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New Strange Chaotic Attractors in Dynamical Systems of Multi-Spin Spacecraft and Gyrostats

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***Abstract*—In this work new strange attractors are found. These strange attractors can correspond to special dynamical regimes in attitude dynamics of multi-spin spacecraft and gyrostat-satellites. The attractors arise in the dynamical systems, which are structurally related to the Newton-Leipnik system. In addition, the complex modes structurally close to chaotic strange attractors are considered.**

Keywords— strange attractor, dynamical chaos, multi-spin spacecraft, attitude dynamics, the Newton-Leipnik system

I. Introduction

Searching and detailed investigating of strange attractors in phase spaces of systems of various nature is one of the main important problems of the modern nonlinear dynamics. It is well-known fact that many dynamical systems contain strange chaotic attractors; among such systems it is possible to indicate the rigid bodies mechanical systems and their applications into space flight dynamics, including tasks of attitude dynamics of compound multibody spacecraft.

One of the important constructional schemes of multibody spacecraft is the scheme of multi-spin spacecraft (MSSC), which uses internal rotors to control the angular motion. As it was shown in previous works [1-4] the strange attractors can be found in the phase space of the MSSC dynamical system, and, moreover, they can be intentionally initiated with the help of usual equipment using (internal rotors' engines, external thrusters, etc.) and MSSC parameters changing/selecting.

The indicated task of the intentional initiation of strange chaotic attractors into systems dynamics is connected with new aspects of dynamical chaos and its application. For example, it is possible to use properties of MSSC chaotic dynamical regimes to implement the attitude reorientation of spacecraft [2, 3]. In this connection, detecting/generating of classical and new strange attractors into MSSC dynamics is the quite important modern task; also this task can be considered as the special task of signal processing.

In this paper we use the MSSC dynamical system as the basic target system, which is described by three ordinal differential equations. In this 3D quadratic continuous time system we will generate some new strange chaotic attractors.

II. Main dynamical systems

The MSSC [1-3] represents the multi-body (multi-rotor) constructional scheme with conjugated pairs of rotors placed on the inertia principle axes of the main body (fig.1). General properties of the MSSC attitude dynamics are connected with the internal redistribution of the angular momentum between the system bodies (the main body, and rotors) due to the internal torques creation by internal rotors engines.

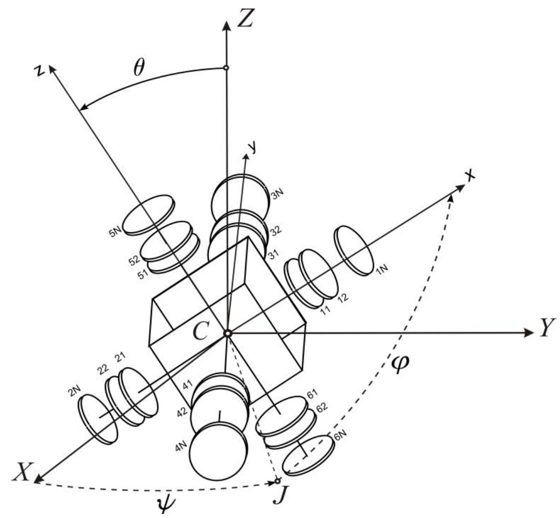

Fig.1. The MSSC as the multirotor system

As it was realized previously [1-3] we can consider the attitude dynamics of the MSSC basing on the dynamical equations for the multi-rotor system with 6*N* rotors (Fig.1) contained into *N* layers on six general directions coinciding with the principle axes of the main body:

$$\begin{cases} \hat{A}\dot{p} + \dot{D}_{12} + \left(\hat{C} - \hat{B}\right)qr + \left[qD_{56} - rD_{34}\right] = M_x^e; \\ \hat{B}\dot{q} + \dot{D}_{34} + \left(\hat{A} - \hat{C}\right)rp + \left[rD_{12} - pD_{56}\right] = M_y^e; \\ \hat{C}\dot{r} + \dot{D}_{56} + \left(\hat{B} - \hat{A}\right)pq + \left[pD_{34} - qD_{12}\right] = M_z^e \end{cases} \quad (1)$$

