Attitude dynamics of a composite nanosatellite with a gravitational damper and with a movable unit on a rail system

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***Abstract* — The paper deals with the dynamics of rotational motion of a composite nanosatellite with a hybrid rotational motion stabilization system installed in it.. As an element of the passive stabilization system a gravity damper installed in the main body is used, an element of the active stabilization system is a movable module on a rail system. The movable module can move linearly relative to the carrier body by means of a rail system. Due to this linear displacement and dissipation of the gravitational damper, the position of the satellite moving in a circular orbit becomes gravitationally stable. In the article a mathematical model is constructed and numerical simulation of the rotational motion is carried out.**

Keywords— nanosatellite, rail system, movable module, gravitational damper.

1. Introduction

At the present time, most modern nanosatellites are equipped with an orientation/stabilization system, which can be either active or passive. The development of passive orientation/stabilization systems is of great interest. They do not consume fuel during active control, but utilize natural properties of external force fields (gravitational, magnetic, aerodynamic) and gyroscopic properties.

The passive system of nanosatellite rotational motion stabilization based on the action of the central gravitational field when the nanosatellite moves in a circular orbit is considered. To stabilize the position of the nanosatellite relative to the orbital coordinate system, it is necessary to reduce its angular velocity to the values of the angular velocity of the center of mass during its orbital motion. To realize this deceleration, the technique of dissipation of kinetic energy and external gravity can be applied.

Let us consider the angular motion of a nanosatellite with one movable unit and with the "gravity damper" described in [1-3]. The gravity damper is installed in the main/carrier body. It is an internal rigid body in a spherical shell, which in turn is inside a spherical cavity of the main body, and the gap between the spherical shell and the spherical cavity is filled with a viscous fluid. The relative rotation of these spheres with the friction of the fluid generates a dissipative moment that slows down the angular motion.

The tensor, of the gravitation damper considered in this paper is three-axis, in contrast to the classical damper scheme [4-6]. This form of the damper body is more effective in comparison with classical one [3].

In addition, to dissipate the kinetic energy can be applied other form and types of dampers [7-10].

The working element of the active rotational motion stabilization system is the moving module and the rail system on which it is mounted (fig.1). With the help of the rail system, the mobile module can move along the axis parallel to the rail system.

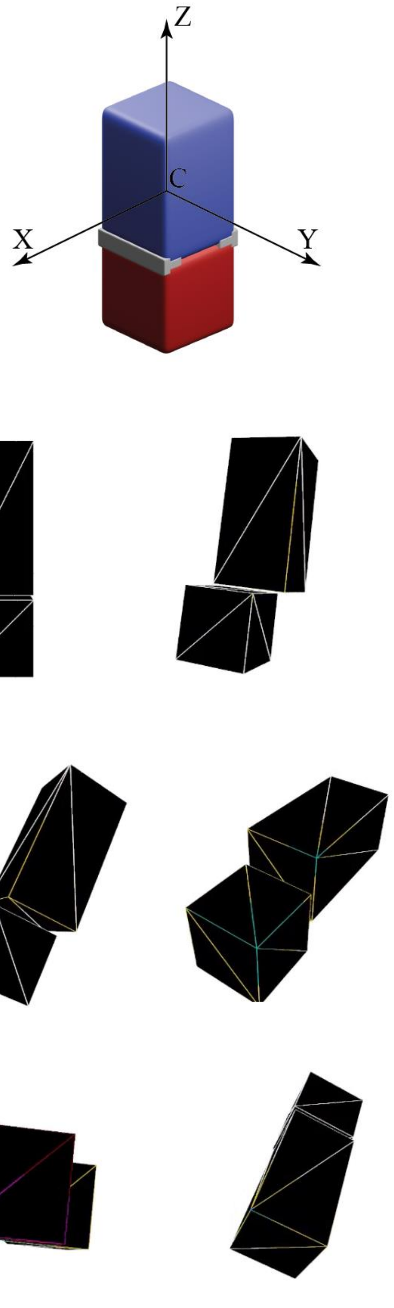
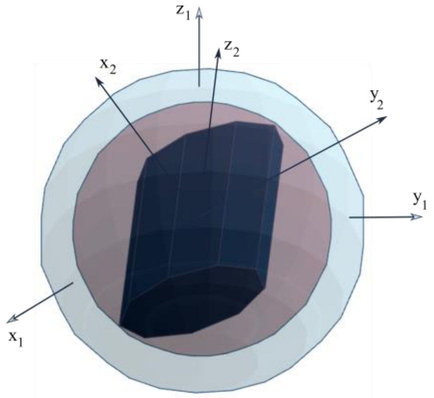


Fig. 1. The composite nanosatellite and its possible angular motion at the linear movability of one unit on the rail system.

