ISSN 0025-6544, Mechanics of Solids, 2023, Vol. 58, No. 8, pp. 2884–2893. © Allerton Press, Inc., 2023.
Russian Text © The Author(s), 2023, published in Prikladnaya Matematika i Mekhanika, 2023, Vol. 87, No. 5, pp. 729–741.

In loving memory of L.D. Akulenko

The Dynamics of Small Satellites with a Three-Axial Gravitational Damper

V. S. Aslanov[a],* and A. V. Doroshin[a]

[a] *Samara National Research University, Samara, 443086 Russia*

**e-mail: aslanov_vs@mail.ru*

Received June 1, 2023; revised August 11, 2023; accepted August 15, 2023

Abstract—The questions of the dynamics of the angular motion of nanosatellites with gravitational dampers are considered. The damper is a solid body rotating in a spherical cavity with a viscous liquid filling and creating internal friction with the dissipation of the kinetic energy of the angular motion. Unlike M.A. Lavrentiev's classical models of similar viscous dampers using with spherical dynamic symmetry of a body-damper, in this work the body-damper has a central triaxial ellipsoid of inertia, which increases the efficiency of interaction with an external gravitational field. This makes it possible to use almost any autonomous nanosatellite assembly as such an internal body-damper, placing it in a sealed spherical shell inside a spherical cavity with a viscous liquid in the center of mass of the main body—satellite body. The presence of a three-axis inertia tensor of the damper body changes and complicates the mathematical model of the angular motion in comparison with the classical one, which can be considered as a certain generalization and development of research in this area.

Keywords: nanosatellite, three-axial gravitational damper, M.A. Lavrentiev's model, angular motion, central gravity field

DOI: 10.3103/S0025654423080034

1. INTRODUCTION

The angular motion of spacecraft with viscous dampers has been and continues to be an important research topic within the framework of rigid body dynamics and its applied issues related to space flight mechanics. The presence of internal dampers in the form of spherical bodies inside spherical cavities with a viscous liquid located as part of a base solid body is studied in the framework of a variety of dynamic aspects, including issues of constructing mathematical models, searching for laws of motion, analyzing the stability of motion modes, and much more. Starting from fundamental works on the problem of the motion of composite solids with internal viscous dampers [1–3], the dynamics of such solid systems with liquid filling were studied in the works of Chernousko et al. [4–16] and, moreover, is currently acquiring new important aspects within the framework of applied issues in the development of satellite systems based on modern ultrasmall form factors, including so-called "nanosatellites."

Nanosatellites in their generally accepted design version have a modular structure, in which the entire spacecraft is composed of independent modules (units) filled with appropriate units and equipment. If we highlight the recognized nanosatellite layout schemes, it is worth emphasizing that they are constructed according to the so-called CubeSat layout, where individual modules are cubic structures with an edge size of 10 cm. Based on such cubic modules, as is well known, specific nanosatellites that receive digital indices reflecting the number of modules they contain, which are usually indicated in the form 3U, 6U, 12U, etc. The modern standard in most space missions is the design of three linearly connected modules forming the CubeSat-3U nanosatellite, which finds itself in educational and research missions of universities and scientific organizations. Due to the fundamental simplicity of the design, these nanosatellites cannot contain any complex units with high functional efficiency and high energy intensity, which is typical for full-featured spacecraft of a large form factor. In this regard, the task of developing such schemes and assemblies of nanosatellites that could be simultaneously simple in terms of design and layout, and also have a multiplicity of functionality of their internal units, which would allow not only to perform the target function of the unit (photography, reception and transmission radio signal, measurement of environmental parameters, etc.), but also be used as a working element of a motion control system.

An example of such a possibility would be the hypothetical CubeSat-3U nanosatellite considered in this article, inside the central module of which there is a spherical cavity with a viscous filling, containing a movable sphere with an internal inertially centered unit, which performs, for example, the role of a radiometer with an autonomous power supply and a radio transmitter, broadcasting measured environmental parameters on the air. In this case, the functional unit (autonomous radiometer—radio transmitter) would fulfill its target mission, at the same time being a viscous damper designed to impart preferable properties to the dynamics of the angular motion of the satellite (achieving and stabilizing a gravitational stabilized position). Thus, in the case in which a centered solid body of an aggregate inside a spherical shell would have an ideal central spherical principal tensor of inertia, the solid-body system of the nanosatellite would correspond to the classical mechanical model of Lavrentiev [1], which has been studied in many aspects of its dynamics in works [1–3, 10–15, 20–22]. The nature of the movement of such a model would be based on the interaction of the main base body, which has a general inertia tensor, with an external central gravitational field, tending to rotate the body into a gravity-oriented position in orbit, while the internal body would continue its movement without the influence of the central one on it. gravitational fields in connection with the spherical inertial-mass arrangement. In this case, relative motion would be created between the bodies, which in turn would cause the interaction of the bodies through liquid friction of the medium between the walls of the cavity and the internal body. The specified classical situation would occur in the case when a three-module nanosatellite would have such an inertial arrangement when its center of mass would be located in the center of mass of the central module, inside which a damper body would be located with exact alignment of its center of mass with the center of mass of the complete mechanical system and would have a spherical inertia tensor. However, as noted above, the internal body-damper in real tasks of a space mission would perform its functionality, being a kind of real unit, and, therefore, its inertia tensor would differ from a spherical one, which, unfortunately, would take the mechanical system out of the classical category, but would be necessary from the point of view of practical target tasks of a nanosatellite. Placing additional masses to give the inertia tensor spherical properties would worsen the mass characteristics of a nanosatellite, minimizing the mass of which is one of the priority goals when designing any space system. It is also worth noting that such a viscous damper scheme can be used in cases in which the interaction with the external environment is of a nature other than gravitational interaction. For example, one can indicate the use of this mechanical type of damper in cases where the preferential interaction occurs with the geomagnetic field, which transforms the type of "gravitational damper" into the type of "magnetic damper," when the damper body, or the main base body of the satellite, has its own dipole magnetic moment, interacting with an external magnetic field. This class of magnetic dampers, as well as magnetic systems for controlling and stabilizing the motion of spacecraft, have also been and remain one of the most relevant objects of research and development [17, 18, 23–26].

