6l}^i\right),\quad M_{56}^e=\sum_{l=1}^{N}\left(M_{5lz}^e+M_{6lz}^e\right)$$

III. THE REDUCTION OF THE MSSC EQUATIONS TO DYNAMICAL SYSTEMS WITH MULTISCROLL CHAOTIC ATTRACTORS

As it was indicated in the recent work [1] the natural

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candidates for the construction of systems with multi-scroll chaotic attractors are 3D quadratic continuous time systems given by

$$\begin{cases} \dot{x} = a_0 + a_1 x + a_2 y + a_3 z + a_4 x^2 + a_5 y^2 + \\ \qquad + a_6 z^2 + a_7 xy + a_8 xz + a_9 yz; \\ \dot{y} = b_0 + b_1 x + b_2 y + b_3 z + b_4 x^2 + b_5 y^2 + \\ \qquad + b_6 z^2 + b_7 xy + b_8 xz + b_9 yz; \\ \dot{z} = c_0 + c_1 x + c_2 y + c_3 z + c_4 x^2 + c_5 y^2 + \\ \qquad + c_6 z^2 + c_7 xy + c_8 xz + c_9 yz; \end{cases} \tag{10}$$

where $\{a_i, b_i, c_i\}_{0 \le i \le 9} \in \mathbb{R}^{30}$ are the parameters (bifurcation parameters).

For example, in the work [1] based on the structure (10) the dynamical system with the three-scroll chaotic attractor (fig.2) was considered/constructed at the following non-nil values of the parameters:

$$\begin{cases} a_1 = 1;\ a_2 = -1;\ a_3 = 0.5;\ a_9 = -3; \\ b_1 = -0.1;\ b_2 = -6;\ b_8 = 1;\ b_9 = -1; \\ c_1 = 0.06;\ c_2 = -10;\ c_3 = -5;\ c_7 = 2;\ c_8 = 0.23 \end{cases}$$

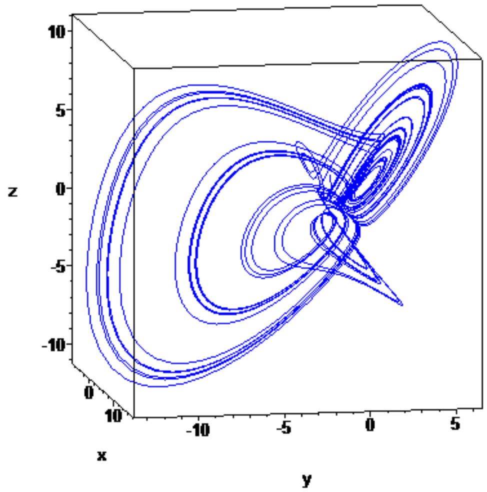

Fig.2. The three-scroll chaotic attractor of the system [1]

Now we solve the task of reducing of the equations (6) to the form of dynamical systems (10).

Let us consider the MSSC case of motion at the presence of only linear control torques (these torques are formed in the internal rotors' engines) proportional to the angular accelerations of the main MSSC-body:

$$\begin{aligned} M_{12}^i &= \alpha_p \dot{p} + \alpha_q \dot{q} + \alpha_r \dot{r}; \\ M_{34}^i &= \beta_p \dot{p} + \beta_q \dot{q} + \beta_r \dot{r}; \\ M_{56}^i &= \gamma_p \dot{p} + \gamma_q \dot{q} + \gamma_r \dot{r}, \end{aligned} \tag{11}$$

$\{\alpha_p \dots \gamma_r\}$ - are the constants corresponding to the amplification factors on channels of angular accelerometers.

Then for the summarized rotors' angular momentums we have following dependencies corresponding to the solutions of equations (7)

$$\begin{aligned} D_{12} &= \alpha_p p + \alpha_q q + \alpha_r r + \alpha_0; \\ D_{34} &= \beta_p p + \beta_q q + \beta_r r + \beta_0; \\ D_{56} &= \gamma_p p + \gamma_q q + \gamma_r r + \gamma_0, \end{aligned} \tag{12}$$

where $\alpha_0, \beta_0, \gamma_0 -$ are the constants, corresponding to the initial conditions.