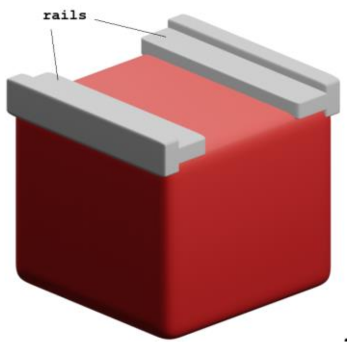
When moving the center of mass of the mobile module will change the geometry of the entire nanosatellite, which will qualitatively affect its dynamics. Knowing this it is possible to determine the type of law controlling the motion of the mobile module, to control the attitude dynamics of the entire nanosatellite. However, since the control torque is produced by the internal forces, we will not be able to change the angular momentum, but only be able to change the direction of the angular momentum vector relatively the satellite. By using active and passive rotational motion stabilization systems, faster stabilization of the nanosatellite rotational motion can be achieved. So, the mobile module acts on the carrier body, the carrier body in response acts even more strongly on the gravity damper, which interact with the external gravitational field. This common action of internal and external torques accelerate the process of the angular momentum unloading.

2. MECANICAL MODEL

Consider the mechanical model of the composite nanosatellite considered in this paper, shown in Fig. 1,2.



(a)



(b)

Fig. 2. Schemes of the gravitational damper (a) and the rail system (b)

Here are the coordinate frames used to calculate the total angular momentum of the composite nanosatellite:

- CXYZ – orbital coordinate frame, its origin is the center of mass of the composite nanosatellite (the axis CZ is directed from the gravity center to the orbital position of the center of mass of the nanosatellite, the axis CY is orthogonal to the orbital plane and CX represents the third right axis);
- $C_1x_1y_1z_1$ are the main coordinate frame of the main body with origin C_1 located in the center of mass of the main body;
- $C_2x_2y_2z_2$ are the main coordinate frame of the gravitational damper with origin C_2 located in the center of mass of the gravitational damper;
- $C_3x_3y_3z_3$ are the main coordinate frame of the movable unit with origin, C_3 located in the center of mass of the movable unit on the rail system.

3. MATHEMATICAL MODEL

Let us use the angular momentum theorem to obtain the differential equations that describe the dynamics of the composite nanosatellite and all its parts. For this purpose, let us calculate the total angular momentum of the co-satellite containing the main body, the movable unit, and the mass point C_2 located at the center of mass of the gravity damper (the effect of the damper rotation is transmitted only through liquid friction):

$$\begin{gathered}\mathbf{K}=\mathbf{K}_1+\mathbf{K}_{C_2}+\mathbf{K}_3;\\ \mathbf{K}_1=\mathbf{I}_1\boldsymbol{\omega}_1+m_1\mathbf{V}_1\times\mathbf{R}_1;\qquad \mathbf{K}_{C_2}=m_2\mathbf{V}_2\times\mathbf{R}_2;\\ \mathbf{K}_3=\mathbf{I}_3\boldsymbol{\omega}_3+m_3\mathbf{V}_3\times\mathbf{R}_3\end{gathered}\tag{1}$$

where $\mathbf{I}_i$ – is the tensor of inertia of the body with number i, $\boldsymbol{\omega}_i$ – is angular velocity of the corresponded body, $\mathbf{V}_i$ - is the linear velocity of the mass center of the body i, $\mathbf{R}_i$ - is the vector of the center of mass of the i-body relatively the center of mass of the complete nanosatellite C, m_i - is the mass of the i-body. Linear velocities of centers of mass of bodies are:

$$\begin{gathered}\mathbf{V}_1=\boldsymbol{\omega}_1\times\mathbf{R}_1;\ \ \mathbf{V}_2=\boldsymbol{\omega}_1\times\mathbf{R}_2;\\ \mathbf{V}_3=\boldsymbol{\omega}_1\times\mathbf{R}_3+\mathbf{V}_y\end{gathered}\tag{2}$$

where $\mathbf{V}_y=\left[0,\dot{y},0\right]^T_{C_1x_1y_1z_1}$ is the relative velocity of the linear displacement of the movable module along the CY axis.

