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# Spinup-Capture Dynamics of Multi-Rotor Nanosatellites and Somersaulting Robots

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***Abstract*—Dynamics of the multi-rotor mechanical drive-kernel is considered. The kernel can be used in the reorientation processes of the small (nano)satellites and, also at the somersaulting motion realization of the roll-walking robots.**

***Keywords*— conjugate spinup, conjugate capture, multi-rotor drive-kernel, nanosatellite, roll-walking robot**

## I. INTRODUCTION

Dynamics of the angular motion of the multi-rotor nanosatellites (NS) and walking robots (WR) with internal multi-rotor drive systems is considered. The indicated multi-rotor drive system consists of pairs of conjugate rotors located on the main axes (Fig.1, 2) – this system allows perform conjugate spinups and captures of the conjugated rotors (for example, rotor #1 and #2) for attitude reorientation of the NS around its center of mass (point *O*) and for somersaulting motion of the WR around its edge ($G_1G_2$).

To perform one step of the WR (Fig.2) we need to perform conjugate spinup of the pair of conjugated rotors, and then make the capture (angular velocity braking) one of them. After this capture the rotor's angular moment is transmitted to the main rigid body of the robot and it makes an angular motion relative to one of its axes. Capture of the second conjugate rotor compensates the angular moment and stops the rotation of the main robot body.

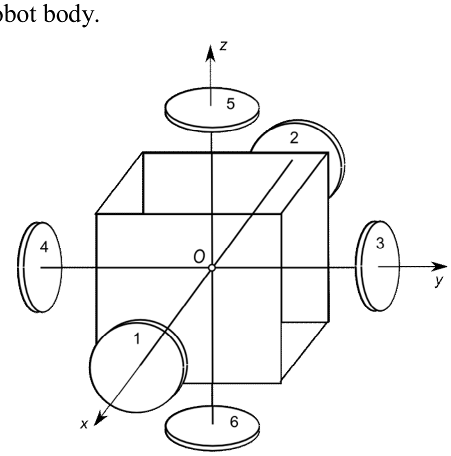


Fig.1

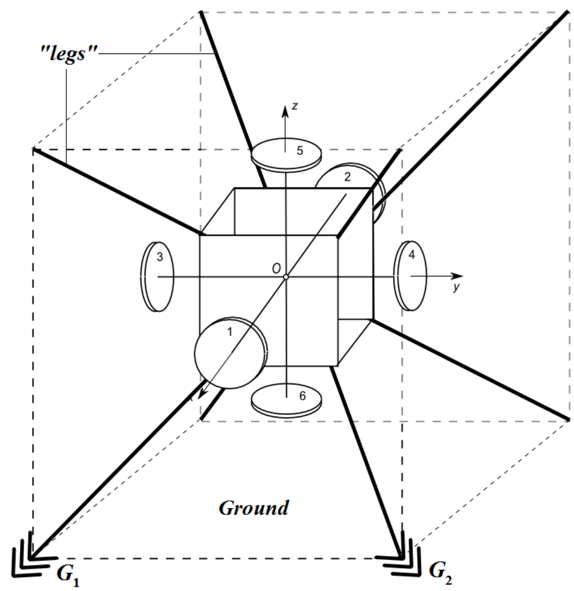


Fig.2

## II. MATHEMATICAL MODEL OF THE MOTION

Let's assume the symmetry of rotors disposition relatively point O and identity of their mass-inertia parameters. The angular momentum of the system (Fig.1) in projections onto the frame axes Oxyz connecting with the main body can be written as [1, 2]

$$\mathbf{K} = \mathbf{K}_m + \mathbf{K}_r \tag{1}$$

$$\mathbf{K}_m = \begin{bmatrix} \tilde{A}p \\ \tilde{B}q \\ \tilde{C}r \end{bmatrix} + \left(4J + 2I\right) \begin{bmatrix} p \\ q \\ r \end{bmatrix}, \quad \mathbf{K}_r = I \begin{bmatrix} \sigma_1 + \sigma_2 \\ \sigma_3 + \sigma_4 \\ \sigma_5 + \sigma_6 \end{bmatrix} \tag{2}$$

where $\mathbf{K}_m$ - is the angular moment of the main rigid body with the "frozen" rotors; $\mathbf{K}_r$ - is the relative angular moment of the rotors; $\omega=[p, q, r]^T$ - is the vector of absolute angular velocity of the main body; $\sigma_i$ - is the relative angular velocity of *i*-th rotor with respect to the main body; $\tilde{A}, \tilde{B}, \tilde{C}$ are the general moments of inertia of the main body; *I* is the longitudinal moments of inertia of the single


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rotor; $J$ is equatorial moments of inertia of the single rotor calculated about point $O$.

