Implementation of regimes with strange attractors in attitude dynamics of multi-rotor spacecraft

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***Abstract*—The implementation of regimes with strange attractors in the attitude dynamics of multi-rotor spacecraft is considered from the point of view of controlling internal rotors relative rotations. This control technique allows to create the predefined strange chaotic attractor in the phase space of spacecraft, and to start the chaotic angular motion.**

Keywords—component, formatting, style, styling, insert (**key words**)

I. Introduction

As it was previously shown [1-8], the chaotic regimes can be presented in the dynamics of the angular motion of the spacecraft. Such chaotic regimes can correspond to arising in the phase space strange chaotic attractors. The strange attractors can be intentionally initiated into the attitude dynamics of spacecraft with internal rotors by the creation of external and internal torques having simple regular forms [3-5]; also, as was shown [8], it is conceptually possible in a case of angular motion of spacecraft with complex systems of vernier engines and toroidal magnetohydrodynamic elements.

Then in the framework of chaotic regimes initiation, the complex effects of the dynamical deterministic chaos are appeared, including the complex aperiodic chaotic oscillations of all dynamical parameters.

To expand the results of the previous research [3, 4] in this work the search of possibilities of creating strange chaotic attractors in the dynamics of multi-rotor spacecraft is fulfilled. In the previous works [3, 4] basing on the strange chaotic attractors of Newton-Leipnik [1], Wang-Sun [5], Chen-Lee [6], the appropriate control laws and all dynamical parameters of the multi-rotor spacecraft are found. Now it is important to construct the clear procedure with concrete steps to implement the strange attractors into the attitude dynamics of multi-spin spacecraft. These steps will be describe below. Basing on these steps the corresponding modeling of generation of strange attractors is also fulfilled in this paper.

II. Mathematical model

As it was described in [3, 4], the multi-rotor or, that is the same, the multi-spin spacecraft (MSSC) has the mechanical structure depicted at the fig.1 and the equations of the attitude motion:

$$\begin{cases} \hat{A}\dot{p} + \dot{D}_{12} + \left(\hat{C} - \hat{B}\right) qr + \left[qD_{56} - rD_{34}\right] = M_x^e; \\ \hat{B}\dot{q} + \dot{D}_{34} + \left(\hat{A} - \hat{C}\right) rp + \left[rD_{12} - pD_{56}\right] = M_y^e; \\ \hat{C}\dot{r} + \dot{D}_{56} + \left(\hat{B} - \hat{A}\right) pq + \left[pD_{34} - qD_{12}\right] = M_z^e; \end{cases} \tag{1}$$

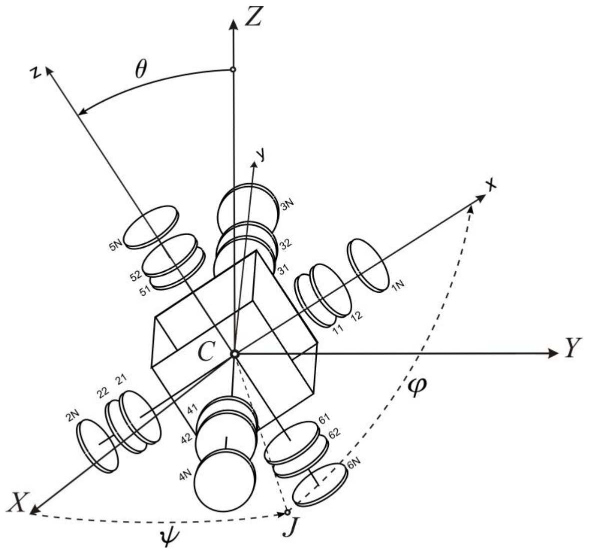


Fig.1. The mechanical structure of the MSSC.