In these equations the following notations are used: $\boldsymbol{\omega}=[p, q, r]^T$ – the vector of the absolute angular velocity of the MSSC main body (in projections on the connected frame *Oxyz*); $\hat{A}$, $\hat{B}$, $\hat{C}$ are the summarized moments of inertia of the MSSC; M_x^e, M_y^e M_z^e – the external torques acting on the system. The summarized rotors' angular momentums in the

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considered case are formed by the control system in the shape:

$$D_{12} = \alpha_p p + \alpha_0;\ D_{34} = \beta_q q + \beta_0;\ D_{56} = \gamma_r r + \gamma_0, \qquad (2)$$

The "external" torques also are created (by thrusters) as follows:

$$M_x^e = m_x + \alpha_1 p;\ M_y^e = m_y + \beta_1 q;\ M_z^e = m_z + \gamma_1 r, \qquad (3)$$

Therefore, we have the following constant "controlling" terms/coefficients:

$$\left\{\alpha_0,\ \beta_0,\ \gamma_0,\ m_x,\ m_y,\ m_z,\ \alpha_1,\ \beta_1,\ \gamma_1,\ \alpha_p,\ \beta_q,\ \gamma_r\right\} \sim \text{const}$$

III. Evaluating the MSSC Parameters Delivering the Dynamics with Strange Chaotic Attractors

As it was indicated [5, 6] the quite general natural candidates for the construction of dynamical systems with multi-scroll chaotic attractors are 3D quadratic continuous time systems given by equations

$$\begin{cases} \dot{x} = a_0 + a_1 x + a_2 y + a_3 z + a_4 x^2 + a_5 y^2 + \\ \qquad + a_6 z^2 + a_7 xy + a_8 xz + a_9 yz; \\ \dot{y} = b_0 + b_1 x + b_2 y + b_3 z + b_4 x^2 + b_5 y^2 + \\ \qquad + b_6 z^2 + b_7 xy + b_8 xz + b_9 yz; \\ \dot{z} = c_0 + c_1 x + c_2 y + c_3 z + c_4 x^2 + c_5 y^2 + \\ \qquad + c_6 z^2 + c_7 xy + c_8 xz + c_9 yz; \end{cases} \qquad (4)$$

where $Coeff = \{a_i, b_i, c_i\}_{0 \le i \le 9} \in \mathbb{R}^{30}$ is the set of constant parameters.

Basing on expressions (2) we can solve the linear algebraic equations (1) relatively $\{\dot{p},\ \dot{q},\ \dot{r}\}$; and with designation of the variables $(p \leftrightarrow x;\ q \leftrightarrow y;\ r \leftrightarrow z)$ it is possible to write the concretized coefficients for the system (4) through the controlling parameters [2]:

$$\begin{cases} a_0 = \dfrac{m_x}{\hat{A} + \alpha_p};\ b_0 = \dfrac{m_y}{\hat{B} + \beta_q};\ c_0 = \dfrac{m_z}{\hat{C} + \gamma_r}; \\ a_1 = \dfrac{\alpha_1}{\hat{A} + \alpha_p};\ \ b_1 = \dfrac{\gamma_0}{\hat{B} + \beta_q};\ \ c_1 = \dfrac{-\beta_0}{\hat{C} + \gamma_r}; \\ a_2 = \dfrac{-\gamma_0}{\hat{A} + \alpha_p};\ \ b_2 = \dfrac{\beta_1}{\hat{B} + \beta_q};\ \ c_2 = \dfrac{\alpha_0}{\hat{C} + \gamma_r}; \\ a_3 = \dfrac{\beta_0}{\hat{A} + \alpha_p};\ \ b_3 = \dfrac{-\alpha_0}{\hat{B} + \beta_q};\ \ c_3 = \dfrac{\gamma_1}{\hat{C} + \gamma_r}; \\ a_4 = a_5 = a_6 = b_4 = b_5 = b_6 = c_4 = c_5 = c_6 \equiv 0; \\ a_7 = 0;\ b_7 = 0;\ c_7 = \left(\hat{A} - \hat{B} - \beta_q + \alpha_p\right) / \left(\hat{C} + \gamma_r\right); \\ a_8 = 0;\ b_8 = \left(\hat{C} - \hat{A} - \alpha_p + \gamma_r\right) / \left(\hat{B} + \beta_q\right);\ c_8 = 0; \\ a_9 = \left(\hat{B} - \hat{C} - \gamma_r + \beta_q\right) / \left(\hat{A} + \alpha_p\right);\ b_9 = 0;\ c_9 = 0. \end{cases} \qquad (5)$$