The purpose of this work is to form the appearance of a triaxial gravitational damper and study the angular motion in circular orbits of a three-module nanosatellite with a central module containing a damper body in a spherical cavity with a viscous filling having a central triaxial inertia tensor. This mechanical system cannot be described on the basis of the classical Lavrentiev model and requires the development of a new mathematical model that develops and, in a sense, generalizes the problems of using viscous dampers within the framework of motion in central gravitational fields. To achieve this goal, the work involves constructing a mathematical model of the motion of a system with a triaxial damper based on the theorem on the change in kinetic moment, as well as studying the possibility of its practical application based on performing a numerical analysis of the dynamics in comparison with the motion of the corresponding classical model.

2. MATHEMATICAL MODEL OF THE ANGULAR MOTION OF A NANOSATELLITE WITH A GRAVITATIONAL DAMPER IN CIRCULAR ORBITS

2.1. Mechanical Model and Operating Principle

Let us consider the CubeSat-3U nanosatellite (Fig. 1), the central module of which contains a spherical cavity with a viscous liquid, inside which a sphere of smaller radius floats freely with an internal rigid body-damper rigidly attached to it, having a triaxial inertia tensor with a center of mass, and with a center of mass coinciding with the center of the sphere. We will assume that the center of mass of the entire system in this case always coincides with the centers of mass of the base body (all three modules without an internal sphere with a damper) and the damper body. In this regard, the interaction of bodies for this rea-

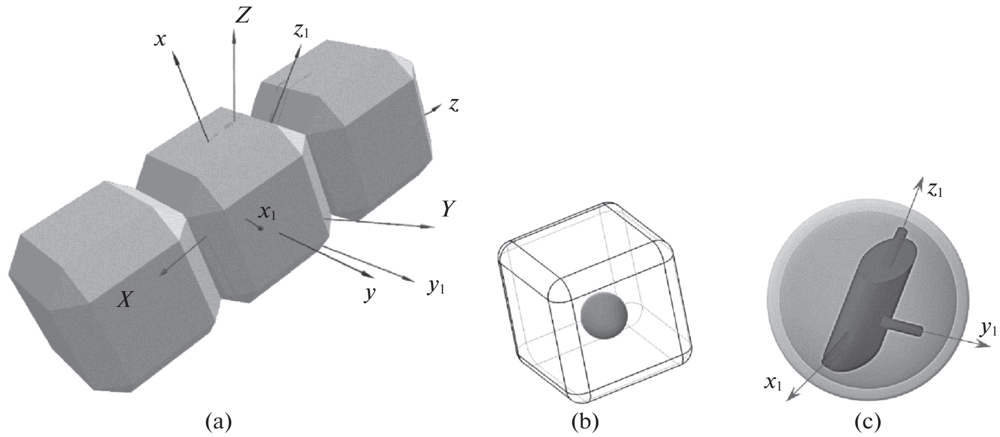

Fig. 1. (a) Diagram of the CubeSAT-3U nanosatellite with (b) a central module, (c) an internal gravitational damper.

son is transmitted only through liquid friction of the viscous filling in a spherical gap, which we will model in the form of a moment of force proportional to the relative speed of rotation of the bodies.

As already noted in the introduction, the dynamics of a satellite with a gravitational damper is based on the difference in the interaction of the main base body and the internal damper body with the central gravitational field, which tends to rotate the bodies into gravity-oriented positions in orbit. Due to differences in the inertial-mass parameters of the bodies, they will perform different angular motions around their own centers of mass, which, in turn, will create the relative angular motion of the bodies and the corresponding fluid friction in the gap of the spherical cavity.

Compared to the classical model, in which the damper body has a spherical inertia tensor, in the case under consideration, the damper body is triaxial, and, therefore, a certain functional unit that performs its independent mission can be used as such a damper body.

2.2. Coordinate Systems and Equations of Motion

We will use the following coordinate systems: *CXYZ* is an orbital coordinate system, the Z axis of which is directed from the center of the Earth to the center of mass of the system in a circular orbit, the Y axis of which orthogonal to the orbital plane and aligned with the angular orbital velocity, and the X axis of which represents the third axis of the right coordinate system. The system *Cxyz* is the central coordinate system associated with the base body of the nanosatellite and is co-directed with the main axes of inertia of the base body. The system $Cx_1y_1z_1$ is the central coordinate system of the damper body, codirectional with its axes of inertia.

