# Attitude dynamics modeling of a dual-spin nanosatellite with an elastic longitudinal axis

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***Abstract*—The paper considers the dynamics of a dual-spin nanosatellite with an internal rotor placed into movable module on elastic connections. The nanosatellite consists of two modules - a carrier body and a mobile module with a rapidly rotating internal rotor, which classifies the nanosatellite as a dual-spin spacecraft. The movable module is connected to the main module by flexible rods. Changing the length of these rods makes it possible to carry out angular displacements of the nanosatellite and, therefore, to control the attitude dynamics of the nanosatellite.**

***Keywords* — nanosatellite, flexible rod system, mobile module**

## 1. Introduction

Currently, the nanosatellite format is increasingly used for a wide variety of space research programs and missions, including the development of satellite systems for remote monitoring of the Earth and the study of the properties of the upper layers of the Earth's atmosphere (for example, these are the Nanosatellite-Gyrostat projects of the MicroMAS-1 and MicroMAS-2A projects [1, 2]).

The design of modern nanosatellites in order to increase their functionality can provide for the installation of mobile modules capable of performing both translational and angular motion relative to the carrier body. The mobile module can contain various functional equipment such as solar panels, communication antennas, optical elements of the Earth remote sensing system. When the mobile module moves relative to the carrier body, the moments of inertia and the angular momentum of the entire nanosatellite change, which in turn affects its dynamics [3-5].

In such mechanical systems, the movable module can be used as an element of a passive rotation stabilization system or as an actuator of the nanosatellite's angular motion control system [5, 6]. In the present work, a mathematical model is constructed for the subsequent analysis of the dynamics of a composite nanosatellite with double rotation and a controlled elastic longitudinal axis, including the study of perturbed motion and the transition to chaotic modes of dynamics.

## 2. Mecanical model

Consider the coordinate systems located at the centers of mass of the components of a nanosatellite with double rotation:

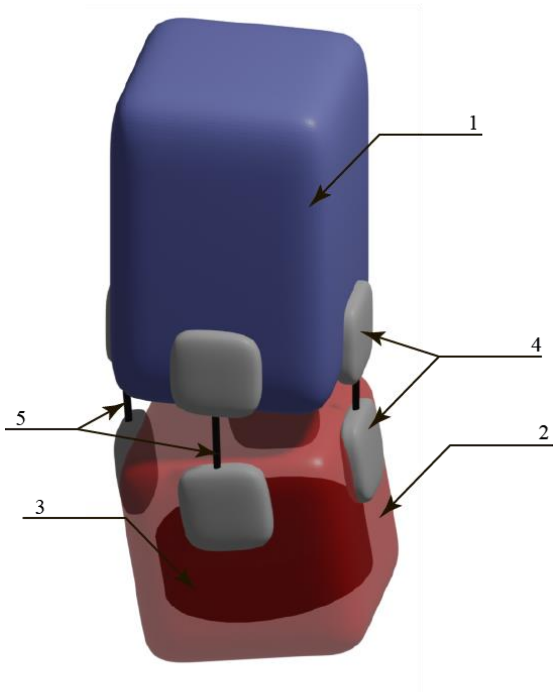
1. CXYZ is the coordinate system located in the center of mass of the entire composite nanosatellite with double rotation, the axes of which are parallel to the main central axes of inertia of the carrier body;

2. $C_1X_1Y_1Z_1$ is the coordinate system located in the center of mass of the carrier body, the axes of which are parallel to the main central axes of inertia of the carrier body;

3. $C_2X_2Y_2Z_2$ is the coordinate system located in the center of mass of the movable module, the axes of which are parallel to the main central axes of inertia of the movable module;

4. $C_3X_3Y_3Z_3$ is the coordinate system located in the center of mass of the rotor, the axes of which are parallel to the main central axes of inertia of the rotor;

The mechanical model of the nanosatellite with double rotation is shown in Figure 1.