The motion equations of the multi-rotor system can be presented as follows [1, 2]

$$\frac{d\mathbf{K}}{dt} + \boldsymbol{\omega} \times \mathbf{K} = \mathbf{M}^e \qquad (3)$$

where $\mathbf{M}^e$ is the principal moment of the external forces. Eq. (3) can be rewritten as

$$\begin{cases} A\dot{p} + I\dot{\sigma}^{12} + (C-B)qr + I\left(q\sigma^{56} - r\sigma^{34}\right) = M_x^e \\ B\dot{q} + I\dot{\sigma}^{34} + (A-C)pr + I\left(r\sigma^{12} - p\sigma^{56}\right) = M_y^e \\ C\dot{r} + I\dot{\sigma}^{56} + (B-A)pq + I\left(p\sigma^{34} - q\sigma^{12}\right) = M_z^e \end{cases} \qquad (4)$$

In the last equations the following terms take place

$$\sigma^{ij} = \sigma_i + \sigma_j, \qquad A = \tilde{A} + 4J + 2I$$
$$B = \tilde{B} + 4J + 2I, \; C = \tilde{C} + 4J + 2I \qquad (5)$$

We need to add the equations for the rotors relative motion. These equations can also be written on the base of the law of the change in the angular momentum

$$\begin{cases} I\left(\dot{p} + \dot{\sigma}_1\right) = M_1^i + M_{1x}^e; \; I\left(\dot{p} + \dot{\sigma}_2\right) = M_2^i + M_{2x}^e \\ I\left(\dot{q} + \dot{\sigma}_3\right) = M_3^i + M_{3y}^e; \; I\left(\dot{q} + \dot{\sigma}_4\right) = M_4^i + M_{4y}^e \\ I\left(\dot{r} + \dot{\sigma}_5\right) = M_5^i + M_{5z}^e; \; I\left(\dot{r} + \dot{\sigma}_6\right) = M_6^i + M_{6z}^e \end{cases} \qquad (6)$$

where $M_j^i$ is the principal moment of the internal forces acting between the main body and $j$-th rotor; $M_{jx}^e, M_{jy}^e, M_{jz}^e$ are the principal moments of external forces acting only at $j$-th rotor.

The equation systems (4) and (6) together completely describe the attitude dynamics of the multi-rotor system.

## III. The Spinup-Capture-Reorientation

Now we can give some definitions.

*Def.1.* Conjugated rotors are the pair of rotors locating in the opposite rays. For example, rotor 3 and rotor 4 (Fig.1) are the conjugate rotors.

*Def.2.* The conjugate spinup is the process of spinning up of the conjugated rotors in opposite directions up to the desired value of the relative angular velocity with help of internal forces moments from the main body. Velocities of the conjugate rotors will equal in absolute value and opposite in sign.

*Def.3.* The rotor capture is the deceleration of the rotor's relative angular velocity with the help of internal forces moment from the main body.

For realization of the NS reorientation it is needed to perform the conjugate spinup (in opposite directions) of the pairs of conjugate rotors, and then make the capture (braking) one of them. After this capture the rotor's angular moment is transmitted to the main rigid body of the NS (or WR), and it makes a rotation about one of axes. The capture of the second conjugate rotor compensates the angular moment and stops the rotation of the main NS' (WR's) body.

This approach of the spatial reorientation is characterized by the minimal motion inertia: it means immediately redistribution of the angular moment between rotors and main NS's body. It is possible at the condition of the immediate rotors' captures on the base of absolute friction generation or with the help of gear meshing (or other type of slip-free engagements). This inertialess property of the NS' angular motion is unique property in comparison with classical case of the attitude control realization by the reaction-wheels.

## IV. Numerical Simulations

Here we need to note that the reorientation process of multi-rotor spacecraft and roll-walking robots at the immediate captures was investigated in works [1, 2]. In this paper we consider the slow captures features in the case of single-axes reorientation at the initial rest of the main body $\left(p(0) = q(0) = r(0) = 0\right)$.