The angular position of the base body relative to the orbital coordinate system will be described by Euler angles $\{\theta_1, \theta_2, \theta_3\}$ according to the following sequence of rotations: $x \to y \to z$. In this case, the following matrices of successive rotations and the final matrix of transition from the coordinate system *CXYZ* to the system *Cxyz* have the form

$$\Theta_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & \sin\theta_1 \\ 0 & -\sin\theta_1 & \cos\theta_1 \end{bmatrix}; \quad \Theta_2 = \begin{bmatrix} \cos\theta_2 & 0 & -\sin\theta_2 \\ 0 & 1 & 0 \\ \sin\theta_2 & 0 & \cos\theta_2 \end{bmatrix}; \quad \Theta_3 = \begin{bmatrix} \cos\theta_3 & \sin\theta_3 & 0 \\ -\sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{2.1}$$

$$\Theta = \Theta_3 \cdot \Theta_2 \cdot \Theta_1 = \begin{bmatrix} c\theta_3 c\theta_2 & s\theta_3 c\theta_1 + c\theta_3 s\theta_2 s\theta_1 & s\theta_3 s\theta_1 - c\theta_3 s\theta_2 c\theta_1 \\ -s\theta_3 c\theta_2 & c\theta_3 \cos\theta_1 - s\theta_3 s\theta_2 s\theta_1 & c\theta_3 s\theta_1 + s\theta_3 s\theta_2 c\theta_1 \\ s\theta_2 & -c\theta_2 s\theta_1 & c\theta_2 c\theta_1 \end{bmatrix}, \tag{2.2}$$

where the letters "c" and "s" denote the functions cos and sin, respectively.

The position of the damper body relative to the orbital coordinate system will be described by analogy by Euler angles $\{\psi_1, \psi_2, \psi_3\}$ and a completely similar transition matrix from the coordinate system *CXYZ* to a connected system $Cx_1y_1z_1$:

$$\Psi = \Psi_3 \cdot \Psi_2 \cdot \Psi_1 = \begin{bmatrix} c\psi_3 c\psi_2 & s\psi_3 c\psi_1 + c\psi_3 s\psi_2 s\psi_1 & s\psi_3 s\psi_1 - c\psi_3 s\psi_2 c\psi_1 \\ -s\psi_3 c\psi_2 & c\psi_3 c\psi_1 - s\psi_3 s\psi_2 s\psi_1 & c\psi_3 s\psi_1 + s\psi_3 s\psi_2 c\psi_1 \\ s\psi_2 & -c\psi_2 s\psi_1 & c\psi_2 c\psi_1 \end{bmatrix}. \tag{2.3}$$

Then we can write the following kinematic systems of equations taking into account the presence of orbital angular velocity ω_0, where the absolute angular velocity of the base body in its related axes is described by vector $w = [p, q, r]^T$, and the angular velocity of the damper body in its associated axes is described by vector $w' = [p', q', r']^T$:

$$\begin{gathered} p = \dot\theta_1 \cos\theta_2 \cos\theta_3 + \dot\theta_2 \sin\theta_3 + \omega_0 \Theta_{12}, \\ q = -\dot\theta_1 \cos\theta_2 \sin\theta_3 + \dot\theta_2 \cos\theta_3 + \omega_0 \Theta_{22}, \\ r = \dot\theta_1 \sin\theta_2 + \dot\theta_3 + \omega_0 \Theta_{32}, \end{gathered} \tag{2.4}$$

$$\begin{gathered} p' = \dot\psi_1 \cos\psi_2 \cos\psi_3 + \dot\psi_2 \sin\psi_3 + \omega_0 \Psi_{12}, \\ q' = -\dot\psi_1 \cos\psi_2 \sin\psi_3 + \dot\psi_2 \cos\psi_3 + \omega_0 \Psi_{22}, \\ r' = \dot\psi_1 \sin\psi_2 + \dot\psi_3 + \omega_0 \Psi_{32}. \end{gathered} \tag{2.5}$$

Let us consider motion with bodies having the following inertia tensors in their associated coordinate systems: $\mathbf{J} = \mathrm{diag}(A, B, C)$ is the central main inertia tensor of the base body (without a damper body) and $\mathbf{J}' = \mathrm{diag}(A', B', C')$ is the central main tensor of inertia of the damper body. In this case, the coincidence of the centers of mass of the system, the base body, and the damper body, the equations of angular motion of these individual bodies will have a standard structure in their associated coordinate systems, where the influence from the second body will be described by the moment of interaction, expressing the presence of fluid friction, formally acting as "external torque" from the second body of the system [19]:

$$\begin{gathered} A\dot p + (C - B) qr = 3\omega_0^2 (C - B) \Theta_{23}\Theta_{33} + M_x, \\ B\dot q + (A - C) pr = 3\omega_0^2 (A - C) \Theta_{33}\Theta_{13} + M_y, \\ C\dot r + (B - A) pq = 3\omega_0^2 (B - A) \Theta_{13}\Theta_{23} + M_z, \end{gathered} \tag{2.6}$$

$$\begin{gathered} A' \dot p' + (C' - B') q' r' = 3\omega_0^2 (C' - B') \Psi_{23}\Psi_{33} + M'_x, \\ B' \dot q' + (A' - C') p' r' = 3\omega_0^2 (A' - C') \Psi_{33}\Psi_{13} + M'_y, \\ C' \dot r' + (B' - A') p' q' = 3\omega_0^2 (B' - A') \Psi_{13}\Psi_{23} + M'_z. \end{gathered} \tag{2.7}$$

