# Gravitational Dampers for Unloading Angular Momentum of Nanosatellites

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## 1 Introduction

As it is well known, the problem of spacecraft attitude control implies the suppression of large values of the SC angular velocity after separation from the last stage of the space-rocket or unloading saturated reaction/momentum wheels. This assumes the large value of the angular momentum of the system (of the main body spacecraft and/or rotors-wheels) and its discharging. Therefore, the task of unloading the angular momentum is one of primary tasks of spacecraft attitude dynamics.

The angular momentum of reaction/momentum wheels can be transferred to the main spacecraft body using internal interaction [1, 2], and after this translation the angular momentum value can be decreased with the help of interaction with the external forces [3–13], e.g., central gravitational forces, which are acting on spacecraft moving along the orbit.

Due to the big importance of modern space missions with nanosatellites applying, it is very important to develop the simplest constructional schemes to the angular momentum unloading, which can be used basing on nanosatellites platforms.

In this work, the scheme of the gravitational unloading is proposed. This scheme (Fig. 1a) uses the internal body with different general inertia moments (Fig. 1c) placed in the spherical shell floating in the spherical cavity with viscous liquid (Fig. 1b). It is clear that at the motion along the orbit this internal body tries to rotate and to place the gravity-oriented spatial position, due to the properties of the gravitational stabilization principles. Therefore, the gravity forces initiate the internal angular motion of the internal sphere relative the cavity with viscous liquid.

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At this internal rotation, the dissipative friction torques arise in the viscous fluid. This friction torque dissipates the kinetic energy and it acts on the main body of satellite and decelerates its angular motion. So, as the result, the angular momentum of satellite will decrease. The suggested spherical damper scheme is similar with the analogous construction of magnetic damper [3] interacting with geomagnetic field, where the internal spherical shell with permanent magnets was placed in the external sphere filled with bismuth: the internal sphere tries to rotate to coincide with the forces lines of geomagnetic field, and, therefore, this rotation relative the main body creates the dissipative torque due to the friction in bismuth.

This scheme allows to use this gravitational damper in cases of nanosatellites, especially if the nanosatellite has a symmetrical construction with three units (Fig. 2), one of which (e.g., central unit) contains this spherical damper.

## 2 Mathematical Model of the Attitude Motion

So, let us consider the orbital motion of the nanosatellite along the circle orbit: the system *CXYZ* is orbital coordinates frame (Figs. 1 and 2), where the axis *Z*

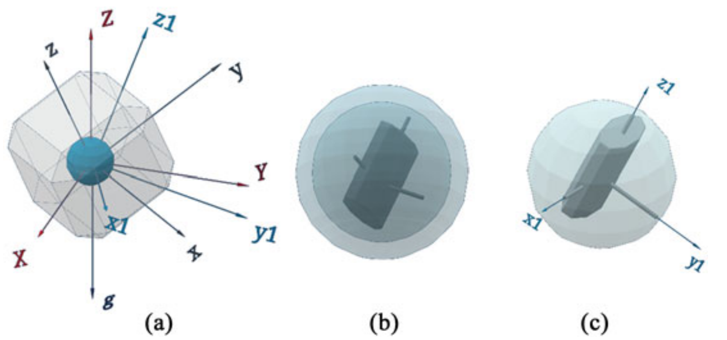
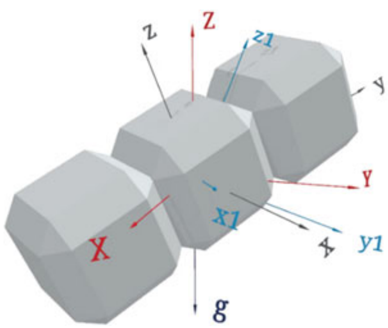



**Fig. 1** A unit of a satellite with the internal spherical cavity (**a**) filled by a viscous fluid and the internal floating sphere (**b**) with the mounted gravitational body-damper (**c**)

