INITIATIONS OF CHAOTIC MOTIONS AS A METHOD OF SPACECRAFT ATTITUDE CONTROL AND REORIENTATIONS

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The possibility of chaotic regimes intentional initiations in attitude dynamics of multi-spin spacecraft and gyrostat-satellites basing on the activation of the homo/heteroclinic chaos and/or strange chaotic attractors is shown. The new simple dynamical system with the new chaotic strange attractor is found basing on the main dynamics of spacecraft and the Newton-Leipnik system. Considered intentional initiations of chaotic regimes can be applied to the attitude reorientation of multi-spin spacecraft in cases of extreme dynamical situations including accidents and failures of main attitude control systems, and/or in cases of cancellations of uncontrolled rotations.

1. Introduction

This paper represents the expanded description of the method of chaotic attitude control/reorientation [1] of spacecraft with multi-rotors gyroscopic systems, including constructional schemes of dual-spin spacecraft (DSSC) and multi-spin spacecraft (MSSC), which also can be called as unbalanced gyrostat-satellites (GS). In an addition to the case of using the homo/heteroclinic chaos as the driver of attitude reorientations (that was considered in the previous paper [1]), this work describes the extension of this method basing on dynamical properties of strange chaotic attractors, which can be generated in a phase space of spacecraft dynamics.

The main feature of the chaotic attitude reorientation of the DSSC/MSSC in the inertial space is the initiation of the transient chaotic regime which allows to implement the attitude/angular motion with sufficiently large spatial angles (the Euler angles, or Tait–Bryan/Cardan angles, etc.). Generally speaking, to obtain the new spacecraft's attitude we should involve DSSC/MSSC into the transient chaotic motion, and after coming into the new area of the phase space we should deactivate the chaotic regime (by removing the disturbing internal torques/forces) and proceed to the regular motion of DSSC/MSSC with new attitude and dynamical parameters.

So, we can consider the initiated chaotic dynamical regime as "a switch" for the commutation of two different regular regimes of systems dynamics (at all, and, in particular, in the connection to attitude dynamics of spacecraft).

2. The spacecraft attitude reorientation/control based on the intentional initiation of heteroclinic chaos

2.1. *Mathematical models of the angular motion of DSSC*

As it was indicated in [1], the main dynamical system (the motion mathematical model) for the free DSSC (Figure 1) motion can be written in the form of dynamical and kinematical Euler equations:

$$\begin{cases} A\dot{p} + \left(C_2 - B\right)qr + q\Delta = 0; \quad B\dot{q} + \left(A - C_2\right)pr - p\Delta = 0; \\ C_2\dot{r} + \dot{\Delta} + \left(B - A\right)pq = 0; \quad \dot{\Delta} = M_\Delta; \end{cases} \tag{1}$$

$$\begin{cases} \dot{\theta} = p\cos\varphi - q\sin\varphi; \quad \dot{\psi} = (p\sin\varphi + q\cos\varphi)/\sin\theta; \\ \dot{\varphi} = r - \mathrm{ctg}\,\theta(p\sin\varphi + q\cos\varphi); \quad \dot{\delta} = \sigma, \end{cases} \tag{2}$$

where $\left[p,q,r\right]^T$ – are components of the absolute angular velocity of the platform-body (the body # 2) in the connected frame *Oxyz*, $\sigma = \dot{\delta}$ is the relative angular velocity of the rotor-body (the body # 1), and is δ the rotor's relative rotation angle; $\Delta = C_1\left(r + \sigma\right)$; $A = A_1 + A_2$, $B = A_1 + B_2$; $diag\left[A_1, A_1, C_1\right]$, $diag\left[A_2, B_2, C_2\right]$ – are the inertia tensors of the DSSC bodies in the own connected frames, M_Δ – the internal torque of the rotor-engine; also here the Euler angles are used: θ - the nutation, ψ - the precession, φ - the intrinsic rotation. If we direct the inertial axis *OZ* along the constant vector of the DSSC angular momentum **K**, then the well-known Serret-Andoyer variables can be linked with angular momentums' components and with Euler angles by the following manner:

$$L = C_2 r + \Delta; \; I_2 = \left|\mathbf{K}\right| = K; \;\; \cos\theta = L/K; \;\; l = \varphi \tag{3}$$

$$K_x = Ap = \sqrt{I_2^2 - L^2}\sin l; \;\; K_y = Bq = \sqrt{I_2^2 - L^2}\cos l; \;\; K_z = C_2 r + \Delta = L \tag{4}$$

where **K** is the DSSC angular momentum. Then, in the Serret-Andoyer variables we have the well-known Hamiltonian form:

$$\mathcal{H} = \mathcal{H}_0 + \varepsilon\mathcal{H}_1; \qquad \mathcal{H}_0 = \frac{I_2^2 - L^2}{2}\left[\frac{\sin^2 l}{A} + \frac{\cos^2 l}{B}\right] + \frac{1}{2}\left[\frac{\Delta^2}{C_1} + \frac{(L-\Delta)^2}{C_2}\right], \tag{5}$$

where $\mathcal{H}_0$ – the "generating" Hamiltonian part, $\mathcal{H}_1$ – a perturbed part of the Hamiltonian and ε – small dimensionless parameter corresponding to perturbations.