where $\{\Theta_{13}, \Theta_{23}, \Theta_{33}\}$, $\{\Psi_{13}, \Psi_{23}, \Psi_{33}\}$ are components of matrices (2.2) and (2.3), corresponding to the direction cosines of the Z axis. Vector $\mathbf{M} = [M_x, M_y, M_z]^T$ is considered formally as an external moment of force acting on the base body from the side of the damper body through friction. Vector $\mathbf{M}' = [M'_x, M'_y, M'_z]^T$ is a moment of force acting on the damper body from the side of the base body. We will consider these moments of forces in the form proportional to the relative angular velocity of the bodies relative to each other with a coefficient of proportionality ν, describing kinematic viscosity:

$$\mathbf{M} = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = -\nu \left[\begin{bmatrix} p \\ q \\ r \end{bmatrix} - \Theta \cdot \Psi^{-1} \cdot \begin{bmatrix} p' \\ q' \\ r' \end{bmatrix} \right], \quad \mathbf{M}' = \begin{bmatrix} M'_x \\ M'_y \\ M'_z \end{bmatrix} = -\nu \left[\begin{bmatrix} p' \\ q' \\ r' \end{bmatrix} - \Psi \cdot \Theta^{-1} \cdot \begin{bmatrix} p \\ q \\ r \end{bmatrix} \right]. \tag{2.8}$$

Thus, the systems of dynamic and kinematic equations (2.4)–(2.7) with the magnitudes of moments of forces (2.8) will completely describe the dynamics of the mechanical system in the parameters of the angular velocities of bodies and Euler angles $\{\theta_1, \theta_2, \theta_3\}$ and $\{\psi_1, \psi_2, \psi_3\}$. When resolved with respect to the first derivatives of the orientation angles, the kinematic equations with respect to the derivatives for the angles will have the form

Table 1. Parameters of a system with a triaxial damper inertia tensor

Parameters and initial conditions of motion of bodies			
	moments of inertia, kg m^2	initial angular velocities, 1/s	starting angles, rad
Base body	$\mathbf{J} = \mathrm{diag}(0.0045, 0.0055, 0.0035)$	$\boldsymbol{\omega}(0) = [0.002, 0.001, -0.002]$	$\{\theta_i\} = \{0.15, 0.1, 0.2\}$
Damper body	$\mathbf{J}' = \mathrm{diag}(0.003, 0.004, 0.0015)$	$\boldsymbol{\omega}'(0) = [0.002, 0.001, 0.005]$	$\{\psi_i\} = \{0.05, 0.02, 0.03\}$
Orbital speed $\boldsymbol{\omega}_0$, 1/s		0.0012	
Kinematic viscosity ν, N m s		0.00001	

$$\begin{aligned}
\dot{\theta}_1 &= -\frac{1}{\cos\theta_2}\left(q\sin\theta_3 - p\cos\theta_3 + \cos\theta_3\omega_0\Theta_{12} - \sin\theta_3\omega_0\Theta_{22}\right),\\
\dot{\theta}_2 &= q\cos\theta_3 + p\sin\theta_3 - \cos\theta_3\omega_0\Theta_{22} - \sin\theta_3\omega_0\Theta_{12},\\
\dot{\theta}_3 &= r + \tan\theta_2\left(q\sin\theta_3 - p\cos\theta_3 + \cos\theta_3\omega_0\Theta_{12} - \sin\theta_3\omega_0\Theta_{22}\right) - \omega_0\Theta_{32},
\end{aligned} \tag{2.9}$$

$$\begin{aligned}
\dot{\psi}_1 &= -\frac{1}{\cos\psi_2}\left(q'\sin\psi_3 - p'\cos\psi_3 + \cos\psi_3\omega_0\Psi_{12} - \sin\psi_3\omega_0\Psi_{22}\right),\\
\dot{\psi}_2 &= q'\cos\psi_3 + p'\sin\psi_3 - \cos\psi_3\omega_0\Psi_{22} - \sin\psi_3\omega_0\Psi_{12},\\
\dot{\psi}_3 &= r' + \tan\psi_2\left(q'\sin\psi_3 - p'\cos\psi_3 + \cos\psi_3\omega_0\Psi_{12} - \sin\psi_3\omega_0\Psi_{22}\right) - \omega_0\Psi_{32}.
\end{aligned} \tag{2.10}$$

The constructed equations (2.6)–(2.10) represent a closed mathematical model of the dynamics of the motion of a nanosatellite with a triaxial gravitational damper.

3. SIMULATION OF MOTION DYNAMICS

To study the effectiveness of the proposed model in comparison with the classical one, as well as to evaluate its practical application, we will conduct a numerical simulation of the dynamics of motion based on the numerical integration of Eqs. (2.6)–(2.10) with hypothetical system parameters and initial conditions given in the table (Table 1).

The simulation results are presented in Figs. 2a–6a, from which it is clear that the internal damper effectively dampens the angular velocities of the nanosatellite. There is clearly a tendency in which the components of the angular velocity vector corresponding to the average and smallest moments of inertia (p, p', r, r') tend to zero (Figs. 2a, 4a), and components q and q', corresponding to the largest moments of inertia, gradually take on absolute values equal to the orbital angular velocity (Fig. 3a).

As can be seen from the modeling results, bodies tend to occupy gravitational equilibrium positions coinciding with their associated axes with the axes of the orbital coordinate system, which corresponds to the principle of gravitational stabilization.