**Fig. 2** The nanosatellite with the central unit with the gravitational damper and corresponding coordinates systems



is directed from the gravity center to the orbital position of nanosatellite, axis $Y$ is orthogonal to the orbital plane, and $X$ represents the third right axis. The coordinates system $Cxyz$ is the central frame connected to the main body of nanosatellite, coinciding with its general axes of inertia; the system $Cx_1y_1z_1$ is the central frame connected to the general axes of the body-damper. In the case of nanosatellite construction with central damper-body unit (Fig. 2) let us to consider that the orbital system $CXYZ$ and connected systems $Cxyz$ and $Cx_1y_1z_1$ are central, i.e., the origin of all indicated systems is common, and it coincides with the center of mass of the satellite $C$.

The attitude position of the coordinates systems can be described by the well-known Euler's angles. For the system $Cxyz$ we will use three angles $\{\theta_1, \theta_2, \theta_3\}$ of subsequent rotations about the corresponding axes $x \to y \to z$ starting from the full coinciding with the system $CXYZ$. Then the following matrixes for subsequent rotations take place:

$$\Theta_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & \sin\theta_1 \\ 0 & -\sin\theta_1 & \cos\theta_1 \end{bmatrix}; \Theta_2 = \begin{bmatrix} \cos\theta_2 & 0 & -\sin\theta_2 \\ 0 & 1 & 0 \\ \sin\theta_2 & 0 & \cos\theta_2 \end{bmatrix}; \Theta_3 = \begin{bmatrix} \cos\theta_3 & \sin\theta_3 & 0 \\ -\sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1}$$

The complete matrix of transition from the orbital system $CXYZ$ to the connected system $Cxyz$ has form:

$$\Theta = \Theta_3 \cdot \Theta_2 \cdot \Theta_1 = \begin{bmatrix} \cos\theta_3\cos\theta_2 & \sin\theta_3\cos\theta_1 + \cos\theta_3\sin\theta_2\sin\theta_1 & \sin\theta_3\sin\theta_1 - \cos\theta_3\sin\theta_2\cos\theta_1 \\ -\sin\theta_3\cos\theta_2 & \cos\theta_3\cos\theta_1 - \sin\theta_3\sin\theta_2\sin\theta_1 & \cos\theta_3\sin\theta_1 + \sin\theta_3\sin\theta_2\cos\theta_1 \\ \sin\theta_2 & -\cos\theta_2\sin\theta_1 & \cos\theta_2\cos\theta_1 \end{bmatrix} \tag{2}$$

By the full analogy the system $Cx_1y_1z_1$ can be translated from the orbital system $CXYZ$ to the concrete final attitude with the help of subsequent rotations by angles $\{\psi_1, \psi_2, \psi_3\}$:

$$\Psi = \Psi_3 \cdot \Psi_2 \cdot \Psi_1 = \begin{bmatrix} \cos\psi_3\cos\psi_2 & \sin\psi_3\cos\psi_1 + \cos\psi_3\sin\psi_2\sin\psi_1 & \sin\psi_3\sin\psi_1 - \cos\psi_3\sin\psi_2\cos\psi_1 \\ -\sin\psi_3\cos\psi_2 & \cos\psi_3\cos\psi_1 - \sin\psi_3\sin\psi_2\sin\psi_1 & \cos\psi_3\sin\psi_1 + \sin\psi_3\sin\psi_2\cos\psi_1 \\ \sin\psi_2 & -\cos\psi_2\sin\psi_1 & \cos\psi_2\cos\psi_1 \end{bmatrix} \tag{3}$$