$$\begin{gathered}\mathbf{R}_1=\left[0,\quad \frac{m_3y}{m},\quad \frac{m_3z_3+m_1z_1-m_2z_2}{m}+z_1\right]^T;\\ \mathbf{R}_2=\left[0,\quad -\frac{m_3y}{m},\quad -\frac{m_1z_1-m_3z_3-m_1z_2-m_3z_2}{m}\right]^T;\\ \mathbf{R}_3=\left[0,\quad \frac{(m_1+m_2)y}{m},\quad -\frac{m_1z_1+m_2z_2-m_1z_3-m_2z_3}{m}\right]^T,\end{gathered}\tag{3}$$

where m_i are masses of parts of composite nanosatellite, m is the total mass of the nanosatellite, z_i are distances between the centers of masses of bodies and bottom plane of the main body case, the value y is the linear displacement of the movable unit along the axis C_3y_3

The main central inertia tensors of bodies are:

$$\mathbf{I}_i=diag[A_i,B_i,C_i],\tag{4}$$

where A_i, B_i, C_i are principal central moments of inertia of the corresponding bodies. Next, we calculate the absolute angular velocities of the parts of the composite nanosatellite in projections on their own connected coordinate frames $C_ix_iy_iz_i$:

$$\boldsymbol{\omega}_i=\left[p_i,q_i,r_i\right]^T.\tag{5}$$

The torques of friction forces from the viscous fluid can be written as follows:

$$\mathbf{M}_b = -\nu\left(\boldsymbol{\omega}_1 - \boldsymbol{\Theta}\cdot\mathbf{B}^{-1}\cdot\boldsymbol{\omega}_3\right);$$
$$\mathbf{M}_d = -\nu\left(\boldsymbol{\omega}_3 - \mathbf{B}\cdot\boldsymbol{\Theta}^{-1}\cdot\boldsymbol{\omega}_1\right); \quad (6)$$

where ν - is the coefficient of the viscose friction, $\boldsymbol{\Theta}$ - is the transition matrix from the orbital coordinate frame CXYZ to the $C_1x_1y_1z_1$ coordinate system, $\mathbf{B}$ - is the transition matrix from the orbital coordinate frame to the $C_3x_3y_3z_3$ coordinate system. Here $\mathbf{M}_b$ - is the torque acting on the main/carrier body from the side of the gravitational damper, $\mathbf{M}_d$ - is the torque acting on the gravitational damper from the side of the carrier body.

To describe the position of bodies (the main body and the gravitational damper) instead of classical Euler angles we will pass to the use of "direction cosines" $\{\mathbf{e}_\tau, \mathbf{e}_n, \mathbf{e}_r\}$, which are unit vectors of axes of the orbital coordinates system CXYZ (fig.3) in corresponded connected coordinates systems of bodies ($C_1x_1y_1z_1$ and $C_3x_3y_3z_3$). Let us write the components of unit vectors in the connected coordinate frames $C_1x_1y_1z_1$ and $C_2x_2y_2z_2$:

$$\mathbf{e}_{\tau i} = \left[\alpha_{i1}, \alpha_{i2}, \alpha_{i3}\right]^T;$$
$$\mathbf{e}_{ni} = \left[\beta_{i1}, \beta_{i2}, \beta_{i3}\right]^T; \quad (7)$$
$$\mathbf{e}_{ri} = \left[\gamma_{i1}, \gamma_{i2}, \gamma_{i3}\right]^T;$$

where the index i=1, 2 indicates the main body and the damper body. The matrixes $\boldsymbol{\Theta}$ and $\mathbf{B}$ in terms of directional cosines will have the following structure:

$$\boldsymbol{\Theta} = \begin{bmatrix} \alpha_{11} & \beta_{11} & \gamma_{11} \\ \alpha_{12} & \beta_{12} & \gamma_{12} \\ \alpha_{13} & \beta_{13} & \gamma_{13} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \alpha_{31} & \beta_{31} & \gamma_{31} \\ \alpha_{32} & \beta_{32} & \gamma_{32} \\ \alpha_{33} & \beta_{33} & \gamma_{33} \end{bmatrix}. \quad (8)$$