$$\begin{cases} \dot{D}_{12} = M_{12}^i + M_{12}^e; \\ \dot{D}_{34} = M_{34}^i + M_{34}^e; \\ \dot{D}_{56} = M_{56}^i + M_{56}^e, \end{cases} \tag{2}$$

where **ω**=[p, q, r]$^{\mathrm{T}}$ – is the vector of the absolute angular velocity of the main body in the connected frame $Cxyz$, $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ – are the general inertia moments of the spacecraft main body, M_x^e, M_y^e M_z^e – are the external torques applied to the spacecraft main body, σ_{kl} is the relative angular velocities of rotors; I_l is the longitudinal and J_l is the equatorial inertia moments of the l-layer-rotor relatively the point O, N–is the quantity of layers of rotors, $M_{jlx}^e, M_{jly}^e, M_{jlz}^e$ are external torques acting only on the jl-th rotor, and M_{jl}^i is the torque from internal forces acting between the main body and the jl-th rotor, created by the internal electro-motors.Also the following expressions for the inertia parameters are actual:

$$\hat{A} = A - 2\sum_{j=1}^{N} I_j; \;\; \hat{B} = B - 2\sum_{j=1}^{N} I_j; \;\; \hat{C} = C - 2\sum_{j=1}^{N} I_j;$$

$$A = \tilde{A} + 4\bar{J} + 2\bar{I}; \;\; B = \tilde{B} + 4\bar{J} + 2\bar{I};$$

$$C = \tilde{C} + 4\bar{J} + 2\bar{I}; \;\; \bar{J} = \sum_{l=1}^{N} J_l; \;\; \bar{I} = \sum_{l=1}^{N} I_l$$

The rotors' summarized angular momentums are the following:

$$\begin{cases} D_{12} = \sum_{j=1}^{N} \left[\Delta_{1j} + \Delta_{2j} \right], \; D_{34} = \sum_{j=1}^{N} \left[\Delta_{3j} + \Delta_{4j} \right], \\ D_{56} = \sum_{j=1}^{N} \left[\Delta_{5j} + \Delta_{6j} \right]; \\ \Delta_{1j} = I_j \left(p + \sigma_{1j} \right); \; \Delta_{2j} = I_j \left(p + \sigma_{2j} \right); \\ \Delta_{3j} = I_j \left(q + \sigma_{3j} \right); \; \Delta_{4j} = I_j \left(q + \sigma_{4j} \right); \\ \Delta_{5j} = I_j \left(r + \sigma_{5j} \right); \; \Delta_{6j} = I_j \left(r + \sigma_{6j} \right). \end{cases} \quad (3)$$

The summarized internal (i) and external (e) torques applied to rotors are:

$$\begin{cases} M_{12}^i = \sum_{l=1}^{N} \left(M_{1l}^i + M_{2l}^i \right); \; M_{34}^i = \sum_{l=1}^{N} \left(M_{3l}^i + M_{4l}^i \right); \\ M_{56}^i = \sum_{l=1}^{N} \left(M_{5l}^i + M_{6l}^i \right); \\ M_{12}^e = \sum_{l=1}^{N} \left(M_{1lx}^e + M_{2lx}^e \right); \; M_{34}^e = \sum_{l=1}^{N} \left(M_{3ly}^e + M_{4ly}^e \right); \\ M_{56}^e = \sum_{l=1}^{N} \left(M_{5lz}^e + M_{6lz}^e \right). \end{cases} \quad (4)$$

To initiate the above noted strange chaotic attractors inside the dynamics of MSSC described by the equations (1) and (2) we will create controlling internal and external torques in the following forms:

$$M_{12}^i = \alpha_p \dot{p}; \qquad M_{34}^i = \beta_q \dot{q}; \qquad M_{56}^i = \gamma_r \dot{r}; \quad (5)$$

$$M_x^e = m_x + \alpha_1 p; \;\; M_y^e = m_y + \beta_1 q; \;\; M_z^e = m_z + \gamma_1 r, \quad (6)$$

with the following set of the controlling constant $\left\{ m_x, m_y, m_z, \alpha_1, \beta_1, \gamma_1, \alpha_p, \beta_q, \gamma_r \right\}$.

Then the summarized angular momentums of rotors take the form:

$$D_{12} = \alpha_p p + \alpha_0; \;\; D_{34} = \beta_q q + \beta_0; \;\; D_{56} = \gamma_r r + \gamma_0, \quad (7)$$

where $\alpha_0, \beta_0, \gamma_0$ – are also the controlling constants, defined by the initial conditions.