Taking into account (12) we can solve the equations (6) for the derivatives of the angular velocities

$$\dot{p} = \frac{\Delta_p}{\Delta_0}; \qquad \dot{q} = \frac{\Delta_q}{\Delta_0}; \qquad \dot{r} = \frac{\Delta_r}{\Delta_0}, \tag{13}$$

where

$$\begin{cases} \Delta_0 = \left(\hat{A} + \alpha_p\right)\left(\hat{B} + \beta_q\right)\left(\hat{C} + \gamma_r\right) + \alpha_q \beta_r \gamma_p + \alpha_r \beta_p \gamma_q - \\ \qquad - \alpha_r \gamma_p \left(\hat{B} + \beta_q\right) - \beta_r \gamma_q \left(\hat{A} + \alpha_p\right) - \alpha_q \beta_p \left(\hat{C} + \gamma_r\right); \\ \Delta_p = \mathfrak{M}_x \left(\hat{B} + \beta_q\right)\left(\hat{C} + \gamma_r\right) + \alpha_q \beta_r \mathfrak{M}_z + \alpha_r \gamma_q \mathfrak{M}_y - \\ \qquad - \alpha_r \left(\hat{B} + \beta_q\right)\mathfrak{M}_z - \beta_r \gamma_q \mathfrak{M}_x - \alpha_q \left(\hat{C} + \gamma_r\right)\mathfrak{M}_y; \\ \Delta_q = \mathfrak{M}_y \left(\hat{A} + \alpha_p\right)\left(\hat{C} + \gamma_r\right) + \alpha_r \beta_p \mathfrak{M}_z + \beta_r \gamma_p \mathfrak{M}_x - \\ \qquad - \alpha_r \gamma_p \mathfrak{M}_y - \beta_p \left(\hat{C} + \gamma_r\right)\mathfrak{M}_x - \beta_r \left(\hat{A} + \alpha_p\right)\mathfrak{M}_z; \\ \Delta_r = \mathfrak{M}_z \left(\hat{A} + \alpha_p\right)\left(\hat{B} + \beta_q\right) + \alpha_q \gamma_p \mathfrak{M}_y + \beta_p \gamma_q \mathfrak{M}_x - \\ \qquad - \gamma_p \left(\hat{B} + \beta_q\right)\mathfrak{M}_x - \alpha_q \beta_p \mathfrak{M}_z - \gamma_q \left(\hat{A} + \alpha_p\right)\mathfrak{M}_y; \end{cases} \tag{14}$$

with quadratic dependencies

$$\begin{cases} \mathfrak{M}_x = \mathfrak{M}_x(p,q,r) = M_x^e(p,q,r) - \gamma_0 q + \beta_0 r + \\ + \left[\beta_q - \gamma_r - \left(\hat{C} - \hat{B}\right)\right] qr - \gamma_p pq + \beta_p pr - \gamma_q q^2 + \beta_r r^2; \\ \mathfrak{M}_y = \mathfrak{M}_y(p,q,r) = M_y^e(p,q,r) - \alpha_0 r + \gamma_0 p + \\ + \left[\gamma_r - \alpha_p - \left(\hat{A} - \hat{C}\right)\right] rp - \alpha_q qr + \gamma_q qp - \alpha_r r^2 + \gamma_p p^2; \\ \mathfrak{M}_z = \mathfrak{M}_z(p,q,r) = M_z^e(p,q,r) - \beta_0 p + \alpha_0 q + \\ + \left[\alpha_p - \beta_q - \left(\hat{B} - \hat{A}\right)\right] pq - \beta_r rp + \alpha_r rq - \beta_p p^2 + \alpha_q q^2; \end{cases} \tag{15}$$

The equations (13) (with linear combinations (14) of the quadratic terms (15)) can be reduced to the form (10) at the redefinition of the main variables $(p \leftrightarrow x;\ q \leftrightarrow y;\ r \leftrightarrow z)$ and at the explicit expressions for $\{a_i, b_i, c_i\}_{0 \le i \le 9}$ in terms $\left\{\hat{A}, \hat{B}, \hat{C}, \alpha_0, \alpha_j, \beta_0, \beta_j, \gamma_0, \gamma_j\right\}_{j=p,q,r} \in \mathbb{R}^{15}$.

Moreover, yet we have not specified the external torques $M_{x,y,z}^e$. Let us consider these torques also as linear dissipative/excitative influences:

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$$M_x^e = m_{x0} + m_{xp}p + m_{xq}q + m_{xr}r;$$
$$M_y^e = m_{y0} + m_{yp}p + m_{yq}q + m_{yr}r; \quad (16)$$
$$M_z^e = m_{z0} + m_{zp}p + m_{zq}q + m_{zr}r,$$

with constants $\left\{m_{ij}\right\}_{i=x,y,z;\ j=0,p,q,r} \in \mathbb{R}^{12}$.