The moments of the gravitational force acting on all bodies of the composite nanosatellite are written as follows:

$$\mathbf{M}_{gd} = 3\omega_0^2\left(\mathbf{e}_{r2}\times\mathbf{I}_2\mathbf{e}_{r2}\right),$$
$$\mathbf{M}_{gb} = 3\omega_0^2\left(\mathbf{e}_{r1}\times\mathbf{I}_1\mathbf{e}_{r1} + \mathbf{e}_{r1}\times\mathbf{I}_2\mathbf{e}_{r1} + \mathbf{e}_{r1}\times\mathbf{J}\mathbf{e}_{r1}\right), \quad (9)$$

where $\mathbf{J}$ – is the inertia tensor of a “gravity dumbbell” rigid body composed by the mass points C_1, C_2, C_3:

$$\mathbf{J} = \begin{bmatrix} J_{xx} & 0 & 0 \\ 0 & J_{yy} & J_{yz} \\ 0 & J_{yz} & J_{zz} \end{bmatrix} \quad (10)$$

$$J_{xx} = m_1(R_{1y}^2 + R_{1z}^2) + m_2(R_{2y}^2 + R_{2z}^2) + m_3(R_{3y}^2 + R_{3z}^2);$$
$$J_{yy} = m_1(R_{1x}^2 + R_{1z}^2) + m_2(R_{2x}^2 + R_{2z}^2) + m_3(R_{3x}^2 + R_{3z}^2);$$
$$J_{zz} = m_1(R_{1y}^2 + R_{1x}^2) + m_2(R_{2y}^2 + R_{2x}^2) + m_3(R_{3y}^2 + R_{3x}^2);$$
$$J_{yz} = -m_1(R_{1y}R_{1z}) - m_2(R_{2y}R_{2z}) - m_3(R_{3y}R_{3z}).$$

As can be seen from the expression (10) inertia tensor $\mathbf{J}$ is not diagonal. Moreover, its components depends on the linear displacement of the mobile module relative to the carrier body. This inertial configuration also affects the dynamics of rotational motion of the nanosatellite.

The vector $\boldsymbol{\omega} = \left[0, \omega_0, 0\right]^T$ represents the angular velocity of the orbital coordinate system (the angular velocity of the orbital motion); in the frame CXYZ it has the following components.

Then the equation of subsystem angular motion can be written as follows:

$$\frac{d\mathbf{K}}{dt} + \boldsymbol{\omega}_1\times\mathbf{K} = \mathbf{M}_b + \mathbf{M}_{gb} \quad (11)$$

A similar equation is used to describe the rotational dynamics of the graphite damper.:

$$\frac{d\mathbf{K}_2}{dt} + \boldsymbol{\omega}_2\times\mathbf{K}_2 = \mathbf{M}_d + \mathbf{M}_{gd}, \quad (12)$$

where $\mathbf{K}_2 = \mathbf{I}_2\boldsymbol{\omega}_2$ - is the angular momentum of the damper body about its center of mass C_2. The equations (11) and (12) form the system of dynamical equations.

As can we see, the dynamical equations depend on the linear displaicment $y(t)$. Therefore, we must add the appropriate control law for the movable unit relative motion. To determine the type of control law of the mobile module motion, let's define the main conroling parameters. These parameters will be the angular velocity component p and the linear displacement of the module y as a feedback parameter. The angular velocity component p is chosen because it can initiate the rotational motion of the satellite, ans, also movable unit on rail will initiate the rotation with angular velosity p. This type of the control law can provide exponential dampeding of the unit motion. The control law in this case will have the following form:

$$\dot{y} = k_p p_3 + k_y(y - y_0), \quad (13)$$

where k_p and k_y are control gain factors, y_0 is a constant defined the final position of the movable module.

To determine the spatial position and closure of the system of differential equations, we add kinematic expressions, taking into account orbital rotation. Let us write differential equations for unit vectors of the orbital frame [e.g., 11]:

$$\begin{cases} \dfrac{d\mathbf{e}_{ri}}{dt} = \mathbf{e}_{ri}\times\boldsymbol{\omega}_i + \omega_0\mathbf{e}_{\tau i}; \\ \dfrac{d\mathbf{e}_{ni}}{dt} = \mathbf{e}_{ni}\times\boldsymbol{\omega}_i; \\ \dfrac{d\mathbf{e}_{\tau i}}{dt} = \mathbf{e}_{\tau i}\times\boldsymbol{\omega}_i - \omega_0\mathbf{e}_{ri}; \end{cases} \quad (14)$$

where i=[1, 3].

So, the systems (11), (12), (13) and (14) fully describe the dynamics of the composite nanosatellite and all its parts.