4. COMPARISON OF THE EFFICIENCY OF DAMPERS WITH TRIAXIAL AND SPHERICAL INERTIA TENSORS

To compare the efficiency of a classical damper with a spherical inertia tensor and a damper with a triaxial inertia tensor, one can carry out numerical modeling based on the same Eqs. (2.6)–(2.10) with the same initial conditions of motion and system parameters, except for the values of the moments of inertia of the damper body, which are assumed to be equal to the values corresponding to the average moments of inertia from the previous simulation in the case of a triaxial tensor (Table 2). In this case, we will obtain the results shown in the figures (Figs. 2b–6b).

As can be seen from Figs. 2–6, in the given time interval, gravitational stabilization by a satellite with a spherical inertia tensor of the damper body is not achieved, and for this a significantly longer time interval is required. To comparatively demonstrate the achievement of the gravitational stabilization position of nanosatellites with triaxial and spherical inertia tensors of the damper body, the equations were integrated over longer time intervals, where the achievement of a gravitational stabilized position is noticeable in both cases (Figs. 7, 8).

As can be seen from the modeling results, the position of gravitational stabilization in the case of a triaxial inertia tensor of the damper body is achieved approximately twice as fast: the stabilization time with

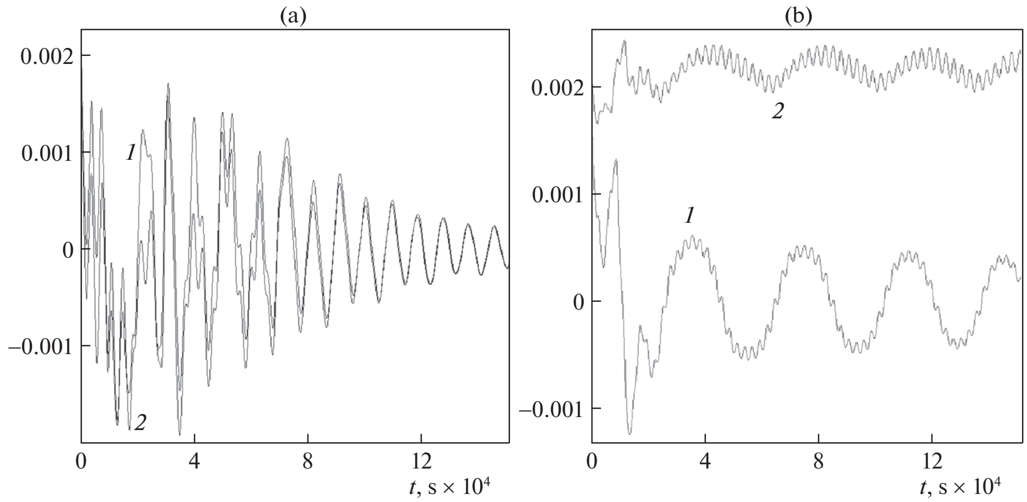

Fig. 2. Dependences of angular velocities (*1*) *p* and (*2*) *p*'.

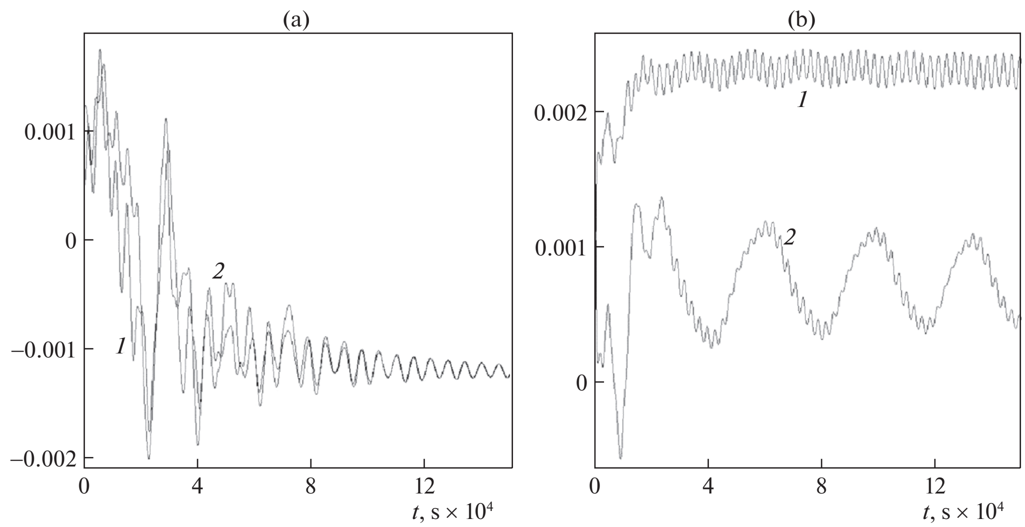

Fig. 3. Dependences of angular velocities (*1*) *q* and (*2*) *q*'.

a triaxial damper body is approximately equal to 2.5×10^5, and with a spherical body-damper it is 5×10^5. The latest comparative simulation shows greater efficiency of the damper with a triaxial inertia tensor. In other words, the model with a triaxial damper body is not only preferable from the point of view of the

Table 2. System parameters with a classic damper

Parameters and initial conditions of motion of bodies			
	moments of inertia, kg m^2	initial angular velocities, 1/s	starting angles, rad
Base body	$\mathbf{J}$ = diag(0.0045, 0.0055, 0.0035)	$\boldsymbol{\omega}(0)$ = [0.002, 0.001, –0.002]	$\{\theta_i\}$ = {0.15, 0.1, 0.2}
Damper body	$\mathbf{J}'$ = diag(0.003, 0.003, 0.003)	$\boldsymbol{\omega}'(0)$ = [0.002, 0.001, 0.005]	$\{\psi_i\}$={0.05, 0.02, 0.03}
Orbital speed ω_0, 1/s		0.0012	
Kinematic viscosity ν, N m s		0.00001	