The kinematical equations for the angular velocity components of the main body $\boldsymbol{\omega} = [p, q, r]^T$ and of the damper $\boldsymbol{\omega}' = [p', q', r']^T$ in projections onto its own connected coordinates systems ($xyz$ and $x_1y_1z_1$, correspondently) have the shape:

$$\begin{cases} p = \dot{\theta}_1\cos\theta_2\cos\theta_3 + \dot{\theta}_2\sin\theta_3 + \omega_0\Theta_{12} \\ q = -\dot{\theta}_1\cos\theta_2\sin\theta_3 + \dot{\theta}_2\cos\theta_3 + \omega_0\Theta_{22} \\ r = \dot{\theta}_1\sin\theta_2 + \dot{\theta}_3 + \omega_0\Theta_{32} \end{cases} \tag{4}$$

$$\begin{cases} p' = \dot{\psi}_1 \cos\psi_2 \cos\psi_3 + \dot{\psi}_2 \sin\psi_3 + \omega_0 \Psi_{12} \\ q' = -\dot{\psi}_1 \cos\psi_2 \sin\psi_3 + \dot{\psi}_2 \cos\psi_3 + \omega_0 \Psi_{22} \\ r' = \dot{\psi}_1 \sin\psi_2 + \dot{\psi}_3 + \omega_0 \Psi_{32} \end{cases} \tag{5}$$

where $\omega_0$ is the value of the orbital angular velocity.

Let us consider the case when the inertia tensor of the main body of the satellite (without the damper) has in the connected system $Cxyz$ the central general diagonal form $\mathbf{J} = \mathit{diag}\ (A, B, C)$, and the inertia tensor of the damper-body also has the central general diagonal form $\mathbf{J}' = \mathit{diag}\ (A', B', C')$ in its connected frame $Cx_1y_1z_1$.

The dynamical equations of the attitude dynamics on the circle orbit can be written for the satellite main body and for damper-body as follows [14]:

$$\begin{cases} A\dot{p} + (C - B)\, qr = 3\omega_0^2\, (C - B)\, \Theta_{23}\Theta_{33} + M_x; \\ B\dot{q} + (A - C)\, pr = 3\omega_0^2\, (A - C)\, \Theta_{33}\Theta_{13} + M_y; \\ C\dot{r} + (B - A)\, pq = 3\omega_0^2\, (B - A)\, \Theta_{13}\Theta_{23} + M_z \end{cases} \tag{6}$$

$$\begin{cases} A'\dot{p}' + \left(C' - B'\right) q'r' = 3\omega_0^2 \left(C' - B'\right) \Psi_{23}\Psi_{33} + M'_x; \\ B'\dot{q}' + \left(A' - C'\right) p'r' = 3\omega_0^2 \left(A' - C'\right) \Psi_{33}\Psi_{13} + M'_y; \\ C'\dot{r}' + \left(B' - A'\right) p'q' = 3\omega_0^2 \left(B' - A'\right) \Psi_{13}\Psi_{23} + M'_z \end{cases} \tag{7}$$

where $\{\Theta_{13}, \Theta_{23}, \Theta_{33}\}$, $\{\Psi_{13}, \Psi_{23}, \Psi_{33}\}$– are components of matrixes (2) and (3) corresponding to the directional cosines of the gravitation direction (i.e., the orbital axis $Z$) in the connected frames. The vector $\mathbf{M} = [M_x, M_y, M_z]^T$ is the torque acting on the main body from the side of the damper-body due to liquid friction between the internal and internal spheres (Fig. 1). The vector $\mathbf{M}' = [M'_x, M'_y, M'_z]^T$ is the analogues torque acting on the damper-body from the side of the main body due to liquid friction.

The interaction of the bodies of the satellite due to liquid friction can be defined by the relative angular velocity of the damper (relative the main body). Then in projections onto the connected axes, the torques acting on the main body and on the damper are equal to the following vectors components:

$$\mathbf{M} = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = -\nu \left[ \begin{bmatrix} p \\ q \\ r \end{bmatrix} - \Theta \cdot \Psi^{-1} \cdot \begin{bmatrix} p' \\ q' \\ r' \end{bmatrix} \right];$$

$$\mathbf{M}' = \begin{bmatrix} M'_x \\ M'_y \\ M'_z \end{bmatrix} = -\nu \left[ \begin{bmatrix} p' \\ q' \\ r' \end{bmatrix} - \Psi \cdot \Theta^{-1} \cdot \begin{bmatrix} p \\ q \\ r \end{bmatrix} \right] \tag{8}$$

where $\nu$ is the damping factor.