So, we assume that conjugate spinup of the conjugated rotors (## 1 and 3) is finished and the rotors take the relative angular velocities $\sigma_1 = \Sigma; \; \sigma_3 = -\Sigma.$

After this conjugate spinup we perform the synchronous captures of both rotors with the help of the viscous frictions with the corresponded internal torques

$$M_1^i = -\nu_1\sigma_1; \quad M_3^i = -\nu_1\sigma_3 \qquad (7)$$

where the coefficients are not equal: $\nu_1 \neq \nu_2$.

Then we obtain the $r$-velocity initiation and slow reorientation process, when the main body rotate on the angle $\varphi = \int r(t)dt$. The numerical simulation results are presented at the figures (Fig.3, 4).

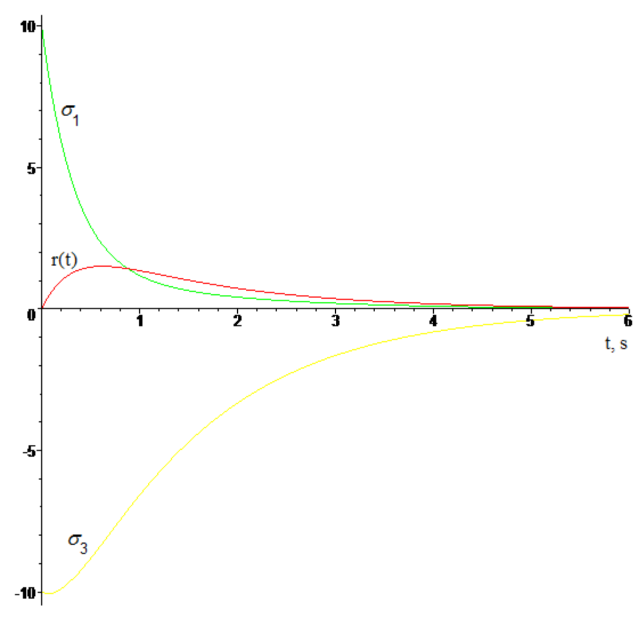


Fig.3



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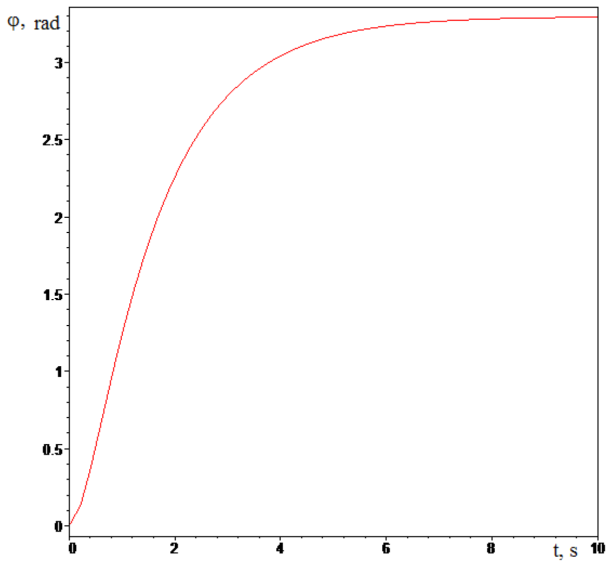


Fig.4

At the simulation we take the following numerical values:

*C*=2 kg*m$^2$; *I*=1 kg*m$^2$; Σ=10 rad/s; ν1=2 kg*m$^2$/s; ν3=0.55 kg*m$^2$/s.

As we can see, the main body takes the rotation about its z-axes on the angle π radian during time about 10 second. This "soft" reorientation method based on the difference of viscous frictions between main body and rotors. This approach allows us using the small ("large-time") torques of the rotors' spinup and relatively small friction coefficients for the reorientation of the NS.

This approach is useful also for the somersaulting motion initiation at the WR movement, but we need to note, that the gravitation here is the important perturbation – it can be take into account by analogy with results [2].

Also it is needed to indicate some interesting aspects of the multi-rotor systems' dynamics such as vehicles synthesis and analysis of the regular/irregular chaotic motion at presence of small perturbations and inertia-mass variability [3-14].