1 - carrier body, 2 - movable module, 3 – internal rotor, 4 - flexible rod control system, 5 - flexible rods.

Figure 1 - composite nanosatellite with double rotation

## 3. Mathematical model

A mathematical model of a composite nanosatellite with double rotation can be built on the basis of the theorem on the change in angular momentum. The total angular momentum of a composite nanosatellite in the CZYZ coordinate system has the form:

$$\mathbf{K} = \mathbf{K}_1 + \boldsymbol{\delta}_{21}\mathbf{K}_2 + \boldsymbol{\delta}_{31}\mathbf{K}_3 \qquad (1)$$

where $\mathbf{K}$ is angular momentum of nanosatelite, $\mathbf{K_1}$ is angular momentum of carrier body, $\mathbf{K_2}$ is angular momentum of mobile module, $\mathbf{K_3}$ is angular momentum of rotor, $\boldsymbol{\delta}_{21}$ – transitional matrix from $C_2X_2Y_2Z_2$ coordinate system to the CXYZ coordinate system, $\boldsymbol{\delta}_{31}$ – transitional matrix from $C_3X_3Y_3Z_3$ to CXYZ coordinate system.

$$\delta_{21} = \begin{bmatrix} \cos(\beta) & \sin(\alpha)\sin(\beta) & -\cos(\alpha)\sin(\beta) \\ 0 & \cos(\alpha) & \sin(\alpha) \\ \sin(\beta) & -\sin(\alpha)\cos(\beta) & \cos(\alpha)\cos(\beta) \end{bmatrix} \quad (2)$$

where $\alpha$ and $\beta$ are the angles of deviation of the movable module relative to the body of the carrier along the $C_2X_2$ and $C_2Y_2$ axes, respectively

$$\delta_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\gamma) & \sin(\gamma) \\ & -\sin(\gamma) & \cos(\gamma) \end{bmatrix} \quad (3)$$

where $\gamma$ is the angle of rotation of the rotor relative to the movable module along the z axis

$$\delta_{31} = [\delta_{32}][\delta_{21}] \quad (4)$$

In the considered mathematical model of a composite nanosatellite, we will consider the angles $\alpha$ and $\beta$ to be small, this makes it possible to linearize the mathematical model of a composite nanosatellite with respect to these parameters. Let us write expressions for determining the angular moments of the parts of the nanosatellite:

$$\mathbf{K}_i = \mathbf{I}_i \boldsymbol{\omega}_i + m_i \mathbf{V}_i \times \mathbf{R}_i \quad (5)$$

where $i$ is number of the part (of the body) of the nanosatelite, $\mathbf{I}_i$ is tensor of inertia, $\boldsymbol{\omega}_i$ is angular velocity, , $\mathbf{V}_i$ is linear velocity of the mass center of the body, $\mathbf{R}_i$ is distance from the center of mass of the part of the composite nanosatellite relative to the center of mass of the entire nanosatellite, $m_i$ is mass of the body.

The expressions for the inertia tensors of the parts of the composite nanosatellite are written in the following form:

$$\mathbf{I}_i = diag\left[A_i, B_i, C_i\right] \quad (6)$$

where $A_i, B_i, C_i$ are principal central moments of inertia of the composite nanosatellite.

Let us determine the angular velocities of the parts of the composite nanosatellite. We can write the expression for the angular velocity of the carrier body:

$$\boldsymbol{\omega}_1 = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad (7)$$

where $p, q, r$ are the components of the angular velocity vector of the carrier body.