Also we introduce into consideration the "gyroscopic" external influences

$$M_x^e = g_{pqx}pq + g_{prx}pr + g_{qrx}qr;$$
$$M_y^e = g_{pqy}pq + g_{pry}pr + g_{qry}qr; \quad (17)$$
$$M_z^e = g_{pqz}pq + g_{prz}pr + g_{qrz}qr,$$

with constants $\left\{g_{ijk}\right\}_{i,j=p,q,r;k=x,y,z} \in \mathbb{R}^9$.

So, it is possible to conclude, that the MSSC equations (6) can be reduced to the quadratic dynamical systems with multi-scroll chaotic attractors.

IV. Examples of reductions

Here we describe some examples of reductions of the MSSC equations to the well known dynamical systems.

An analysis of the systems' parameters allows to give us the following correspondences.

Example 1 – The Dequan Li attractor.

The MSSC parameters, which correspond to the Dequan Li attractor [6] realization in the angular velocity components phase space, are:

$$\forall \hat{C};\ \ \hat{B} = \hat{C};\ \ \hat{A} = \frac{2\hat{C}}{1+\delta^2};\ \ \varepsilon = \frac{\hat{A}}{\hat{C}}\delta;$$
$$D_{34} = \hat{A}\delta p;\quad M_x^e = \hat{A}\alpha\left(q-p\right); \quad (18)$$
$$M_y^e = \hat{B}\left(\rho p + \zeta q\right);\ M_z^e = \hat{C}\beta r + \hat{A}\delta^2 pq;$$

Then from the MSSC equations we obtain the Dequan Li system [6] with the three-scroll chaotic attractor (Fig.3):

$$\begin{cases} \dot{x} = \alpha\left(y-x\right) + \delta xz, \\ \dot{y} = \rho x + \zeta y - xz, \\ \dot{z} = \beta z + xy - \varepsilon x^2, \end{cases}$$

$\alpha = 40;\ \beta = 1.833;\ \delta = 0.16;$

$\varepsilon = 0.65;\ \rho = 55;\ \zeta = 20.$

Example 2 – The Wang-Sun attractor.

The MSSC parameters, which correspond to the Wang-Sun attractor [11] realization in the angular velocity components phase space, are:

$$\forall \hat{A};\quad \hat{B} > \hat{A};\quad \hat{C} = \hat{B} - \hat{A};\ \ (\xi = 1;\ \zeta = -1);$$
$$D_{ij} = 0;\quad M_x^e = \hat{A}\alpha p; \quad (19)$$
$$M_y^e = \hat{B}\left(\beta p + \delta q\right);\ M_z^e = \hat{C}\varepsilon r;$$

Then from the MSSC equations we obtain the Wang-Sun system [11] with the four-scroll chaotic attractor (Fig.4):

$$\begin{cases} \dot{x} = \alpha x + \xi yz, \\ \dot{y} = \beta x + \delta y - xz, \\ \dot{z} = \varepsilon z + \zeta xy, \end{cases}$$

$\alpha = 0.2;\ \beta = -0.01;\ \xi = 1;$

$\delta = -0.4;\ \varepsilon = -1;\ \zeta = -1.$

Example 3 – The Chen-Lee attractor.

The MSSC parameters, which correspond to the Chen-Lee attractor [20] realization in the angular velocity components phase space, are:

$$\forall \hat{C};\quad \hat{B} = \frac{\hat{C}}{3};\quad \hat{A} = \frac{2\hat{C}}{3};$$
$$D_{ij} = 0;\quad M_x^e = \hat{A}\alpha p; \quad (20)$$
$$M_y^e = \hat{B}\beta p;\ M_z^e = \hat{C}\delta r;$$

Then from the MSSC equations we obtain the Chen-Lee system [20] with the two-scroll chaotic attractor (Fig.5):

$$\begin{cases} \dot{x} = \alpha x - yz, \\ \dot{y} = \beta y + xz, \\ \dot{z} = \delta z + \frac{1}{3}xy, \end{cases}$$