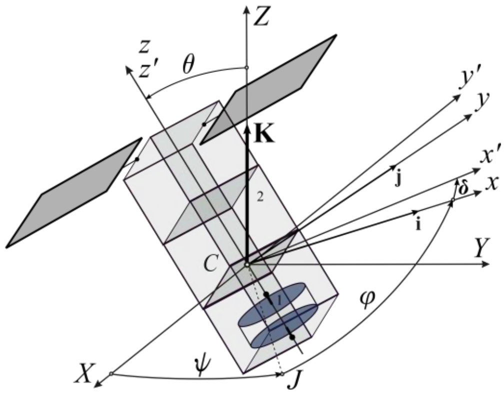

Figure 1.The dual-spin spacecraft and coordinates systems

Taking into account (5) we have the following equations for the positional part of the Serret-Andoyer coordinates (l, L):

$$\dot{L} = f_L(l,L) + \varepsilon g_L(t); \qquad \dot{l} = f_l(l,L) + \varepsilon g_l(t);$$

$$f_L(l,L) = -\frac{\partial\mathcal{H}_0}{\partial l} = \left(\frac{1}{B} - \frac{1}{A}\right)\left(I_2^2 - L^2\right)\sin l \cos l; \quad g_L = -\frac{\partial\mathcal{H}_1}{\partial l};$$

$$f_l(l,L) = \frac{\partial\mathcal{H}_0}{\partial L} = L\left[\frac{1}{C_2} - \frac{\sin^2 l}{A} - \frac{\cos^2 l}{B}\right] - \frac{\Delta}{C_2}; \quad g_l = \frac{\partial\mathcal{H}_1}{\partial L} \tag{6}$$

2.2. *The method of attitude reorientation based on the initiation of homo/heteroclinic chaos*

From the last equation (1) the formal exact solution follows:

$$\Delta(t) = \bar{\Delta} + \int_0^t M_\Delta dt \tag{7}$$

At absence of perturbations $\left(M_\Delta \equiv 0,\ \mathcal{H}_1 = 0\right)$ we will have the torques free motion with corresponding "generating" regular homo/heteroclinc solutions [2-4], which can be applied to the homo/heteroclinic chaotic regimes investigation. And, as it is known, the homo/heteroclinic chaos will arise in the DSSC dynamical system at the action of small harmonic internal torque $\left(M_\Delta = \varepsilon \cos \nu t\right)$ [2, 4] due to homo/heteroclinic separatrix splitting-intersecting; also as the effect of separatrix splitting-intersecting the "chaotic layer" in the system phase space is generated. Let us repeat [1] the case with initiating the following complex internal "perturbing" torque:

$$\begin{aligned} M_\Delta(t) = \left[u\left(t-T_1\right) - u\left(t-T_2\right)\right] M - \left[u\left(t-T_5\right) - u\left(t-T_6\right)\right] M + \\ +\varepsilon M \left[u\left(t-T_3\right) - u\left(t-T_4\right)\right] \sin\left(\nu\left[t-T_3\right]\right) \end{aligned} \tag{8}$$

where M=const and $u(\tau)$ is Heaviside's unit step function; T_1 and T_2 – are time-moments of initiating and stopping piecewise-constant spin-up torque (the aim of this time-interval is to involve the current dynamical regime into the separatrix/heteroclinic area, which is liable to heteroclinic chaos); T_3 and T_4 – are time-moments of initiating and stopping harmonic disturbing torque (the aim of this time-interval is to generate heteroclinic chaos); T_5 and T_6 – are time-moments of initiating and stopping piecewise-constant spin-down torque (the aim of this time-interval is to leave the separatrix/heteroclinic area, and to spin-down the rotor-body).