To prove the stability of the considered controlled motion, we can use the Lyapunov function approach. For this purpose, let us introduce relative (relative to the orbital coordinate system) angular parts of the nanosatellite(i=1, 3):

$$\tilde{\boldsymbol{\omega}}_i = \boldsymbol{\omega}_i - \omega_0\mathbf{e}_{ni}. \quad (15)$$

To build the Lyapunov functions we will use the Beletsky integral [2]:

$$L = V - V(0);$$

$$V = \frac{1}{2}\left[\tilde{\boldsymbol{\omega}}_1 \mathbf{I}_s \tilde{\boldsymbol{\omega}}_1 + 3\omega_0^2 \mathbf{e}_{r1} \mathbf{I}_s \mathbf{e}_{r1} - \omega_0^2 \mathbf{e}_{n1} \mathbf{I}_s \mathbf{e}_{n1}\right] + \frac{1}{2}\left[\tilde{\boldsymbol{\omega}}_3 \mathbf{I}_3 \tilde{\boldsymbol{\omega}}_3 + 3\omega_0^2 \mathbf{e}_{r3} \mathbf{I}_3 \mathbf{e}_{r3} - \omega_0^2 \mathbf{e}_{n3} \mathbf{I}_3 \mathbf{e}_{n3}\right] \quad (16)$$

Below we will carry out numerical simulation of dynamics and present numerical dependencies for all motion parameters.

TABLE 1. INITIAL CONDITIONS

Parameter	**Unit**	**Value**
p_1	rad/s	*0.0012*
q_1	rad/s	*0.001*
r_1	rad/s	*-0.0025*
p_2	rad/s	*0.0022*
q_2	rad/s	*0.001*
$\alpha_{i2} = \alpha_{i3}$	rad	*0*
$\beta_{i1} = \beta_{i3}$	rad	*0*
$\gamma_{i1} = \gamma_{i2}$	rad	*0*
$\alpha_{i1} = \beta_{i2} = \gamma_{i3}$	rad	*1*

TABLE 2. INERTIAL-MASS PARAMETERS

Parameter	**Unit**	**Value**
A_1	$kg \cdot m^2$	*0.0045*
B_1	$kg \cdot m^2$	*0.0055*
C_1	$kg \cdot m^2$	*0.0035*
A_2	$kg \cdot m^2$	*0.003*
B_2	$kg \cdot m^2$	*0.004*
C_2	$kg \cdot m^2$	*0.0015*
$A_3 = B_3$	$kg \cdot m^2$	*0.0035*
C_3	$kg \cdot m^2$	*0.0015*

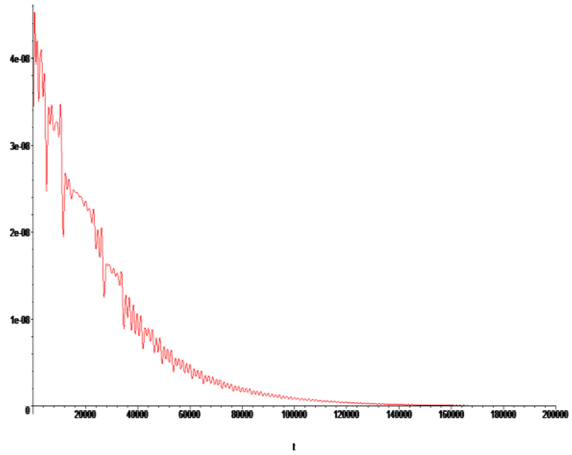


Fig. 3. The Lyapunov function L(t)