$\alpha = 5;\ \beta = -10;\ \delta = -3.8.$

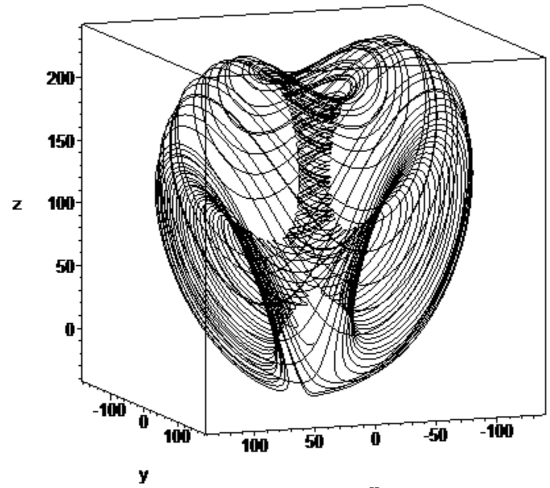

Fig.3. The Dequan Li [6] chaotic attractor

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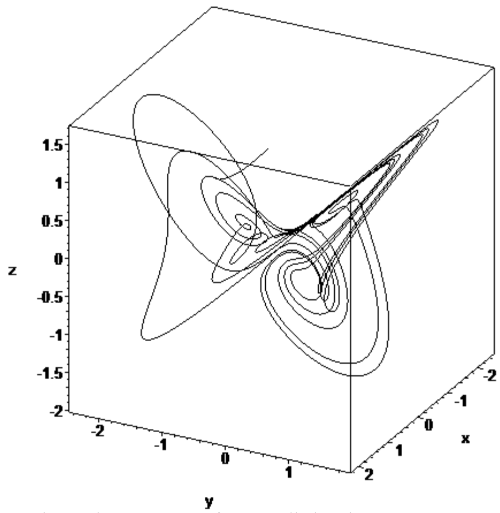

Fig.4. The Wang-Sun four-scroll chaotic attractor [11]

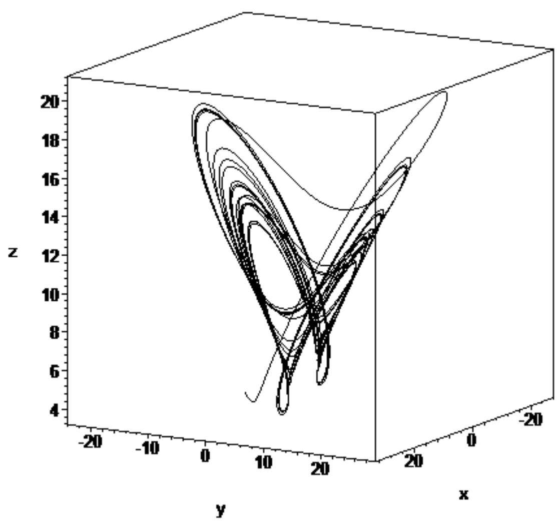

Fig.5. The Chen-Lee system [20] with the two-scroll chaotic attractor

So, the considered models/systems/calculations represent the important cases of the MSSC dynamics which can be regular or chaotic depending on parameters combinations. It is the important aspect of the real motion realization in the frameworks of various space missions and programs [30].

REFERENCES

[1] Z.Elhadj, J.C. Sprott, Simplest 3D continuous-time quadratic systems as candidates for generating multiscroll chaotic attractors, International Journal of Bifurcation and Chaos, Vol. 23, No. 7 (2013).

[2] J. Lü, G. Chen, A new chaotic system and beyond: the generalized Lorenz-like system, International Journal of Bifurcation and Chaos, Vol. 14, No. 5 (2004) 1507-1537.

[3] A.V. Doroshin, Homoclinic solutions and motion chaotization in attitude dynamics of a multi-spin spacecraft, Communications in Nonlinear Science and Numerical Simulation, Volume 19, Issue 7 (2014), 2528–2552.

[4] A.V. Doroshin, "Attitude Control of Spider-type Multiple-rotor Rigid Bodies Systems", Proceedings of the World Congress on Engineering 2009, London, U.K. Vol II, pp.1544-1549.

[5] A.V. Doroshin, Modeling of chaotic motion of gyrostats in resistant environment on the base of dynamical systems with strange attractors. Communications in Nonlinear Science and Numerical Simulation, Volume 16, Issue 8 (2011) 3188–3202.

[6] Li, D. Q. A three-scroll chaotic attractor, Phys.Lett. A 372, (2008) 387–393.