Now with the help of connections (5) we can equate the MSSC dynamical system in the form (4), that allows to initiate into the MSSC dynamics some possible strange attractors by creation of natural controlling torques (which realize at the concrete values of *Coeff*, e.g. Lorenz, Rössler, Newton–Leipnik, Wang-Sun, Chen-Lee, etc.). Here it is important to underline that we will consider the systems without the quadratic terms $\left(a_i = b_j = c_k = 0\right)_{i,j,k=4..6}$.

However, in our natural dynamical system (1) at the torques/momentums (2) and (3) we have the set of the natural controlling constants with another dimension (in comparison with dim(*Coeff*)):

$$Control = \left\{\alpha_p, \alpha_0, m_x, \alpha_1, \beta_q, \beta_0, m_y, \beta_1, \gamma_r, \gamma_0, m_z, \gamma_1\right\} \in \mathbb{R}^{12} \qquad (6)$$

Due to the incompatibility of the indicated sets *Coeff* and *Control* we cannot define exact correspondences between theirs coefficients – so we have to use the optimization algorithm (e.g. the well-known gradient descent procedure for minimizing a misalignment function) to find an optimal parameters (6) which deliver given numerical values of the coefficients *Coeff* exactly or approximately. Further, we will use the gradient descent procedure for the following quadratic misalignment function [2]:

$$\begin{aligned} &\Psi\left(\alpha_p, \alpha_0, m_x, \alpha_1, \beta_q, \beta_0, m_y, \beta_1, \gamma_r, \gamma_0, m_z, \gamma_1\right) = \\ &= \left(a_0 - \frac{m_x}{\hat{A} + \alpha_p}\right)^2 + \left(b_0 - \frac{m_y}{\hat{B} + \beta_q}\right)^2 + \left(c_0 - \frac{m_z}{\hat{C} + \gamma_r}\right)^2 + \\ &+ \left(a_1 - \frac{\alpha_1}{\hat{A} + \alpha_p}\right)^2 + \left(b_1 - \frac{\gamma_0}{\hat{B} + \beta_q}\right)^2 + \left(c_1 + \frac{\beta_0}{\hat{C} + \gamma_r}\right)^2 + \\ &+ \left(a_2 + \frac{\gamma_0}{\hat{A} + \alpha_p}\right)^2 + \left(b_2 - \frac{\beta_1}{\hat{B} + \beta_q}\right)^2 + \left(c_2 - \frac{\alpha_0}{\hat{C} + \gamma_r}\right)^2 + \\ &+ \left(a_3 - \frac{\beta_0}{\hat{A} + \alpha_p}\right)^2 + \left(b_3 + \frac{\alpha_0}{\hat{B} + \beta_q}\right)^2 + \left(c_3 - \frac{\gamma_1}{\hat{C} + \gamma_r}\right)^2 + \\ &+ \left(c_7 - \left(\hat{A} - \hat{B} - \beta_q + \alpha_p\right) / \left(\hat{C} + \gamma_r\right)\right)^2 + \\ &+ \left(b_8 - \left(\hat{C} - \hat{A} - \alpha_p + \gamma_r\right) / \left(\hat{B} + \beta_q\right)\right)^2 + \\ &+ \left(a_9 - \left(\hat{B} - \hat{C} - \gamma_r + \beta_q\right) / \left(\hat{A} + \alpha_p\right)\right)^2 \end{aligned} \qquad (7)$$