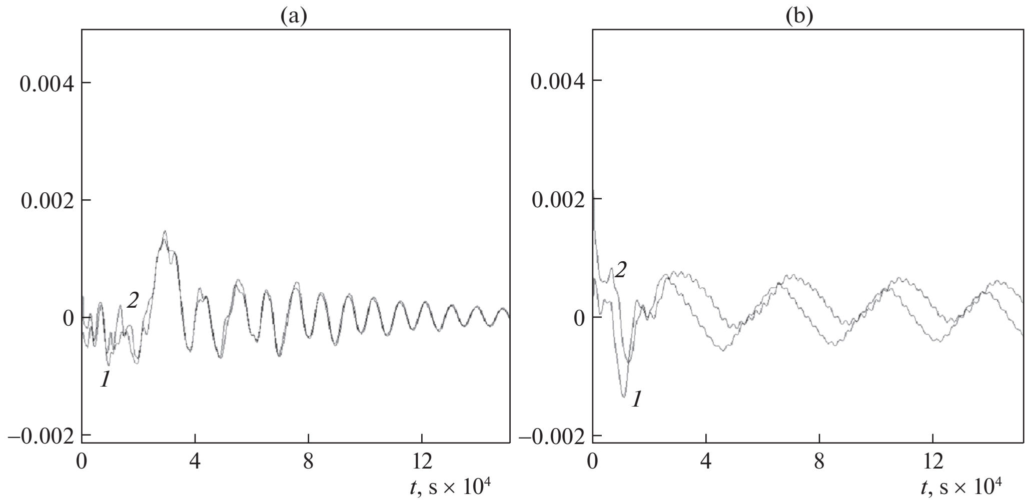

Fig. 4. Dependences of angular velocities (*1*) r and (*2*) r'.

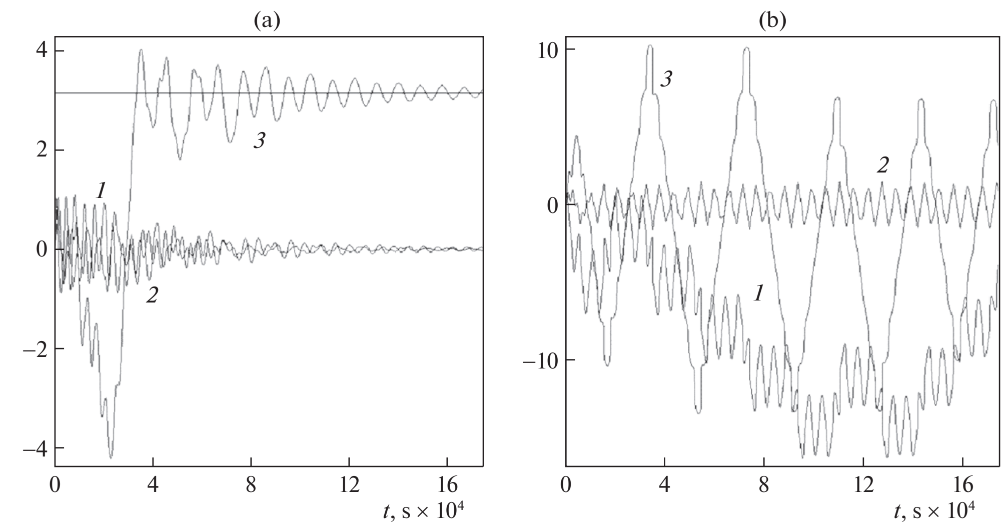

Fig. 5. Time dependence of spatial angles (*1*) θ_1, (*2*) θ_2, and (*3*) θ_3.

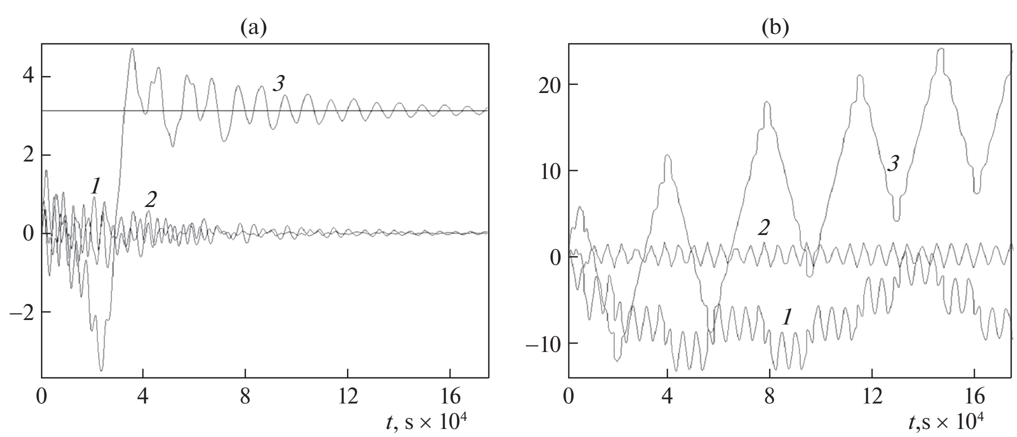

Fig. 6. Time dependence of angles (*1*) ψ_1, (*2*) ψ_2, and (*3*) ψ_3.