[7] Liu, W. B. & Chen, G. R. "Can a threedimensional smooth autonomous quadratic chaotic system generate a single four scroll attractor?" Int.J. Bifurcation and Chaos 14, (2004) 1395–1403.

[8] Lü, J., Murali, K., Sinha, S., Leung, H. & Aziz-Alaoui, M. A. "Generating multi-scroll chaotic attractors by thresholding," Phys. Lett. A 372, (2008) 3234–3239.

[9] Qi, G. Y., Chen, G. R., van Wyk, M. A., van Wyk, B. J. & Zhang, Y. H. (2008) "A four-wing chaotic attractor generated from a new 3-D quadratic autonomous system," Chaos Solit. Fract. 38, 705–721.

[10] Wang, L. "3-scroll and 4-scroll chaotic attractors generated from a new 3-D quadratic autonomous system," Nonlin. Dyn. 56, (2008) 453–462.

[11] Wang, Z., Sun, Y., van Wyk, B. J., Qi, G. & van Wyk, M. A. "A 3-D four-wing attractor and its analysis," Brazilian J. Phys. 39, (2009) 547–553.

[12] Wang, X. & Chen, G. "Constructing a chaotic system with any number of equilibria," Nonlin. Dyn. 71, (2012) 429–436.

[13] Yu, S. M., Lu, J. H., Tang, W. K. S. & Chen, G. "A general multiscroll Lorenz system family and its DSP realization," Chaos 16 (2006), 033126-1–10.

[14] Yu, S. M., Tang, W. K. S., Lü, J. & Chen, G. "Generation of n×m-wing Lorenz-like attractors from a modified Shimizu–Morioka model," IEEE Trans.Circuits Syst.-II 55, (2008) 1168–1172.

[15] Yu, S. M., Tang, W. K. S., Lü, J. & Chen, G. "Generating 2n-wing attractors from Lorenz-like systems," Int. J. Circuit Th. Appl. 38, (2010) 243–258.

[16] Yu, S., Lü, J., Chen, G. & Yu, X. "Design and implementation of grid multiwing butterfly chaotic attractors from a piecewise Lorenz system," IEEE Trans. Circuits Syst.-II : Exp. Briefs 57, (2010) 803–807.

[17] Arneodo, A., Coullet, P. H., Spiegel, E. A. & Tresser, C. (1985) Asymptotic chaos," Physica D14, 327-347.

[18] Celikovsky, S. & Chen, G. (2002) On a generalized Lorenz canonical form of chaotic systems," Int. J. Bifurcation and Chaos 12, 1789-1812.

[19] Chen, G. & Lai, D. (1998) Anticontrol of chaos via feedback,Int. J. Bifurcation and Chaos 8, 1585-1590.

[20] Chen HK, Lee CI, Anti-control of chaos in rigid body motion. Chaos, Solitons & Fractals 2004; 21: 957-65

[21] Festa, R., Mazzino, A. & Vincenzi, D. (2002) Lorenz-like systems and classical dynamical equations with memory forcing: An alternate point of view for singling out the origin of chaos, Phys. Rev. E65, 046205.

[22] Linz, S. J. & Sprott, J. C. (1999) Elementary chaotic flow, Phys. Lett. A259, 240-245.

[23] Liu, W. B. & Chen, G. (2003) A new chaotic system and its generation, Int. J. Bifurcation and Chaos 12, 261-267.

[24] Lofaro, T. (1997) A model of the dynamics of the Newton-Leipnik attractor," Int. J. Bifurcation and Chaos 7, 2723-2733.

[25] Lü, J. & Chen, G. [2002] A new chaotic attractor coined," Int. J. Bifurcation and Chaos 12, 659-661.

[26] Lü, J., Chen, G., Cheng, D. & _Celikovsk_y, S. [2002] Bridge the gap between the Lorenz system and the Chen system, Int. J. Bifurcation and Chaos 12, 2917-2926.

[27] Lü, J., Chen, G. & Zhang, S. [2002] Dynamical analysis of a new chaotic attractor," Int. J. Bifurcation and Chaos 12, 1001-1015.

[28] Sprott, J. C. [1994] Some simple chaotic flows, Phys. Rev. E50.

[29] Sprott, J. C. [1997] Simplest dissipative chaotic flow, Phys. Lett. A228, 271-274.

[30] A.V. Doroshin, Chaos and its avoidance in spinup dynamics of an axial dual-spin spacecraft. Acta Astronautica, Volume 94, Issue 2, February 2014, Pages 563-576.

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