Implementing the following iterations [2] (with some values *d* of the mean-square distance between the coefficients *Control* and the assigned target set *Coeff*):

$$\begin{aligned} &while\ \left|\sqrt{\Psi(X_i)}\right| > d:\ X_{i+1} = X_i - h \cdot \nabla\Psi(X_i); \\ &X_i \in Control;\ \ X_0 - the\ defined\ initial\ point \end{aligned} \qquad (8)$$

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it is possible to obtain the numerical values for the set *Control* that can provide the realization of some strange attractors which will be near to target attractors.

Here we must note that in the previous work [2] this iterative algorithm was efficiently used to the synthesis of the classical well-known Wang-Sun four-scroll chaotic attractor [9] and the Chen-Lee two-scroll chaotic attractor [10] in the MSSC dynamical system.

IV. NEW STRANGE CHAOTIC ATTRACTORS

Now we can implement the indicated above procedure [2] to synthesize some new strange chaotic attractors. It is possible to fulfill the indicated above iterative algorithm [2] taking the Newton-Leipnik system [11] as the target system. For the Newton-Leipnik system we have the following concretized set *Coeff* of non-zero numerical coefficients:

$$Coeff = \begin{Bmatrix} a_1 = -0.4;\ a_2 = 1;\ a_9 = 10; \\ b_1 = -1;\ b_2 = -0.4;\ b_8 = 5; \\ c_3 = 0.175;\ c_7 = -5 \end{Bmatrix}$$

Now we can implement some calculations of new sets of coefficients of systems with strange attractors.

1). Firstly, we present modeling results for the starting parameters:

$$\hat{A} = 1000;\quad \hat{B} = 2500;\quad \hat{C} = 3000\ [\mathrm{kg \cdot m^2}]$$

$$X_0 = (\alpha_p, \alpha_0, m_x, \alpha_1, \beta_q, \beta_0, m_y, \beta_1, \gamma_r, \gamma_0, m_z, \gamma_1) = \\ = (-684,\ 0,\ 0,\ -125,\ 1428,\ 0,\ 0, -106,\ -2245,\ -336,\ 0,\ 91).$$

In this case the iterative algorithm gives the first strange attractor at the following final values:

$$X_{final} = (\alpha_p, \alpha_0, m_x, \alpha_1, \beta_q, \beta_0, m_y, \beta_1, \gamma_r, \gamma_0, m_z, \gamma_1) = \\ = (-692.7387,\ 0,\ 0,\ -122.9331,\ 1319.2399,\ 0,\ 0, \\ -943.7322,\ -2265.7542,\ -329.9222,\ 0,\ 128.6660);$$

$$d = \sqrt{\Psi(X_{final})} = 4.9925,$$

that results in the values of coefficients for the system (4):

$$SysA = \begin{Bmatrix} a_1 = -0.4000;\ a_2 = 1.0738;\ a_9 = 10.0403; \\ b_1 = -0.0864;\ b_2 = -0.2471;\ b_8 = 0.1118; \\ c_3 = 0.1752;\ c_7 = -4.7831 \end{Bmatrix}$$

As the result we have the new system

$$SysA = \begin{cases} \dot{x} = -0.4000x + 1.0738y + 10.0403yz; \\ \dot{y} = -0.0864x - 0.2471y + 0.1118xz; \\ \dot{z} = 0.1752z - 4.7831xy; \end{cases} \qquad (9)$$

with the new strange chaotic attractor (fig. 2) at the initial values $x(0) = 0.05;\ y(0) = 0.1;\ z(0) = 1.5.$ Also this attractor (red) is depicted (fig.3) together with the classical Newton-Leipnik attractor (black).