The angular velocity of the movable module and the rotor will consist of relative and external components. For the movable module, the relative angular velocity will be the rotation of the movable module relative to the carrier body, and the exteral one will be the angular velocity of the carrier body. The expression for determining the angular velocity for the moving module is:

$$\boldsymbol{\omega}_2 = \begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \\ 0 \end{bmatrix} + \boldsymbol{\delta}_{21}\boldsymbol{\omega}_1 \quad (8)$$

By analogy with the expression (8), we calculate the angular velocity of the rotor:

$$\boldsymbol{\omega}_3 = \begin{bmatrix} 0 \\ 0 \\ \dot{\gamma} \end{bmatrix} + \boldsymbol{\delta}_{32}\boldsymbol{\omega}_2 \quad (9)$$

Linear velocities of parts of the composite nanosatellite are calculated as follows:

$$\mathbf{V}_i = \boldsymbol{\omega}_i \times \mathbf{R}_i \quad (10)$$

## 4. Modeling results

Let us consider the case of the nanosatellite attitude motion at small oscillations of the movable module with the following hypothetical harmonic time-functions of the $\alpha$ and $\beta$ angles:

$$\begin{cases} \alpha = \varepsilon_1 \cos(\Omega_1 t) \\ \beta = \varepsilon_2 \cos(\Omega_2 t) \end{cases} \quad (11)$$

where $\varepsilon_1$ and $\varepsilon_2$ are the small amplitudes of the oscillations, $\Omega_1$ and $\Omega_2$ - are the oscillation frequencies.

The previous research in the framework of satellites and dual-spin satellites dynamics [3-4, 11-14] demonstrates the possibilities of the chaotic dynamics initiation at small internal perturbations. Let us here also to present the chaotic modes in the motion of the composite nanosatellite with double rotation. To do this, it is advisable to use the well-known Andoyer-Deprit variables, which are associated with the components of the system's angular momentum:

$$\begin{cases} K_{(1)} = \sqrt{G^2 - L^2}\sin l \\ K_{(2)} = \sqrt{G^2 - L^2}\cos l \\ K_{(3)} = L \end{cases} \qquad (12)$$

To write the dynamical equations in the Andoyer Deprit variables, it is necessary to write the Hamiltonian of our mechanical system:

$$\mathbf{H} = T + P \qquad (13)$$

where $T$ is the kinetic energy of the mechanical system, $P$ is the potential energy of the mechanical system. Since there are no potential forces in the mechanical system, the potential energy will be equal to zero.

$$\mathbf{H} = T \qquad (14)$$

Let us calculate the kinetic energy of the nanosatellite with double rotation:

$$T = \frac{\mathbf{K}_1\boldsymbol{\omega}_1 + \mathbf{K}_2\boldsymbol{\omega}_2 + \mathbf{K}_3\boldsymbol{\omega}_3}{2} \qquad (15)$$

The general notation of dynamic equations in Andoyer-Deprit variables will take the form:

$$\dot{L} = -\frac{\partial \mathbf{H}}{\partial l} \qquad \dot{l} = \frac{\partial \mathbf{H}}{\partial L} \qquad (16)$$

To determine the dynamic equation for the angle of rotation of the rotor, we write the Lagrange equation of the second kind

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\gamma}}\right) - \frac{\partial T}{\partial \gamma} = 0 \qquad (17)$$

After writing the perturbed equations of dynamics in a specific form, taking into account the angular displacements of the movable module with the rotor on the controlled elastic longitudinal axis, as well as possible external force factors, we obtain the Poincaré sections in the Andoer-Deprit's phase space {*l*, *L/K*} (figures 3 - 4). The corresponding inertial-mass parameters of the composite nanosatellite used in the simulation are given in Table 1.

Let's divide expression (14) into parts: the part corresponding to the kinetic energy of a solid body and parts depending on the angles alpha and beta

$$\mathbf{H} = \mathbf{H_0} + \alpha \mathbf{H}_1 + \beta \mathbf{H}_2 \qquad (18)$$