Research of the perturbed regimes can be provide with the help of Hamilton canonical equations for the well-known Andoyer-Deprit variables [12-14]:

$$q_i = \langle l, \varphi_2, \varphi_3, \delta_1, ..., \delta_6 \rangle$$
$$p_i = \langle L, G, H, \Delta_1, ..., \Delta_6 \rangle$$

where $\delta_i, \Delta_i -$ are the rotors' relative rotations angles and corresponding momentums. The system kinetic energy has the following form:

$$\begin{aligned} T &= \left(G^2 - L^2\right)\left[\frac{\sin^2 l}{\hat{A}} + \frac{\cos^2 l}{\hat{B}}\right] + \frac{1}{\hat{C}}\left(L - [\Delta_5 + \Delta_6]\right)^2 - \\ &-2\sqrt{G^2 - L^2}\left\{\frac{\sin l}{\hat{A}}\cdot[\Delta_1 + \Delta_2] + \frac{\cos l}{\hat{B}}\cdot[\Delta_3 + \Delta_4]\right\} + \\ &+\frac{1}{\hat{A}}\left(\Delta_1 + \Delta_2\right)^2 + \frac{1}{\hat{B}}\left(\Delta_3 + \Delta_4\right)^2 + \sum_{i=1}^{6}\frac{\Delta_i^2}{I} \end{aligned} \qquad (8)$$

where $\hat{A} = A - 2I;\ \hat{B} = B - 2I;\ \hat{C} = C - 2I.$

Let's consider the system motion with the constant angular moments of the rotors:

$$\Delta_i = \Delta_i\big|_{t=0} = \Delta_i(0) = \text{const}_i$$

Now we assume the presence of the small (ε<<1) perturbation in the internal engines of the rotor #3:

$$M_{\delta_3} = -\varepsilon \sin\left(\nu t + \psi\right)$$

Then the following equation takes place:

$$\dot{\Delta}_{31} = -\varepsilon \sin\left(\nu t + \psi\right) \qquad (9),$$

or we have the Hamiltonian system with the small potential

$$H = T + P; \quad P = \varepsilon \sin\left(\nu t + \psi\right)\delta_3. \qquad (10)$$

From Eq.(9) the solution follows

$$\Delta_3(t) = \varepsilon f(t) + \Delta_3(0);$$
$$f(t) = \frac{1}{\nu}\left[\cos\left(\nu t + \psi\right) - \cos\psi\right]$$

Based on the Hamiltonian (10) we can write the independent second order dynamical system for two positional coordinates {*l*, *L*}:

$$\begin{aligned} \dot{L} &= -\frac{G^2 - L^2}{2}\left[\frac{1}{\hat{A}} - \frac{1}{\hat{B}}\right]\sin 2l - \\ &-\sqrt{G^2 - L^2}\,D\sin(l - s) - \varepsilon f(t)\sin l; \end{aligned} \qquad (11)$$

$$\begin{aligned} \dot{l} &= L\left[\frac{1}{\hat{C}} - \left(\frac{1}{2\hat{A}} + \frac{1}{2\hat{B}}\right) - \left(\frac{1}{2\hat{B}} - \frac{1}{2\hat{A}}\right)\cos 2l + \frac{D\cos(l - s)}{\sqrt{G^2 - L^2}}\right] - \\ &-\frac{D_{56}}{\hat{C}} + \frac{\varepsilon L}{\sqrt{G^2 - L^2}} f(t)\cos l \end{aligned}$$

where

$$D_{12} = \Delta_1(0) + \Delta_2(0),$$
$$D_{34} = \Delta_3(0) + \Delta_4(0),$$
$$D_{56} = \Delta_5(0) + \Delta_6(0),$$
$$D = \sqrt{\frac{D_{12}^2}{\hat{A}^2} + \frac{D_{34}^2}{\hat{B}^2}},$$
$$\cos(s) = \frac{D_{34}}{\hat{B}D},\ G = \text{const}$$

The dynamical system (11) can be applied to investigation of main features of the system dynamics including chaos phenomenon.


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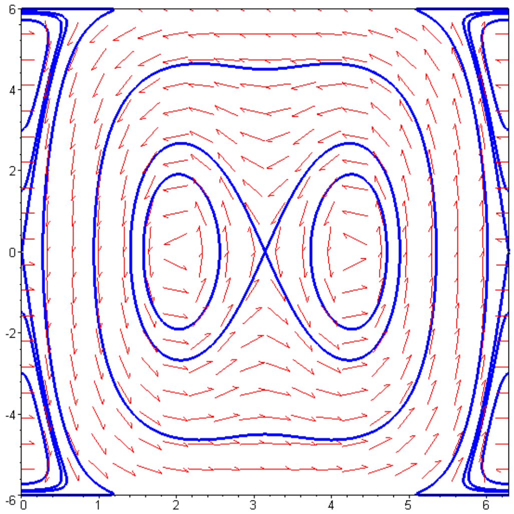