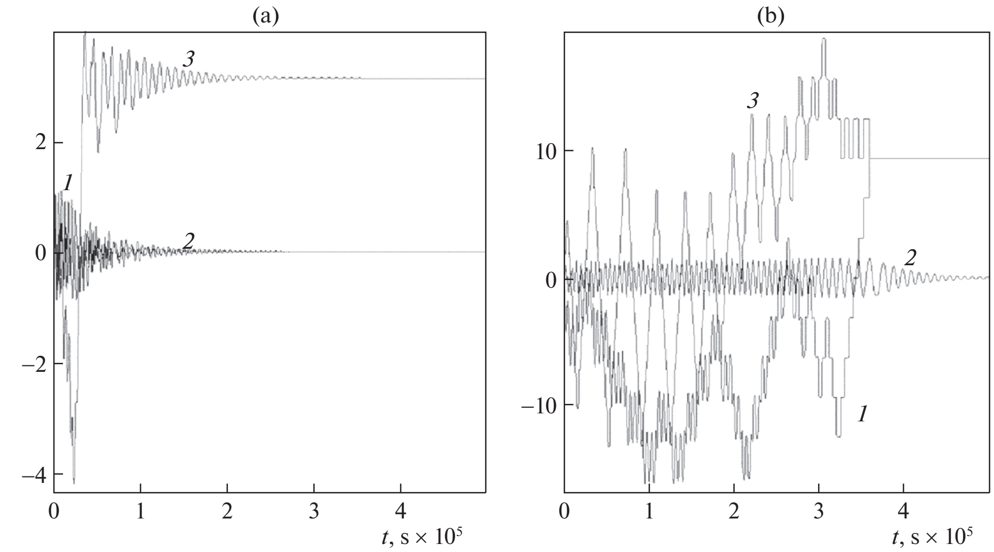

Fig. 7. Time dependence of spatial angles (*1*) θ_1, (*2*) θ_2, and (*3*) θ_3: (a) triaxial inertia tensor of the damper; (b) spherical inertia tensor of the damper.

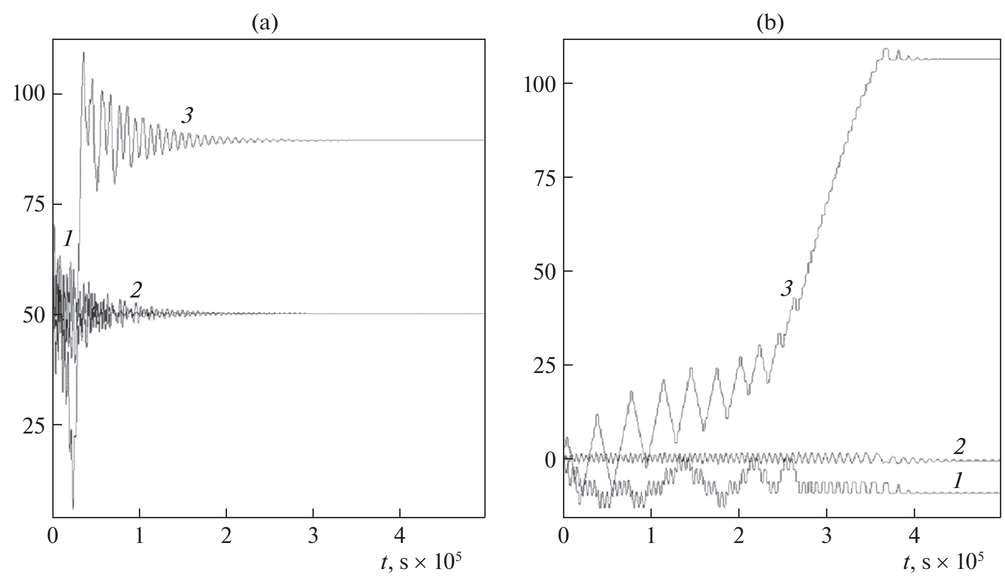

Fig. 8. Time dependence of angles (*1*) ψ_1, (*2*) ψ_2, and (*3*) ψ_3: (a) triaxial inertia tensor of the damper; (b) spherical inertia tensor of the damper.

layout of a multifunctional nanosatellite, but is also a more effective damping device compared to the classic model of M.A. Lavrentiev.

5. CONCLUSIONS

The paper proposes mechanical and mathematical models of a gravitational triaxial damper, developing the classical model of M.A. Lavrentiev and creating the prerequisites for its applied use within the framework of solving modern problems of space flight mechanics of small spacecraft.

The constructed models make it possible to "reset" the angular velocity of the satellite due to dissipative moments of forces arising during the interaction of the base body and the triaxial damper body through the moments of fluid friction forces. The final position occupied by the base body and the damper body corresponds to the position of gravitational stabilization when the associated axes of the bodies coincide with the orbital axes in accordance with the principle of gravitational stabilization.

As modeling has shown, the model with a triaxial damper body is not only preferable from the point of view of the design of a nanosatellite with multifunctional units, the inertial geometry of which is spherically symmetrical, but is also a more efficient system in comparison with the classical damper of M.A. Lavrentiev.

FUNDING

The work was supported by the Russian Science Foundation, project no. 19-19-00085 A.

CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest.