So, if find the appropriate values of the controlling constants, it is possible to initiate the strange chaotic attractors. The corresponded method to find such values is described in the paper [3]. E.g., to initiate the Wang-Sun strange chaotic attractor, the following values of the controlling constants were found [3]:

$$\begin{Bmatrix} m_x = 0.0079, \; m_y = 0.0182, \; m_z = 0; \\ \alpha_1 = 16.0277, \; \beta_1 = -32.3408, \; \gamma_1 = -0.0220; \\ \alpha_p = -3.6975, \; \beta_q = 16.3246, \; \gamma_r = -49.9780; \\ \alpha_0 = 0, \; \beta_0 = 0, \; \gamma_0 = -0.4247. \end{Bmatrix} \quad (8)$$

The values like (8) can be created by only one layer of rotors, and everywhere below we will consider the single-layer (N=1) structure of the MSSC, that allows to neglect the second indexes in the designations of relative angular velocities of rotors: $\sigma_{kl} = \sigma_k$. Now the question only is how we can implement the found constants in the framework of the natural dynamics of the MSSC.

III. The implementation of the controlling constants

To supply the predefined values of the controlling constants (e.g. the found values (8)), it is possible to suggest the following steps of the algorithm of the initiation of the motion of MSSC.

Firstly, the main rigid body of the MSSC must be preliminary stopped in its rotation relative inertial space, that corresponds to nullification of the angular velocity (p=q=r=0). This stopping can be fulfilled with the help of the creation of torques M_x^e, M_y^e M_z^e, formed by the main jet-engines of MSSC. After stopping the main body, we can take the initial condition of motion of the main body of MSSC as:

$$p(0) = p_0 = 0; \quad q(0) = q_0 = 0; \quad r(0) = r_0 = 0. \quad (9)$$

Secondly, we must stabilize the rest of the main body (p=q=r=0) by the main jet-engines, and must spin the rotors by the internal electro-motors $\left(M_j^i, \; j = 2,4,6 \right)$ up to the following relative angular velocities, which further we will consider as initial:

$$\begin{cases} \sigma_1(0) = \sigma_{10} = 0; \\ \sigma_3(0) = \sigma_{30} = 0; \\ \sigma_5(0) = \sigma_{50} = 0; \\ \sigma_2(0) = \sigma_{20} = \alpha_0; \\ \sigma_4(0) = \sigma_{40} = \beta_0; \\ \sigma_6(0) = \sigma_{60} = \gamma_0. \end{cases} \quad (10)$$

Thirdly, the following laws of the relative rotation of the rotors are realized with the help of internal electro-motors and sensors of the angular velocity of the main body:

$$\begin{cases} \sigma_1(t) = k_1 p(t); \\ \sigma_3(t) = k_3 q(t); \\ \sigma_5(t) = k_5 r(t); \\ \sigma_2(t) \equiv \alpha_0; \\ \sigma_4(t) \equiv \beta_0; \\ \sigma_6(t) \equiv \gamma_0. \end{cases} \quad (11)$$

where

$$k_1 = \frac{1}{I}\left(\alpha_p - 2I \right); \quad k_3 = \frac{1}{I}\left(\beta_q - 2I \right); \quad k_5 = \frac{1}{I}\left(\gamma_r - 2I \right) \quad (12)$$

The checking the suggested algorithm can be realized by the direct substitution of the expressions (9)-(12) into formulae (5)-(7). So, starting with the initial values (10) and (9), the laws of controlling the rotors relative velocities (11) allow to implement the time-dependencies of summarized angular momentums (7), and to create the external torque (6) formed by the main jet-engines of the MSSC. Then at the preliminary found controlling constants (e.g. (8)) the MSSC will fulfill such angular motion, that has in the 3D-phase space {p, q, r} the trajectory, which corresponds to strange chaotic attractor.

IV. Modelling the strange attractors arising

In the previous section, the conditions of the strange attractor initiation in the dynamical system (1) was found. The conditions (11) represent the control laws for relative angular velocities of internal rotors. So, if we control the rotation velocities of rotors according the laws (11), then the spacecraft will have the strange attractor in the phase space of components of the angular velocity of spacecraft.