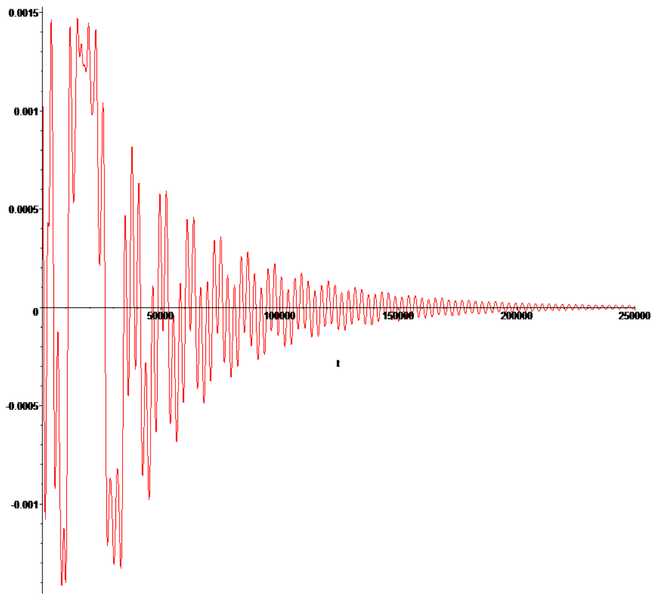


Fig. 4. The angular velocity component $p_1(t)$ of the nanosatellite

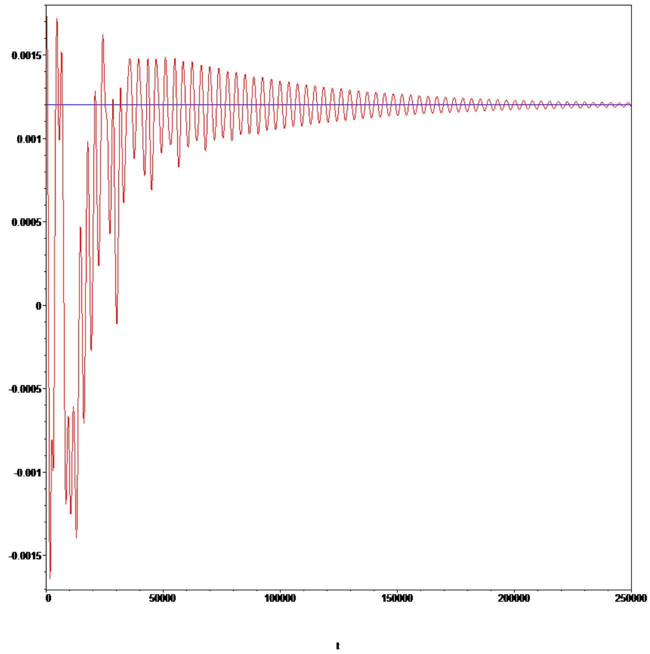


Fig. 5. The angular velocity component $q_1(t)$ of the nanosatellite

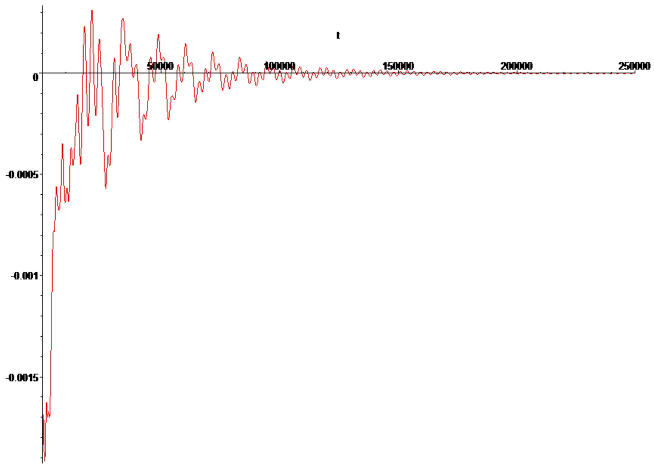


Fig. 6. The angular velocity component $r_1(t)$ of the nanosatellite

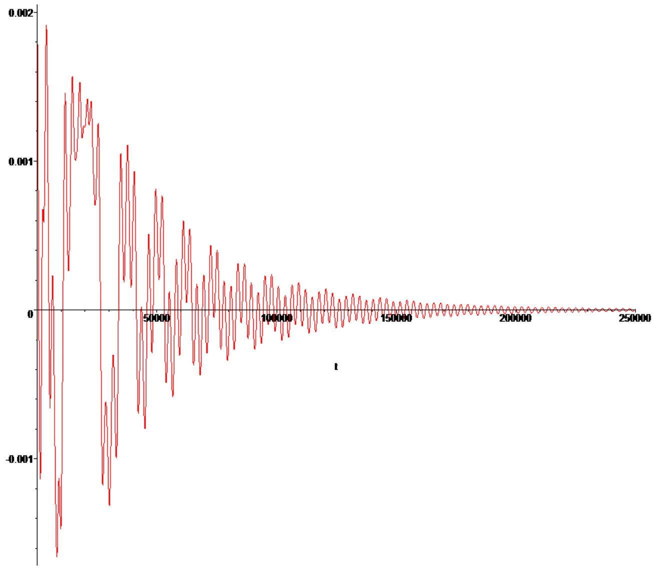


Fig. 7. The angular velocity component $p_2(t)$ of the gravitational damper

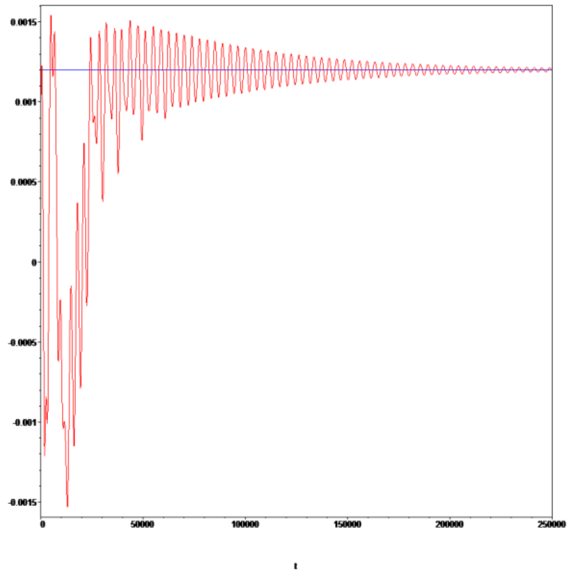