As it was shown in details in the previous work [1], the initiation of the positive chaotic regime can be implemented, which results in the spatial reorientation of the DSSC (Figure 2). The corresponding numerical simulation (Figure 2) was fulfilled at the following system/motion parameters, including the following inertia moments [kg·m^2]: A_1=5, C_1=4, A_2=15, B_2=8, C_2=6; the times-moments [s]: T_1=20, T_2=30, T_3=50, T_4=80, T_5=101, T_6=111; initial velocities [rad/s]: p_0=1.4, q_0=0.2, r_0=0.25; initial angles [rad]: θ_0=1.48, φ_0=1.57, ψ_0=0, at the following parameters of perturbations: M=1.5 [kg·m^2/s^2]; ν=1 [1/s] and dimensionless ε=1. So, the modeling results clearly show the possibility of the implementation of the suggested method of the DSSC reorientation.

In the next section we will develop the analogous method of the spacecraft chaotic reorientation basing on chaotic properties of strange chaotic attractors which also can be initiated in the phase space of MSSC [5, 6].

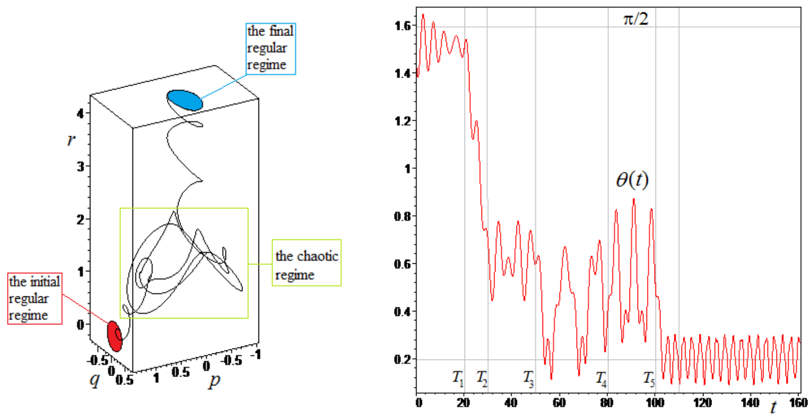

Figure 2.The numerical simulation results for the attitude reorientation of DSSC basing on dynamical properties of the heteroclinic chaos

3. The method of spacecraft attitude control/reorientations based on chaotic properties of strange chaotic attractors

It is well known fact that in phase spaces of dynamical systems on a par with regular phase trajectories many complex structures and objects can be presented, including perturbed split homo/heteroclinic bundles, fractal geometrical areas/basins, strange attractors and repellers. In this sense, certainly, dynamical systems for DSSC/MSSC/GS are not exceptions. The chaotic strange attractors in the phase space of MSSC angular (attitude) motion is e.g. shown in [5], were also the well-known dynamical systems (Lorenz, Sprott, Wang, Qi, Li, Chen, Lü, Liu, Čelikovský, Burke, Shaw, Arneodo, Coullet, etc.) are observed.

In this section we consider the intentional creation of strange chaotic attractors in the dynamics of MSSC/GS for further implementation of corresponding chaotic properties in purposes of attitude control/reorientation.

Here it is important to remind the mechanical structure and main properties of multi-spin spacecraft. The MSSC [5, 6] represents the multi-body (multi-rotor) constructional scheme with conjugated pairs of rotors placed on the inertia principle axes of the main body (Figure 3). General properties of the MSSC attitude dynamics are connected with the internal redistribution of the angular momentum between the system bodies (the main body, and rotors) due to the internal torques action. These properties of dynamics can be applied to the implementation of spatial (attitude/attitude) reorientations of the MSSC and also to the roll-walking motions of multi-rotor walking robots [5, 6].