Here it is not out of place to mention that the *SysA* system corresponds to the natural attitude dynamics equation of MSSC, and therefore, this new *SysA*-attractor can be intentionally initiated in the MSSC dynamics.

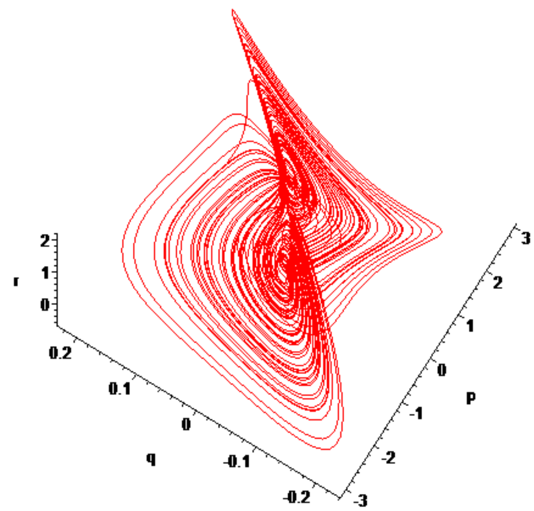

Fig.2. The new strange *SysA*-attractor in the system (9)

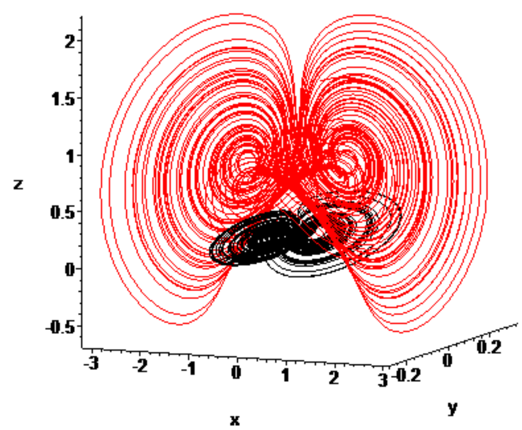

Fig.3. The *SysA* attractor (red) and the *Newton-Leipnik* attractor (black)

2). The second case of calculations corresponds to the following starting parameters:

$$\hat{A} = 1000;\quad \hat{B} = 2500;\quad \hat{C} = 3000\ [\mathrm{kg \cdot m^2}]$$

$$X_0 = (\alpha_p, \alpha_0, m_x, \alpha_1, \beta_q, \beta_0, m_y, \beta_1, \gamma_r, \gamma_0, m_z, \gamma_1) = \\ = (-696,\ 0,\ 0,\ -121,\ 1280,\ 0,\ 0, -1467,\ -2271,\ -326,\ 0,\ 228).$$

The iterative algorithm gives the second strange attractor at the following final values:

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$$X_{final} = (\alpha_p, \alpha_0, m_x, \alpha_1, \beta_q, \beta_0, m_y, \beta_1, \gamma_r, \gamma_0, m_z, \gamma_1) =$$
$$= (-695.9057,\ 0,\ 0,\ -121.5977,\ 1281.2392,\ 0,\ 0,$$
$$-1467.3693,\ -2272.0667,\ -326.3300,\ 0,\ 199.3635);$$
$$d = \sqrt{\Psi(X_{final})} = 4.9793,$$

The corresponding values of coefficients for the system (4) are:

$$SysB = \begin{Bmatrix} a_1 = -0.3999;\ a_2 = 1.0731;\ a_9 = 10.0407; \\ b_1 = -0.0863;\ b_2 = -0.3881;\ b_8 = 0.1121; \\ c_3 = 0.2739;\ c_7 = -4.7767 \end{Bmatrix}$$

So we have the new system

$$SysB = \begin{cases} \dot{x} = -0.3999x + 1.0731y + 10.0407yz; \\ \dot{y} = -0.0863x - 0.3881y + 0.1121xz; \\ \dot{z} = 0.2739z - 4.7767xy; \end{cases} \quad (10)$$

with the new second strange chaotic attractor (fig. 4) at the initial values $x(0) = 0.05;\ y(0) = 0.1;\ z(0) = 1.5.$ This second attractor (red) is depicted (fig.5) together with the classical Newton-Leipnik attractor (black).