REFERENCES

1. A. Yu. Ishlinskii, “Activities of Mikhail Alekseevich Lavrentiev in the Academy of Sciences of the Ukrainian SSR,” Prikl. Mekh. Tekh. Fiz., No. 3, 16–19 (1960).
2. F. L. Chernous’ko, *Motion of a Rigid Body with Cavities Containing a Viscous Fluid* (Computing Centre of the USSR Acad. Sci., Moscow, 1968) [in Russian].
3. F. L. Chernous’ko, “On the motion of a rigid body containing a spherical damper,” Prikl. Mekh. Tekh. Fiz., No. 1, 73–79 (1968).
4. F. L. Chernous’ko, L. D. Akulenko, and D. D. Leshchenko, *Evolution of Motions of a Rigid Body Relatively to the Center of Mass* (Institute of Computer Science, Izhevsk, 2015) [in Russian].
5. L. D. Akulenko, D. D. Leshchenko, and F. L. Chernous’ko, “Rapid motion around a fixed point of a heavy rigid body in resisting medium,” Izv. Akad. Nauk SSSR, Mekh. Tverd. Tela, No. 3, 5–13 (1982).
6. L. D. Akulenko, *Asymptotic Methods of Optimal Control* (Nauka, Moscow, 1987) [in Russian].
7. L. D. Akulenko, D. D. Leshchenko, A. L. Rachinskaya, and Yu. S. Shchetinina, “Evolution of perturbed rotations of an asymmetric gyro in a gravitational field and a resisting medium,” Mech. Solids **51** (4), 406–414 (2016).
8. L. D. Akulenko, D. D. Leshchenko, and A. L. Rachinskaya, “Evolution of the rotations of a satellite with a cavity filled with a viscous fluid,” Mekh. Tverd. Tela, No. 37, 126–139 (2007).
9. L. D. Akulenko, Ya. S. Zinkevich, D. D. Leshchenko, and A. L. Rachinskaya, “Rapid rotation of a satellite with a cavity filled with a viscous fluid under the action of moments of gravitational forces and light pressure,” Kosm. Issled. **49** (5), 453–463 (2011).
10. N. I. Amel’kin and V. V. Kholoshchak, “Stability of the steady rotations of a satellite with internal damping in a central gravitational field,” J. Appl. Math. Mech. **81** (2), 85–94 (2017).
11. V. V. Kholoshchak, “Dynamics of rotational motion of a satellite with a damper in a central gravitational field,” Tr. Mosk. Fiz-Tekh. Inst. **9** (4(36)), 106–119 (2017).
12. N. I. Amel’kin and V. V. Kholoshchak, “Rotational motion of a non-symmetrical satellite with a damper in a circular orbit,” Mech. Solids **54** (2), 190–203 (2019).
13. N. I. Amel’kin and V. V. Kholoshchak, “Evolution of the rotational movement of a dynamically symmetric satellite with inner damping in a circular orbit,” Prikl. Mat. Mekh. **54** (2), 204–223 (2019). https://doi.org/10.3103/S0025654419030014
14. N. I. Amel’kin, “The asymptotic properties of the motions of satellites in a central field due to internal dissipation,” J. Appl. Math. Mech. **75** (2), 140–153 (2011).
15. N. I. Amel’kin and V. V. Kholoshchak, “Stability of the steady rotations of a satellite with internal damping in a central gravitational field,” J. Appl. Math. Mech. **81** (2), 85–94 (2017).
16. V. V. Sidorenko, “Evolution of the rotational motion of a planet with a liquid core,” Astron. Vestn. **27** (2), 119–127 (1993).
17. *Applied Celestial Mechanics and Motion Control. Collection of Articles Dedicated to the 90th anniversary of D. E. Okhotsimskii,* Ed. by T. M. Eneev, M. Yu. Ovchinnikov, and A. R. Golikov (Keldysh Institute of Applied Mathematics, Moscow, 2010) [in Russian].
18. V. M. Morozov and V. I. Kalenova, “Satellite control using magnetic moments: controllability and stabilization algorithms,” Kosm. Issled. **58** (3), 199–207 (2020).
19. A. V. Doroshin, “Gravitational dampers for unloading angular momentum of nanosatellites,” in *Advances in Nonlinear Dynamics. NODYCON Conf. Proc. Series,* Ed. by W. Lacarbonara, B. Balachandran, et al. (Springer, 2022), Vol. 1, pp. 257–266.

20. N. I. Amel'kin and V. V. Kholoshchak, "Rotational motion of a non-symmetrical satellite with a damper in a circular orbit," Mech. Solids **54**, 190–203 (2019).
21. N. I. Amel'kin, "Evolution of rotational motion of a planet in a circular orbit under the influence of internal elastic and dissipative forces," Mech. Solids **55**, 234–247 (2020).
22. P. K. Winfree and J. E. Cochran, Jr., "Nonlinear attitude motion of a dual-spin spacecraft containing spherical dampers," J. Guid., Control, Dyn. **9** (6), 681–690 (1986).
23. L. K. Davis, U.S. Patent No. 3,399,317 (U.S. Patent and Trademark Office, Washington, 1968).
24. D. S. Ivanov, M. Y. Ovchinnikov, V. I. Penkov, and T. A. Ivanova, "Modeling a nanosatellite's angular motion damping using a hysteresis plate," Math. Models Comput. Simul. **12**, 816–823 (2020).
25. M. Y. Ovchinnikov and D. S. Roldugin, "A survey on active magnetic attitude control algorithms for small satellites," Progr. Aerospace Sci. **109**, 100546 (2019).
26. D. S. Roldugin and M. Y. Ovchinnikov, "Wobble of a spin stabilized satellite with cross products of inertia and magnetic attitude control," Adv. Space Res. **71** (1), 408–419 (2023).

Publisher's Note. Allerton Press remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.