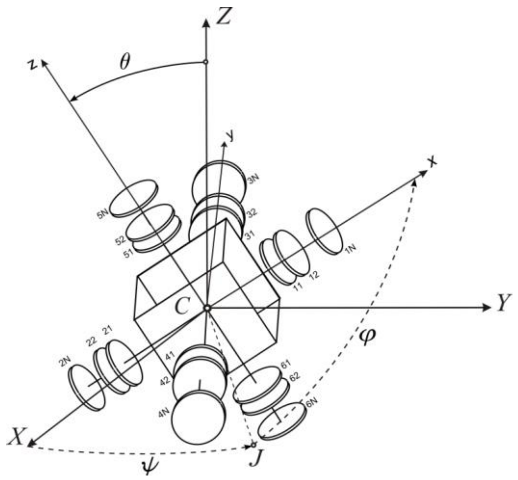

Figure 3. The MSSC and main coordinates systems

3.1. *Mathematical models of the angular motion of MSSC/GS*

Basing on the works [5, 6] we can present the main equations of the MSSC attitude dynamics as the dynamical equations for the multi-rotor system with 6*N* rotors (Figure 3) contained into *N* layers on six general directions coinciding with the principle axes of the main body. In these equations the following notations are used: $\boldsymbol{\omega}=[p, q, r]^{\mathrm{T}}$ – the vector of the absolute angular velocity of the main body (in projections on the connected frame *Cxyz*); $\tilde{A}, \tilde{B}, \tilde{C}$ are the general moments of inertia of the main body; $M_x^e, M_y^e\ M_z^e -$ are the external torques acting on the system (in general applied to the main body); σ_{kl} is the relative angular velocity of the *kl*-th rotor (relatively the main body); I_l and J_l are the longitudinal and the equatorial inertia moments of the *l*-layer-rotor relatively the point *O*; $M_{jlx}^e, M_{jly}^e, M_{jlz}^e$ are external torques acting only on the *jl*-th rotor, and M_{jl}^i is the torque from internal forces acting between the main body and the *jl*-th rotor (the internal engines torques). Then the indicated equations have the form [5, 6]:

$$\begin{cases} A\dot{p}+\sum_{l=1}^{N} I_l\left(\dot{\sigma}_{1l}+\dot{\sigma}_{2l}\right)+\left(C-B\right)qr+ \\ \qquad +\left[q\sum_{l=1}^{N} I_l\left(\sigma_{5l}+\sigma_{6l}\right)-r\sum_{l=1}^{N} I_l\left(\sigma_{3l}+\sigma_{4l}\right)\right]=M_x^e \\ B\dot{q}+\sum_{l=1}^{N} I_l\left(\dot{\sigma}_{3l}+\dot{\sigma}_{4l}\right)+\left(A-C\right)pr+ \\ \qquad +\left[r\sum_{l=1}^{N} I_l\left(\sigma_{1l}+\sigma_{2l}\right)-p\sum_{l=1}^{N} I_l\left(\sigma_{5l}+\sigma_{6l}\right)\right]=M_y^e \\ C\dot{r}+\sum_{l=1}^{N} I_l\left(\dot{\sigma}_{5l}+\dot{\sigma}_{6l}\right)+\left(B-A\right)qp+ \\ \qquad +\left[p\sum_{l=1}^{N} I_l\left(\sigma_{3l}+\sigma_{4l}\right)-q\sum_{l=1}^{N} I_l\left(\sigma_{1l}+\sigma_{2l}\right)\right]=M_z^e \end{cases} \tag{9}$$

with the addition of the relative motion equations of the rotors $\left(l=1..N\right)$:

$$\begin{cases} I_l\left(\dot{p}+\dot{\sigma}_{1l}\right)=M_{1l}^i+M_{1lx}^e;\ I_l\left(\dot{p}+\dot{\sigma}_{2l}\right)=M_{2l}^i+M_{2lx}^e; \\ I_l\left(\dot{q}+\dot{\sigma}_{3l}\right)=M_{3l}^i+M_{3ly}^e;\ I_l\left(\dot{q}+\dot{\sigma}_{4l}\right)=M_{4l}^i+M_{4ly}^e; \\ I_l\left(\dot{r}+\dot{\sigma}_{5l}\right)=M_{5l}^i+M_{5lz}^e;\ I_l\left(\dot{r}+\dot{\sigma}_{6l}\right)=M_{6l}^i+M_{6lz}^e \end{cases} \tag{10}$$

Also presented equations can be rewritten in the unbalanced-gyrostat-form:

$$\begin{cases} \hat{A}\dot{p}+\dot{D}_{12}+\left(\hat{C}-\hat{B}\right)qr+\left[qD_{56}-rD_{34}\right]=M_x^e; \\ \hat{B}\dot{q}+\dot{D}_{34}+\left(\hat{A}-\hat{C}\right)rp+\left[rD_{12}-pD_{56}\right]=M_y^e; \\ \hat{C}\dot{r}+\dot{D}_{56}+\left(\hat{B}-\hat{A}\right)pq+\left[pD_{34}-qD_{12}\right]=M_z^e; \end{cases} \tag{11}$$

$$\dot{D}_{12}=M_{12}^i+M_{12}^e;\quad \dot{D}_{34}=M_{34}^i+M_{34}^e;\quad \dot{D}_{56}=M_{56}^i+M_{56}^e, \tag{12}$$

where

$$\hat{A}=A-2\sum_{j=1}^{N} I_j;\quad \hat{B}=B-2\sum_{j=1}^{N} I_j;\quad \hat{C}=C-2\sum_{j=1}^{N} I_j;$$

$$A=\tilde{A}+4\overline{J}+2\overline{I};\ \ B=\tilde{B}+4\overline{J}+2\overline{I};\ \ C=\tilde{C}+4\overline{J}+2\overline{I};\ \ \overline{J}=\sum_{l=1}^{N} J_l;\ \ \overline{I}=\sum_{l=1}^{N} I_l;$$