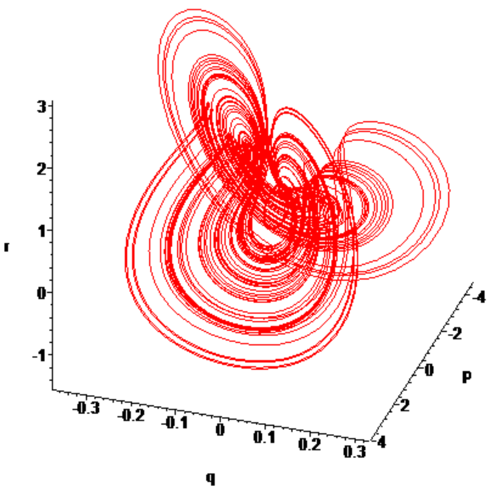

Fig.4. The new strange *SysB*-attractor in the system (10)

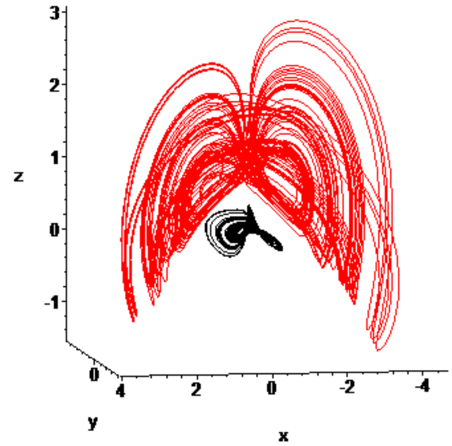

Fig.5. The *SysB* attractor (red) and the *Newton-Leipnik* attractor (black)

3). The third case of calculations corresponds to the following starting parameters:

$$\hat{A} = 1000;\quad \hat{B} = 2500;\quad \hat{C} = 3000\ [\mathrm{kg \cdot m^2}]$$

$$X_0 = (\alpha_p, \alpha_0, m_x, \alpha_1, \beta_q, \beta_0, m_y, \beta_1, \gamma_r, \gamma_0, m_z, \gamma_1) =$$
$$= (-696,\ 0,\ 0,\ -121,\ 1280,\ 0,\ 0, -1467,\ -2271,\ -326,\ 0,\ 1228).$$

The iterative algorithm gives the third strange attractor at the following final values:

$$X_{final} = (\alpha_p, \alpha_0, m_x, \alpha_1, \beta_q, \beta_0, m_y, \beta_1, \gamma_r, \gamma_0, m_z, \gamma_1) =$$
$$= (-682.4176,\ 0,\ 0,\ -126.8955,\ 1451.8728,\ 0,\ 0,$$
$$-1473.7799,\ -2237.0650,\ -340.5281,\ 0,\ 292.1299);$$

$$d = \sqrt{\Psi(X_{final})} = 4.9827,$$

The corresponding values of coefficients for the system (4) are:

$$SysC = \begin{Bmatrix} a_1 = -0.3996;\ a_2 = 1.0723;\ a_9 = 10.0413; \\ b_1 = -0.0862;\ b_2 = -0.3729\ \ b_8 = 0.1127; \\ c_3 = 0.3829;\ c_7 = -4.7636 \end{Bmatrix}$$

So we have the third new system

$$SysC = \begin{cases} \dot{x} = -0.3996x + 1.0723y + 10.0413yz; \\ \dot{y} = -0.0862x - 0.3729y + 0.1127xz; \\ \dot{z} = 0.3829z - 4.7636xy; \end{cases} \quad (11)$$

with the new strange chaotic attractor (fig. 6) at the initial values $x(0) = 0.05;\ y(0) = 0.1;\ z(0) = 1.5.$ This attractor (red) is depicted (fig.7) together with the classical Newton-Leipnik attractor (black).