Fig. 8. The angular velocity component $q_2(t)$ of the gravitational damper

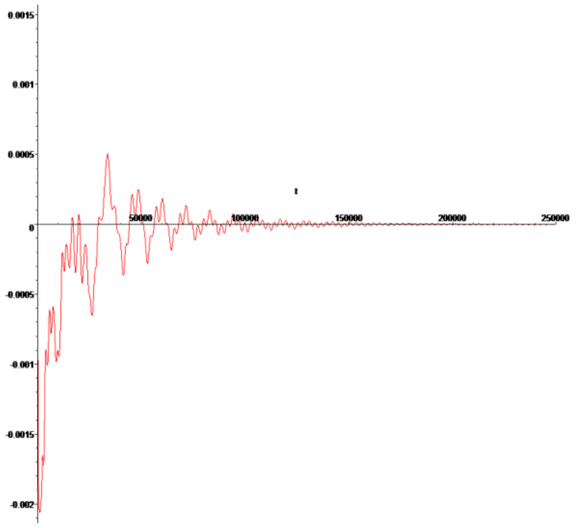


Fig. 9. The angular velocity component $r_2(t)$ of the gravitational damper

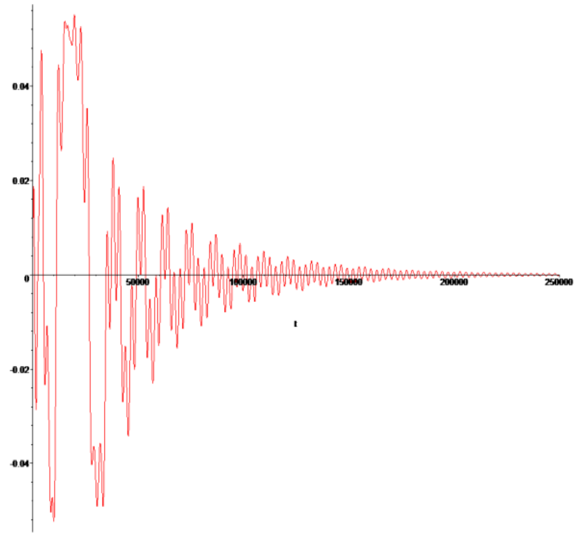


Fig. 10. The linear displacement $y(t)$ of the movable unit

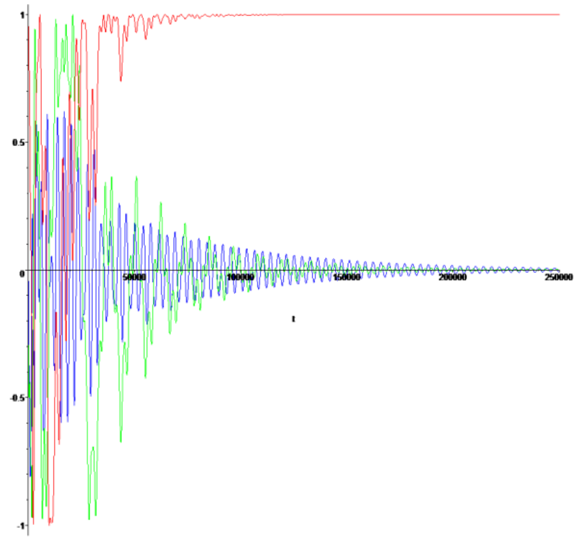


Fig. 11. The directional cosines $\alpha_{11}(t)$ (red), $\beta_{11}(t)$ (green), $\gamma_{11}(t)$ (blue) of the main body of the nanosatellite