$$\begin{cases} D_{12} = \sum_{j=1}^{N} \left[\Delta_{1j} + \Delta_{2j} \right]; \; D_{34} = \sum_{j=1}^{N} \left[\Delta_{3j} + \Delta_{4j} \right]; \; D_{56} = \sum_{j=1}^{N} \left[\Delta_{5j} + \Delta_{6j} \right]; \\ \Delta_{1j} = I_j \left(p + \sigma_{1j} \right); \;\; \Delta_{2j} = I_j \left(p + \sigma_{2j} \right); \; \Delta_{3j} = I_j \left(q + \sigma_{3j} \right); \\ \Delta_{4j} = I_j \left(q + \sigma_{4j} \right); \;\; \Delta_{5j} = I_j \left(r + \sigma_{5j} \right); \;\; \Delta_{6j} = I_j \left(r + \sigma_{6j} \right). \end{cases} \tag{13}$$

The summarized rotors' internal (*i*) and external (*e*) torques are:

$$\begin{cases} M_{12}^{i} = \sum_{l=1}^{N} \left(M_{1l}^{i} + M_{2l}^{i} \right); \quad M_{34}^{i} = \sum_{l=1}^{N} \left(M_{3l}^{i} + M_{4l}^{i} \right); \quad M_{56}^{i} = \sum_{l=1}^{N} \left(M_{5l}^{i} + M_{6l}^{i} \right); \\ M_{12}^{e} = \sum_{l=1}^{N} \left(M_{1lx}^{e} + M_{2lx}^{e} \right); \; M_{34}^{e} = \sum_{l=1}^{N} \left(M_{3ly}^{e} + M_{4ly}^{e} \right); \; M_{56}^{e} = \sum_{l=1}^{N} \left(M_{5lz}^{e} + M_{6lz}^{e} \right). \end{cases} \tag{14}$$

Also for the kinematical description of the attitude motion of the MSSC main body we have to add the Euler kinematical equations (2). Let us as in the work [6] consider the case of the MSSC controlled attitude motion at the creation of the artificial torques:

$$M_{12}^{i} = \alpha_p \dot{p}; \qquad M_{34}^{i} = \beta_q \dot{q}; \qquad M_{56}^{i} = \gamma_r \dot{r}; \tag{15}$$

$$M_x^e = m_x + \alpha_1 p; \;\; M_y^e = m_y + \beta_1 q; \;\; M_z^e = m_z + \gamma_1 r, \tag{16}$$

with the constant controlling terms/coefficients:

$$\left\{ m_x, \, m_y, \, m_z, \, \alpha_1, \, \beta_1, \, \gamma_1, \, \alpha_p, \, \beta_q, \, \gamma_r \right\} \sim \text{const}$$

Hear we can underline the linear structure of internal control torques proportional to the angular accelerations of the main MSSC-body (15) (these torques are formed by the internal rotors' engines); also the linear structure have external control torques applied to the main body (16) which can be formed by thrusters (e.g. this torques can be formed by electrically powered propulsion systems – this type of engines is characterized by the extremely low consumption of the working body (usually compressed gases and/or plasma), that corresponds to the practically constant mass system). Taking into account equations (12) and torques (15) we can obtain analytical forms for the summarized rotors' angular momentums:

$$D_{12} = \alpha_p p + \alpha_0; \;\; D_{34} = \beta_q q + \beta_0; \;\; D_{56} = \gamma_r r + \gamma_0, \tag{17}$$

where $\alpha_0, \beta_0, \gamma_0$ – are the constants following from initial conditions.

3.2. *The possible synthesis of the MSSC parameters delivering the dynamics along strange chaotic attractors*

As it was indicated in the work [7] the natural candidates for the construction of dynamical systems with multi-scroll chaotic attractors are 3D quadratic continuous time systems given by equations

$$\begin{cases} \dot{x} = a_0 + a_1 x + a_2 y + a_3 z + a_4 x^2 + a_5 y^2 + a_6 z^2 + a_7 xy + a_8 xz + a_9 yz; \\ \dot{y} = b_0 + b_1 x + b_2 y + b_3 z + b_4 x^2 + b_5 y^2 + b_6 z^2 + b_7 xy + b_8 xz + b_9 yz; \\ \dot{z} = c_0 + c_1 x + c_2 y + c_3 z + c_4 x^2 + c_5 y^2 + c_6 z^2 + c_7 xy + c_8 xz + c_9 yz; \end{cases} \tag{18}$$

where $\{a_i, b_i, c_i\} \in \mathbb{R}^{30}$ are the constant parameters.