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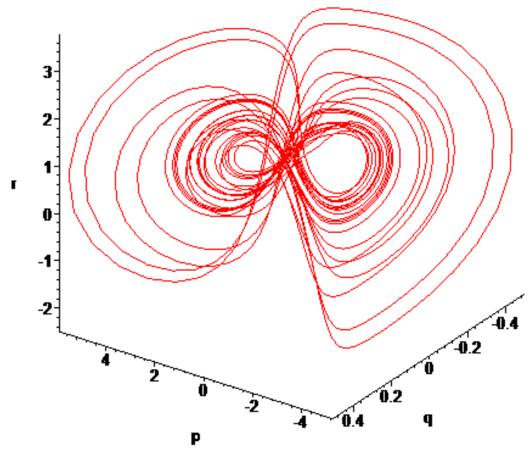

Fig.6. The new strange *SysC*-attractor in the system (11)

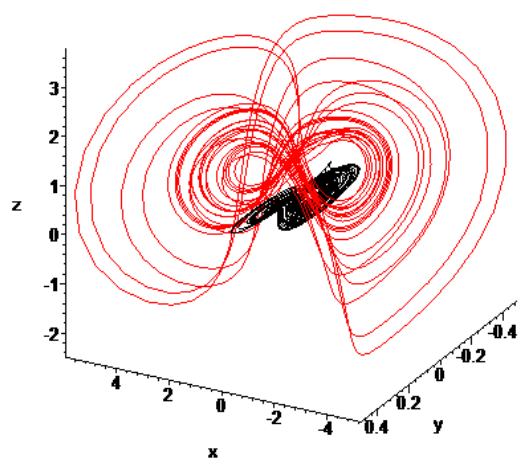

Fig.7. The *SysC* attractor (red) and the *Newton-Leipnik* attractor (black)

V. Systems with Complex Behavior Close to Strange Chaotic Attractors

In this section we present two cases of systems which do not include the strange attractors, but have the complex dynamics of phase trajectories, that can be close to the type of strange attractors.

1). The first case of complex dynamics corresponds to parameters:

$$\hat{A}=100;\ \ \hat{B}=250;\ \ \hat{C}=300\ [\mathrm{kg\cdot m^2}]$$

$$X_0=\left(\alpha_p,\alpha_0,m_x,\alpha_1,\beta_q,\beta_0,m_y,\beta_1,\gamma_r,\gamma_0,m_z,\gamma_1\right)=$$
$$=\left(-6,\ 0,\ 0,\ -1.5,\ 13,\ 0,\ 0,-15,\ -23,\ 32,\ 0,\ 13\right);$$

$$X_{final}=\left(\alpha_p,\alpha_0,m_x,\alpha_1,\beta_q,\beta_0,m_y,\beta_1,\gamma_r,\gamma_0,m_z,\gamma_1\right)=$$
$$=\left(-2886.4968,\ 0,\ 0,\ 361.7618,\ 409.0296,\ 0,\ 0,\ -263.7884,\ 467.3039,-476.9110,\ 0,\ 133.8367\right);$$
$$d=\sqrt{\Psi\left(X_{final}\right)}=10.0578;$$

Then we take the system:

$$Complex1=\begin{cases}\dot{x}=\text{-}0.1298x-0.1712y+0.03886yz;\\ \dot{y}=\text{-}0.7237x-0.4003y+5.3925xz;\\ \dot{z}=0.1744z-4.4904xy\end{cases}$$

At the indicated parameters the complex behavior of the system *Complex1* realizes. Corresponding phase trajectory depicted at the figure (fig.8).