Taking into account (18), we rewrite the system of equations(16) in the following form:

$$\begin{aligned} \dot{L} &= f_L + \alpha g_{L1} + \beta g_{L2} \\ \dot{l} &= f_l + \alpha g_{l1} + \beta g_{l2} \end{aligned} \qquad (19)$$

where

$$\begin{aligned} f_L &= -\frac{\partial \mathbf{H_0}}{\partial L} \quad & g_{L1} &= -\frac{\partial \mathbf{H_1}}{\partial L} \quad & g_{L2} &- \frac{\partial \mathbf{H_2}}{\partial L} \\ f_l &= -\frac{\partial \mathbf{H}_0}{\partial l} \quad & g_{l1} &= -\frac{\partial \mathbf{H_1}}{\partial L} \quad & g_{l2} &- \frac{\partial \mathbf{H_2}}{\partial L} \end{aligned} \qquad (20)$$

To prove the presence of chaotic regimes in the dynamics of a nanosatellite, we will use the classical Melnikov method, which was developed [7] and generalized [8-10] by well-known authors. The expression for the Melnikov function is written as follows:

$$M(t_0) = \int_{-\infty}^{+\infty} \left[ f_L g_l - f_l g_L \right]_{(\bar{L}(t), \bar{l}(t), t + t_0)} dt \qquad (21)$$

where the functions $g_l$ and $g_L$ are defined as follows:

$$\begin{aligned} g_L &= \alpha g_{L1} + \beta g_{L2} \\ g_l &= \alpha g_{l1} + \beta g_{l2} \end{aligned} \qquad (22)$$

Since $\alpha$ and $\beta$ are trigonometric functions (11), then substituting $t = t + t_0$ into equation (21) and then integrating it, we obtain expressions of the following form:

$$\begin{aligned} M(t_0) = {} & \varepsilon_1\lambda_1 \cos(\Omega_1 t_0) - \varepsilon_1\lambda_2 \sin(\Omega_1 t_0) + \\ & + \varepsilon_2\lambda_3 \cos(\Omega_2 t_0) - \varepsilon_2\lambda_4 \sin(\Omega_2 t_0) \end{aligned} \qquad (23)$$

where $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are integration constants.

As can be seen from expression (23), the Melnikov function will be trigonometric and the graph of this function will cross the abscissa axis an infinite number of times, which indicates the presence of chaotic modes in the dynamics of the nanosatellite. The graph of the Melnikov function is shown in (fig.2).

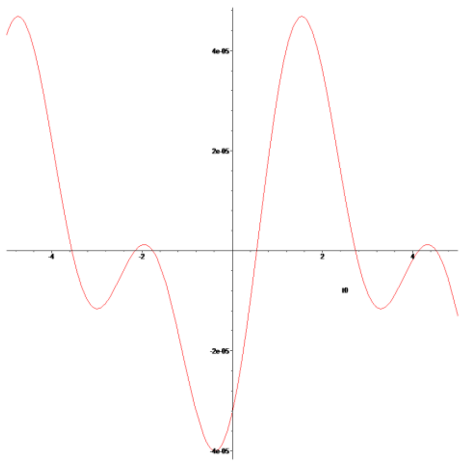


Figure 2.The Melnikov function

($\varepsilon_1 = 0.1, \varepsilon_2 = 0.1, \Omega_1 = 1, \Omega_2 = 1.5$)

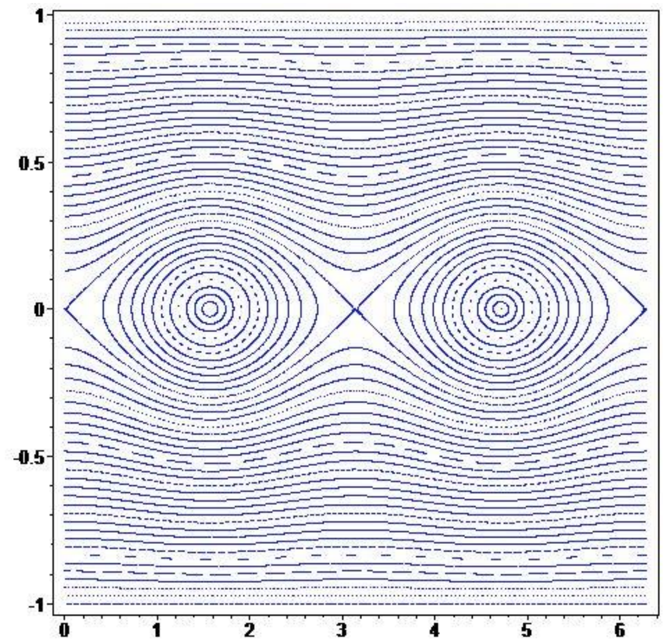


Figure 3. The Poincaré section of unperturbed dynamics ( $\varepsilon_1 = 0$ , $\varepsilon_2 = 0$ )