For example, let us consider the angular motion of spacecraft with the inertia moments $\hat{A}=90,\ \hat{B}=70,\ \hat{C}=50,\ I=0.5\,[\mathrm{kg\cdot m^2}]$, at the zero initial conditions (9)-(11) and with the control constants (8). In this case, we obtain the phase trajectory depicted at the fig.2 – the trajectory corresponds to the Wang-Sun strange attractor, which prove the described above procedure.

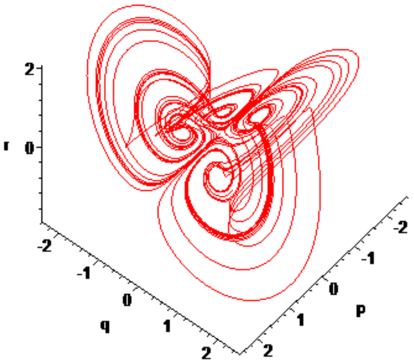


Fig. 2. The phase trajectory – Wang-Sun strange attractor.

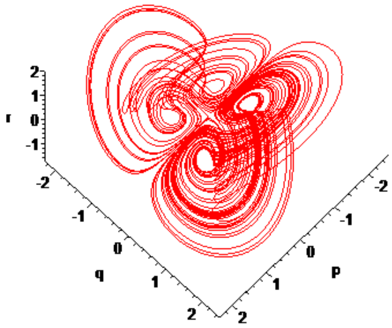
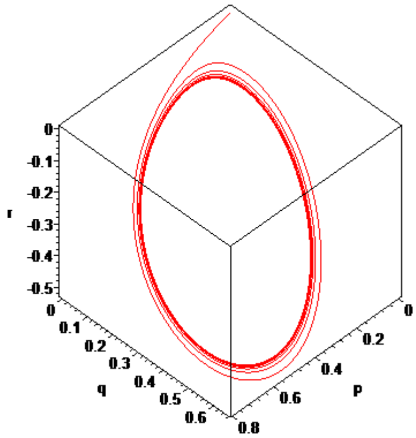


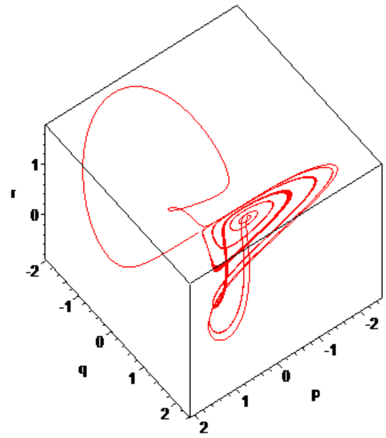
Fig. 3. The phase trajectory at perturbed initial conditions (p_0=0.3, q_0=0.3, r_0=0.3 rad/s).

Due to the structural stability we obtain the same attractor even at perturbations of the initial conditions (9), that is presented at fig.3 (p_0=0.3, q_0=0.3, r_0=0.3 rad/s). However, the perturbations in the initial conditions (10) of internal rotors can destroy the attractor, as depicted at fig.4: case (a) at $\sigma_4(t)\equiv\beta_0+0.01$; case (b) $\sigma_2(t)\equiv\alpha_0-0.01$; case (c) at $\sigma_2(t)\equiv\alpha_0+0.008,\ \sigma_4(t)\equiv\alpha_0+0.00015$; case (d) at $\sigma_2(t)\equiv\alpha_0+0.01,\ \sigma_4(t)\equiv\alpha_0+0.0005,\ \sigma_6(t)\equiv\alpha_0-0.01.$

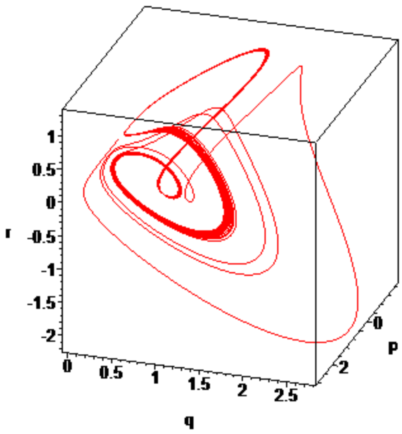
Moreover, the small perturbation in calculated values (10) can generate the new form of strange attractor, as it is presented at fig.5, where the Wang-Sun attractor changes its own form on another one: the generated attractor has two scrolls (fig.5-a) and (fig.5-b) one scroll, instead four scrolls in the initial case of Wang-Sun attractor. These new form of strange attractors are also structurally stable – it can be demonstrated by the plotting at perturbed initial conditions (9). As can we see from fig.6, forms of strange attractors presented at fig.5, are not changed. Only starting pieces of phase trajectories take place; and in process of time the phase trajectories come and lie down on corresponded strange attractors.