Fig.5

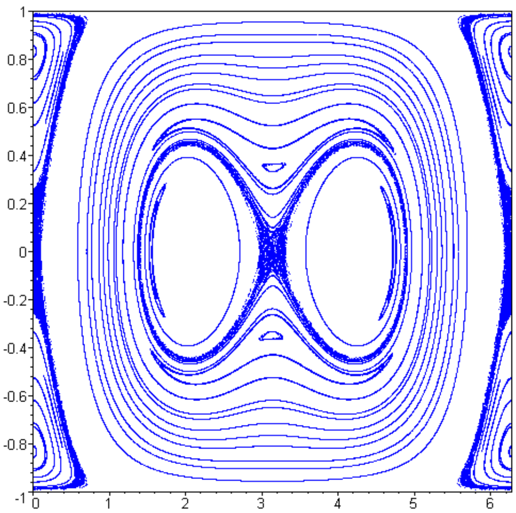


Fig.6

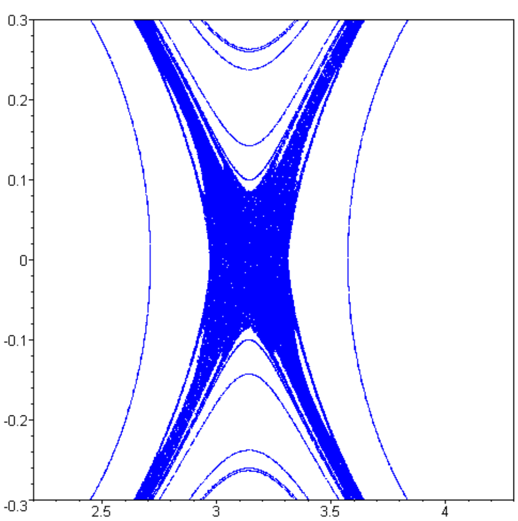


Fig.7

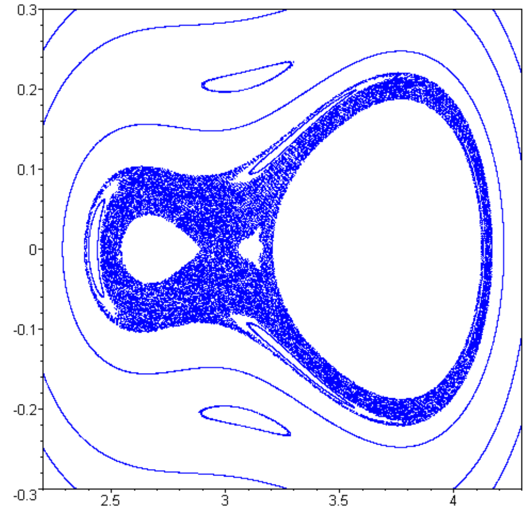


Fig.8

We can show the system phase portraits (fig.5-8) which demonstrate the chaos presence close to the homoclinic orbits in the phase space $\{l, L/G\}$. The phase portraits (fig.6-8) correspond to the Poincaré section $(\nu t \bmod 2\pi) = 0$.

As can we see, at presence of the small perturbation the "chaotic layer" formed in the region close to the heteroclinic separatrix-trajectory. This chaotic layer is generated as the result of multiple intersections of the stable and unstable split manifolds of the heteroclinic separatrix-trajectory. Inside the chaotic layer phase trajectories can performed complex "chaotic" evolutions including complicated alternations of rotating and oscillating regimes with variable characteristics – it is the main reason of the NS captures in complex tilting NS motion.

The numerical modeling results (fig.5-8) were obtained at the following parameters: $\hat{A} = 0.5$; $\hat{B} = 0.6$; $\hat{C} = 0.7$, ν=2, ψ=0, ε=0.06.

For the cases fig.5-7 we take the momentums' values $G$=6, $D_{12}$=0, $D_{34}$=0.5, $D_{56}$=0; and for case fig.8 – $G$=6, $D_{12}$=0.3, $D_{34}$=1, $D_{56}$=0.

Advanced study of the chaos phenomena in the NS dynamics is the important independent research task, which can be considered in further separate publications.

So, the considered models and results can be used in the real space-flight and vehicles-movement tasks, especially in the cases of small mechanical devices, such as small roll-walking robots and communication-nanosatellites with multi-rotor attitude control systems.

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