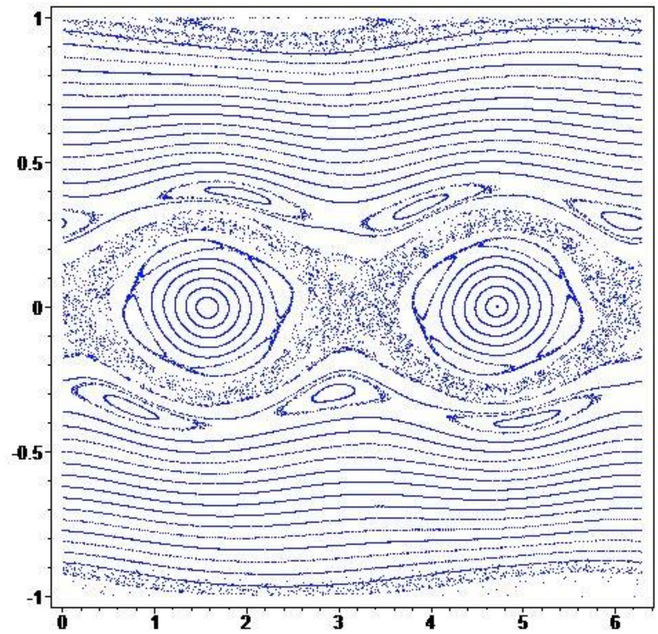


Figure 4.The Poincaré section of the perturbed dynamics ( $\varepsilon_1 = 0.1, \varepsilon_2 = 0.1, \Omega_1 = 1, \Omega_2 = 1$ )

As can we see from the Poincaré sections, the unperturbed dynamics (fig.3) is characterized by the invariant forms of the phase-trajectories, which are separated curves on the phase space. In this case, we have the classical form of the "pendulum" phase portrait.

In the case of the perturbed dynamics (fig.4) we can see the so-called chaotic layers near the separatrixes region and also in the upper part of the phase space. This proves the initiation of the corresponding chaotic dynamics of the nanosatellites with the heteroclinical initial conditions for the motion parameters in the separatixes zone.

TABLE1 INERTIAL-MASS PARAMETERS

| Parameter name | Parameter value |
|---|---|
| $A_1$ | 0.013 [kg·m$^2$] |
| $B_1$ | 0.009 [kg·m$^2$] |
| $C_1$ | 0.006 [kg·m$^2$] |
| $A_2$=$B_2$ | 0.0045 [kg·m$^2$] |
| $C_2$ | 0.0035 [kg·m$^2$] |
| $A_3$=$B_3$ | 0.002 [kg·m$^2$] |
| $C_3$ | 0.003 [kg·m$^2$] |
| $m_1$ | 3 [kg] |
| $m_2$ | 2 [kg] |
| $m_3$ | 1 [kg] |
| $\varepsilon_1$=$\varepsilon_2$ | 0.1 |
| $\Omega_1$=$\Omega_2$ | 1 [1/s] |

## 5. CONCLUSION

The attitude motion of the dual-spin nanosatellite was considered at small oscillations of the elastic axis of the internal rotor. The possibilities of the chaotic regimes initiation in the attitude dynamics of the dual-spin nanosatellite with the elastic axis is demonstrated.
The further research will be directed on the analysis of the chaotic modes arising and avoiding due to presence in the system small distributions and damping factors.


## ACKNOWLEDGMENTS

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