Basing on expressions (17) we can solve the linear algebraic equations (11) relatively $\{\dot{p}, \dot{q}, \dot{r}\}$; and implying the resignation $(p \leftrightarrow x;\ q \leftrightarrow y;\ r \leftrightarrow z)$ it is possible to write the following correspondences for the system (18) coefficients $\{a_i, b_i, c_i\}$ and the MSSC parameters:

$$\begin{cases} a_0 = \dfrac{m_x}{\hat{A} + \alpha_p};\ \ b_0 = \dfrac{m_y}{\hat{B} + \beta_q};\ \ c_0 = \dfrac{m_z}{\hat{C} + \gamma_r}; \\ a_1 = \dfrac{\alpha_1}{\hat{A} + \alpha_p};\ \ b_1 = \dfrac{\gamma_0}{\hat{B} + \beta_q};\ \ c_1 = \dfrac{-\beta_0}{\hat{C} + \gamma_r}; \\ a_2 = \dfrac{-\gamma_0}{\hat{A} + \alpha_p};\ \ b_2 = \dfrac{\beta_1}{\hat{B} + \beta_q};\ \ c_2 = \dfrac{\alpha_0}{\hat{C} + \gamma_r}; \\ a_3 = \dfrac{\beta_0}{\hat{A} + \alpha_p};\ \ b_3 = \dfrac{-\alpha_0}{\hat{B} + \beta_q};\ \ c_3 = \dfrac{\gamma_1}{\hat{C} + \gamma_r}; \\ a_4 = a_5 = a_6 = b_4 = b_5 = b_6 = c_4 = c_5 = c_6 \equiv 0; \\ a_7 = 0;\ \ b_7 = 0;\ \ c_7 = \left(\hat{A} - \hat{B} - \beta_q + \alpha_p\right) \Big/ \left(\hat{C} + \gamma_r\right); \\ a_8 = 0;\ \ b_8 = \left(\hat{C} - \hat{A} - \alpha_p + \gamma_r\right) \Big/ \left(\hat{B} + \beta_q\right);\ c_8 = 0; \\ a_9 = \left(\hat{B} - \hat{C} - \gamma_r + \beta_q\right) \Big/ \left(\hat{A} + \alpha_p\right);\ \ b_9 = 0;\ \ c_9 = 0. \end{cases} \tag{19}$$

The obtained correspondences (19) can be used for the synthesis of the control constants, which provide the coincidence of the MSSC dynamical system with a dynamical system containing a strange chaotic attractor. In other words, we have to find such values of constants

$$Control = \left\{ \alpha_p, \alpha_0, m_x, \alpha_1, \beta_q, \beta_0, m_y, \beta_1, \gamma_r, \gamma_0, m_z, \gamma_1 \right\} \in \mathbb{R}^{12} \tag{20}$$

which deliver appropriate values of dynamical systems with strange chaotic attractors (at the concretized numerical values, certainly):

$$Coeff = \left\{ a_i, b_i, c_i \right\}_{0 \le i \le 9} \in \mathbb{R}^{30} \tag{21}$$

The correspondences (19) unfortunately cannot be considered as correct compatible linear algebraic equations system relative "unknown" parameters (20). Also from expressions (19) the form of possible dynamical systems with strange chaotic attractors follows: we have to find the concretized cases of systems (18) with null-coefficients from the set

$$Zeros = \left\{ \begin{matrix} a_4 = a_5 = a_6 = b_4 = b_5 = b_6 = c_4 = c_5 = c_6 = 0; \\ a_7 = 0;\ b_7 = 0;\ a_8 = 0;\ c_8 = 0;\ b_9 = 0;\ c_9 = 0 \end{matrix} \right\} \tag{22}$$

To fulfill the last conditions (22) we must limit oneself in using only the "natural gyroscopic systems" (e.g. in the work [6] the well-known systems with the Wang-Sun four-scroll chaotic attractor and with the Chen-Lee two-scroll chaotic attractor were presented as systems, which are correspond to conditions (22)). The indicated class of "natural gyroscopic systems" includes, certainly, dynamical systems of the angular motion of usual rigid-bodies systems, DSSC, MSSC and GS.