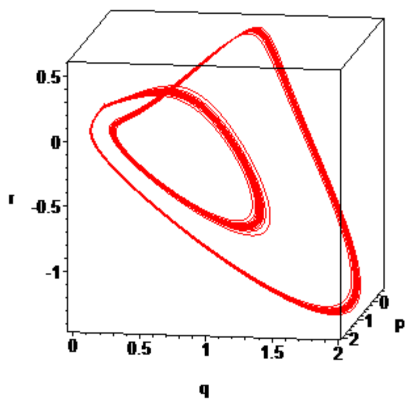
(a)



(b)

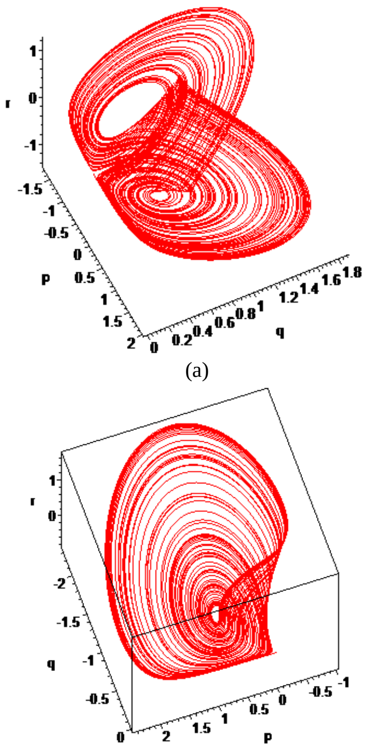


(c)



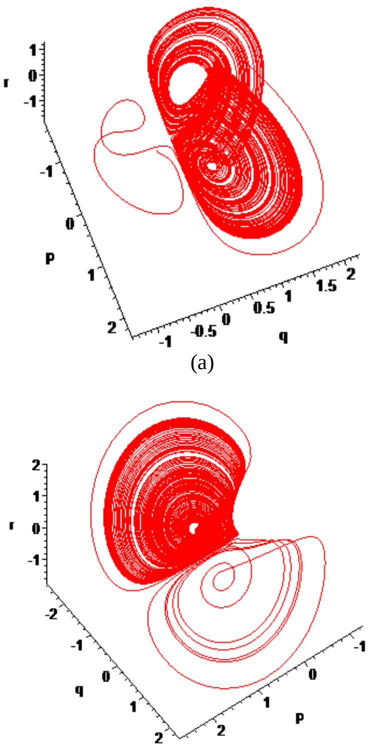
(d)

Fig. 4. The collapse of the Wang-Sun strange attractor.



(a) (b)

Figure 5. New forms of the strange attractor at changes in (10):
(a): $\sigma_2(t) \equiv \alpha_0 + 0.002, \quad \sigma_4(t) \equiv \alpha_0 + 0.0005, \quad \sigma_6(t) \equiv \alpha_0 - 0.01;$
(b): $\sigma_2(t) \equiv \alpha_0 + 0.01$



(a) (b)

Fig. 6. Structural stability of strange attractors.

At the fig.6 the structural stability of strange attractors (fig.5) is sown at perturbed initial conditions (9): Case (a): p_0=0.3, q_0= -0.3, r_0=0.3 rad/s; case (b): p_0=0.3, q_0= 0.3, r_0=0.3 rad/s.

So, the important question about the structural stability of the strange attractors at the changing the conditions (10) takes place. This question should be considered separately in the future research.

V. Conclusion

The initiation of the strange chaotic attractors in the phase space of the attitude dynamics of the multi-spin spacecraft was demonstrated in the paper. The fact of the existing of strange attractors in the motion of spacecraft is known, but, in this paper, the new algorithm of the implementation of strange chaotic attractors was described. The motion of spacecraft in this corresponds to chaotic regimes, which can be applied to solutions of non-trivial tasks of attitude control [e.g. 7].

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