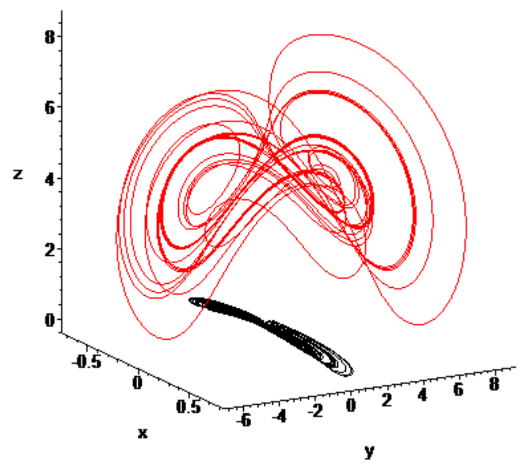

Fig.8. The *Complex1* phase trajectory (red) and the *Newton-Leipnik* attractor (black)

2) An interesting case of the complex dynamics of MSSC is possible at the following parameters:

$$\hat{A}=100;\ \ \hat{B}=250;\ \ \hat{C}=300\ [\mathrm{kg\cdot m^2}]$$

$$X_0=\left(\alpha_p,\alpha_0,m_x,\alpha_1,\beta_q,\beta_0,m_y,\beta_1,\gamma_r,\gamma_0,m_z,\gamma_1\right)=$$
$$=\left(-6,\ 0,\ 0,\ -1.5,\ 13,\ 0,\ 0,-15,\ -23,\ 32,\ 0,\ 13\right);$$

$$X_{final}=\left(\alpha_p,\alpha_0,m_x,\alpha_1,\beta_q,\beta_0,m_y,\beta_1,\gamma_r,\gamma_0,m_z,\gamma_1\right)=$$
$$=\left(-2947.8679,\ 0,\ 0,\ 23.5201,\ 430.1965,\ 0,\ 0,\ -77.3623,\ 500.7775,-105.2338,\ 0,\ 38.9642\right);$$
$$d=\sqrt{\Psi\left(X_{final}\right)}=10.0837;$$

Then we take the system:

$$Complex2=\begin{cases}\dot{x}=\ \text{-}0.0083x-0.0369y+0.0423yz;\\ \dot{y}=\text{-}0.1547x-0.1137y+5.3641xz;\\ \dot{z}=0.0487z-4.4058xy\end{cases}$$

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The corresponding complex phase trajectory with two additional dissipative scrolls is presented below.

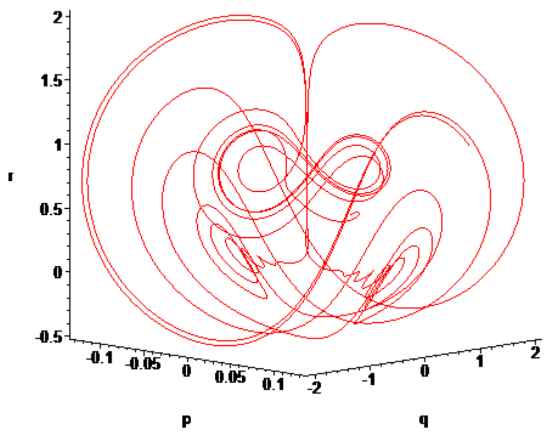

Fig.9. The *Complex2* phase trajectory

VI. CONCLUSION

In the paper basing on the multi-spin spacecraft mathematical models [1] and on the iterative algorithm [2] some new strange attractors were found; and their intentional initiation in the MSSC attitude dynamics was considered. The detected in this paper new strange chaotic attractors are structurally related with the Newton-Leipnik system, but they have different coefficients and locations in the phase space. Also the complex modes structurally close to chaotic strange attractors were numerically investigated.

These strange chaotic attractors can be used for the fulfillment of parameters changing of the motion of MSSC, space and underwater robots, including the chaotic reorientation and chaotic maneuvering.

The considered dynamical systems with strange attractors also can be connected with tasks of signal processing, including an investigation and an application of all main properties of chaotic signals.

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