Thus, our task now reduces to find the concrete control "unknown" parameters (20), which can generate one of possible strange chaotic attractors in the phase space of the MSSC motion. As it was realized in the previous work [6], we can select the one of known dynamical system with concrete coefficients (21) with strange chaotic attractor (it can be call as the "basic" system), and find the parameters (20) which provides equalities (19) exactly or approximately. Due to the incompatibility of systems (19) relatively the MSSC parameters (20) we can suggest to use the well-known first-order optimization algorithm to find a local minimum of a connected function using gradient descent procedure. And then for solving our task we will use the gradient descent procedure for the following quadratic function, where constants a_i, b_i, c_i correspond to concrete numerical values of the selected "basic" dynamical system:

$$\Psi\left(\alpha_p,\alpha_0,m_x,\alpha_1,\beta_q,\beta_0,m_y,\beta_1,\gamma_r,\gamma_0,m_z,\gamma_1\right)=$$
$$=\left(a_0-\frac{m_x}{\hat{A}+\alpha_p}\right)^2+\left(b_0-\frac{m_y}{\hat{B}+\beta_q}\right)^2+\left(c_0-\frac{m_z}{\hat{C}+\gamma_r}\right)^2+\left(a_1-\frac{\alpha_1}{\hat{A}+\alpha_p}\right)^2+$$
$$+\left(b_1-\frac{\gamma_0}{\hat{B}+\beta_q}\right)^2+\left(c_1+\frac{\beta_0}{\hat{C}+\gamma_r}\right)^2+\left(a_2+\frac{\gamma_0}{\hat{A}+\alpha_p}\right)^2+\left(b_2-\frac{\beta_1}{\hat{B}+\beta_q}\right)^2+$$
$$+\left(c_2-\frac{\alpha_0}{\hat{C}+\gamma_r}\right)^2+\left(a_3-\frac{\beta_0}{\hat{A}+\alpha_p}\right)^2+\left(b_3+\frac{\alpha_0}{\hat{B}+\beta_q}\right)^2+\left(c_3-\frac{\gamma_1}{\hat{C}+\gamma_r}\right)^2+$$
$$+\left(c_7-\left(\hat{A}-\hat{B}-\beta_q+\alpha_p\right)\Big/\left(\hat{C}+\gamma_r\right)\right)^2+\left(b_8-\left(\hat{C}-\hat{A}-\alpha_p+\gamma_r\right)\Big/\left(\hat{B}+\beta_q\right)\right)^2+$$
$$+\left(a_9-\left(\hat{B}-\hat{C}-\gamma_r+\beta_q\right)\Big/\left(\hat{A}+\alpha_p\right)\right)^2 \tag{23}$$

As it quite understandable, if the function (23) have the local zero-minimum, then the parameters of this zero-minimum will provide the equalities (19), that in its turn will correspond to the natural transition of the MSSC-equations to the “basic” dynamical system with strange attractors. Finding this convergent (within the limits of accuracy) zero-minimum is quite complicated task, and, moreover the indicated optimization algorithm can result in divergent iterations; and therefore not every selected “basic” dynamical system can give the convergent zero-minimum-solution for the MSSC control parameters (20). Also very important and useful is to find the possibility of searching “approximate” near-zero-minimums of the function (23) instead the “exact” zero-minimum. Such near-zero-minimums can give quite appropriate results in the sense of obtaining dynamical systems with strange attractors.

So, the indicated gradient descent procedure is executed basing on following iterations:

$$while\ \left|\sqrt{\Psi\left(X_i\right)}\right|>\varepsilon:\ \ X_{i+1}=X_i-h\cdot\nabla\Psi\left(X_i\right); \tag{24}$$

where $X_0 \in Control$ is the initial approximation of finding parameters, and ε is the tolerance. And, as we mentioned above, we do not guarantee the local convergence of iterations (24) to the zero-minimum.

3.3. *Initiating of new strange chaotic attractors in the MSSC phase space*

Now we can use the method from the previous section to find the MSSC coefficients providing the generation of the "basic" strange chaotic attractor of the "basic" dynamical system, and/or to find the MSSC coefficients delivering a new "close to the basic" strange chaotic attractor.

The calculation of the MSSC coefficients in cases of "basic" systems Wang-Sun and Chen-Lee was in details considered as bright example in the work [6]; these cases of "basic" systems allowed to achieve the zero-minimums at the fulfillment of the gradient descent procedure and to initiate in the phase space of MSSC the corresponding four-scroll Wang-Sun attractor and two-scroll Chen-Lee attractor. Therefore more important is to show results of generating coefficients for a new system with a new strange chaotic attractor – this case will be implemented at the converging of the gradient descent procedure to a non-zero-minimum, or even at the non-convergent algorithm.