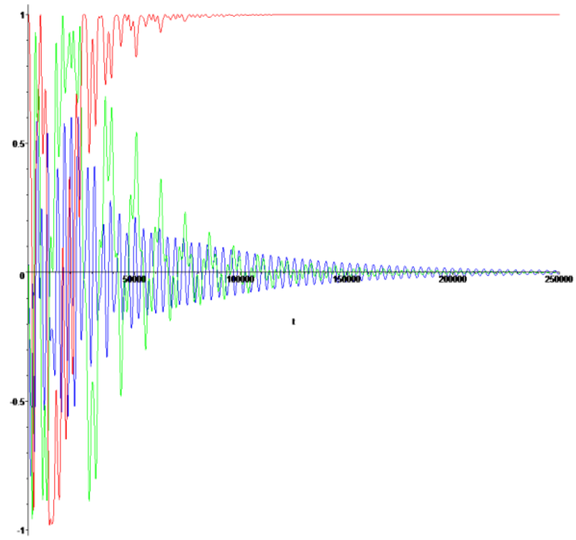


Fig. 12. The directional cosines $\alpha_{21}(t)$ (red), $\beta_{21}(t)$ (green), $\gamma_{21}(t)$ (blue) of the gravitational damper

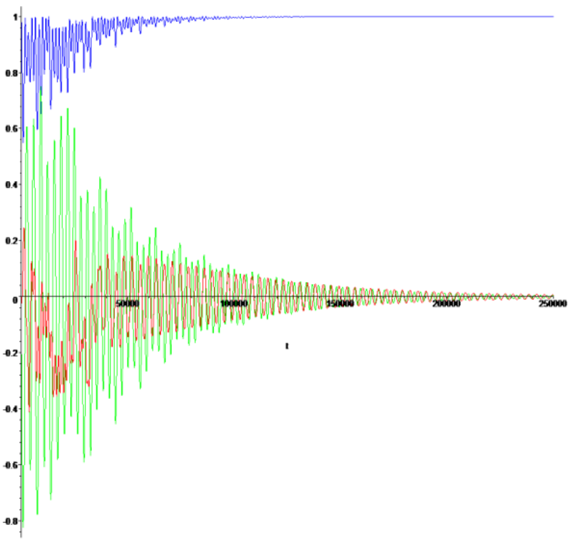


Fig. 13. The directional cosines α_{13}(t) (red), β_{13}(t) (green), γ_{13}(t) (blue) of the nain body of the nanosatellite

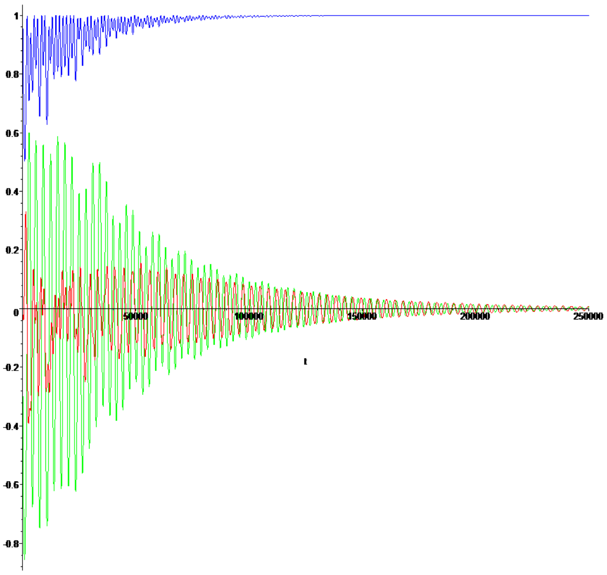


Fig. 14. The directional cosines α_{23}(t) (red), β_{23}(t) (green), γ_{23}(t) (blue) of the gravitational damper

CONCLUSION

As can be seen from Fig. 3, the Lyapunov function has an exponential decreasing trend in its mean values. This occurs due to the fact that the kinetic energy tends to zero due to the interaction between the carrier body and the gravitational damper.

The oscillatory peaks of this function are explained, firstly, by the fact that the function corresponds to the simplest case of a "solid body" (and very roughly describes the stability properties), and, secondly, by the fact that the control is not strictly stable at the initial stage. However, the chosen form of the Lyapunov function allows us to evaluate the stability of the control law, and as can be seen from Fig. 3, the stability properties of the chosen control law (13) grow with time: from about time 50000 [s], the control law (13) can be considered as asymptotically stable. Numerical simulations confirm (Fig. 4-12) that the dynamic motion process is carried out in the correct way in full compliance with the principle of gravitational stabilization. The angular velocity components $p_{1,2}$(t) and $r_{1,2}$(t) of the main body and the damper decrease to zero, and the components $q_{1,2}$(t) tend to the value of the orbital velocity ω_0.

The derictional cosines of the main body and the damper body (fig.11-14) have the time-dependecies, which show the transition to the gravitational oriented attitude position on the orbit.

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