Let us consider the case of the calculation of MSSC parameters with using the Newton-Leipnik dynamical system [8] as the "basic" system. Then, writing the Newton-Leipnik system in the form (18) we take the following set *NL* of non-zero numerical coefficients (21):

$$NL = \left\{ a_1 = -0.4;\ a_2 = 1;\ a_9 = 10;\ b_1 = -1;\ b_2 = -0.4;\ b_8 = 5;\ c3 = 0.175;\ c7 = -5 \right\}$$

As MSSC inertia moments values let us take $\hat{A} = 90,\ \ \hat{B} = 70,\ \ \hat{C} = 50[\mathrm{kg}\cdot\mathrm{m}^2]$. In the considered case the iterations algorithm (24) converges to the following non-zero local point $\Psi\left(X_{final}\right) = 4.9783$ at $\varepsilon = 0.0045$:

$$X_{final} = \left\{ \begin{matrix} \alpha_p = \text{-}80.6893;\ \alpha_0 = 0;\ \ m_x = 0;\ \ \alpha_1 = \text{-}3.7243; \\ \beta_q = 45.7309;\ \ \beta_0 = 0;\ \ \ m_y = 0;\ \ \beta_1 = \text{-}46.2952; \\ \gamma_r = \text{-}27.7522;\ \gamma_0 = \text{-}9.9951;\ m_z = 0;\ \gamma_1 = 3.8934 \end{matrix} \right\} \tag{25}$$

As the result we formally obtain the new dynamical system of the MSSC attitude motion (11) with summarized rotors' angular momentums (17) and torques (16), (15) at the defined numerical values of parameters (25). This new dynamical system in the form (18) can be written as follows

$$\begin{cases} \dot{x} = -0.4x + 1.0735y + 10.0403yz; \\ \dot{y} = -0.0864x - 0.4y + 0.1118xz; \\ \dot{z} = 0.1750z - 4.7834xy \end{cases} \tag{26}$$

The system (26) has the new two-scroll chaotic attractor (Figure 4).

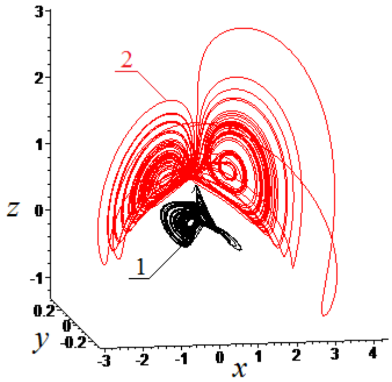

Figure 4. Chaotic attractors: 1 – the Newton-Leipnik attractor; 2 – the new attractor

So, we have to finally say, the new detected above chaotic attractor can be initiated in the attitude dynamics of MSSC. The initiation of this chaotic attractor (as well as other chaotic attractors [6]) allows to implement the chaotic reorientation of MSSC – the corresponding angular motion time-history is presented at the Figure 5.

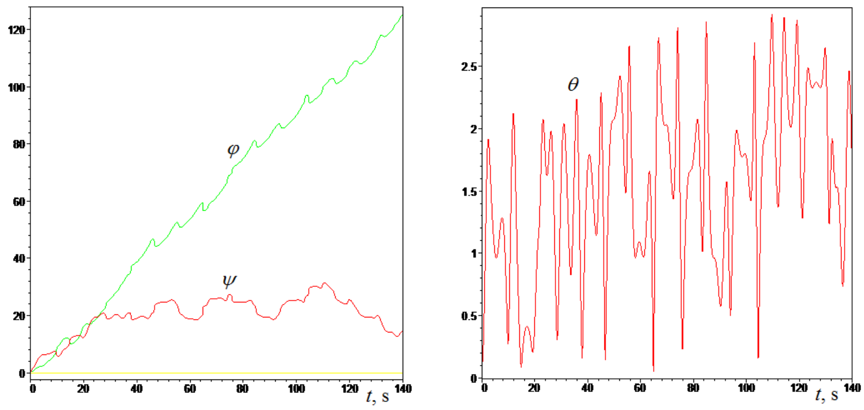

Figure 5. The time-history of Euler angles corresponding to the chaotic attractor of the system (26)

4. Conclusion

The possibility of chaotic regimes intentional initiations in attitude dynamics of multi-spin spacecraft and gyrostat-satellites basing on the activation of the homo/heteroclinic chaos and/or strange chaotic attractors was shown. Moreover, in this work the new simple dynamical system with the new chaotic strange attractor was found basing on the main dynamics of MSSC and the Newton-

Leipnik system. Considered intentional initiations of chaotic regimes can be applied to the attitude reorientation of DSSC/MSSC/GS in cases of extreme dynamical situations including accidents and failures of main attitude control systems, and/or in cases of cancellations of uncontrolled rotations. Also in these chaotic regimes it is possible to fulfill tasks of a fast random observation of surroundings, task of a random search of objects.

Acknowledgments

This work is partially supported by the Russian Foundation for Basic Research (RFBR#15-08-05934-A), and by the Ministry education and science of the Russian Federation in the framework of the design part of the State Assignments to higher education institutions and research organizations in the field of scientific activity (project #9.1004.2014/K).

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