It is useful to add the kinematical equations for angles $\{\theta_1, \theta_2, \theta_3\}$ and $\{\psi_1, \psi_2, \psi_3\}$ in form resolved relative the derivatives:

$$\begin{cases} \dot{\theta}_1 = -\frac{1}{\cos\theta_2}\left(q\sin\theta_3 - p\cos\theta_3 + \cos\theta_3\omega_0\Theta_{12} - \sin\theta_3\omega_0\Theta_{22}\right); \\ \dot{\theta}_2 = q\cos\theta_3 + p\sin\theta_3 - \cos\theta_3\omega_0\Theta_{22} - \sin\theta_3\omega_0\Theta_{12}; \\ \dot{\theta}_3 = r + \mathrm{tg}\theta_2\left(q\sin\theta_3 - p\cos\theta_3 + \cos\theta_3\omega_0\Theta_{12} - \sin\theta_3\omega_0\Theta_{22}\right) - \omega_0\Theta_{32} \end{cases} \quad (9)$$

$$\begin{cases} \dot{\psi}_1 = -\frac{1}{\cos\psi_2}\left(q'\sin\psi_3 - p'\cos\psi_3 + \cos\psi_3\omega_0\Psi_{12} - \sin\psi_3\omega_0\Psi_{22}\right); \\ \dot{\psi}_2 = q'\cos\psi_3 + p'\sin\psi_3 - \cos\psi_3\omega_0\Psi_{22} - \sin\psi_3\omega_0\Psi_{12}; \\ \dot{\psi}_3 = r' + \mathrm{tg}\psi_2\left(q'\sin\psi_3 - p'\cos\psi_3 + \cos\psi_3\omega_0\Psi_{12} - \sin\psi_3\omega_0\Psi_{22}\right) - \omega_0\Psi_{32} \end{cases} \quad (10)$$

So, the Eqs. (6), (7), (8), (9), and (10) form the complete systems to modeling the angular motion of the satellite with the internal gravitational damper relative the orbital coordinates frame.

## 3 Modeling Results

Let us present the results of numerical modeling for the satellite with the internal gravitational damper with parameters indicated in Table 1.

As we see from the modeling results (Figs. 3, 4, 5, 6, 7, 8, and 9), the internal damper-body can effectively unload the initial angular momentum of the satellite. It follows from the fact that the equatorial angular velocity components ($p$, $p'$, $r$, $r'$) take near-zero values after first 150,000 seconds (Figs. 3 and 5), and after we have decreasing oscillations with near-zero small amplitudes. The absolute values of $q$ and $q'$ will be finally equal to orbital angular velocity (Fig. 4).

The attitude of bodies is evolutionarily coming near to a position along the axes of the orbital frame, in full accordance with gravitational stabilization principle. The initial rotational motion relative the orbital frame, as we can see, is stopped due to the kinetic energy dissipation with the help liquid friction in the internal damper.

**Table 1** The modeling parameters

| Bodies parameters | | | |
|---|---|---|---|
| | Inertia tensor [kg*m$^2$] | Initial angular velocity [1/s] | Initial attitude [rad] |
| Main body | $\boldsymbol{J} = \mathrm{diag}(0.0045, 0.0055, 0.0035)$ | $\boldsymbol{\omega}(0) = [0.0012, 0.001, -0.0025]$ | $\{\theta_i\} = \{0.015, 0.01, 0.02\}$ |
| Damper-body | $\boldsymbol{J}' = \mathrm{diag}(0.003, 0.004, 0.0015)$ | $\boldsymbol{\omega}'(0) = [0.0022, 0.001, 0.0015]$ | $\{\psi_i\} = \{0.015, 0.01, 0.02\}$ |
| Orbital angular velocity $\boldsymbol{\omega}_0$ [1/s] | | 0.0012 | |
| Damping factor $\nu$ [N*m*s] | | 0.00001 | |

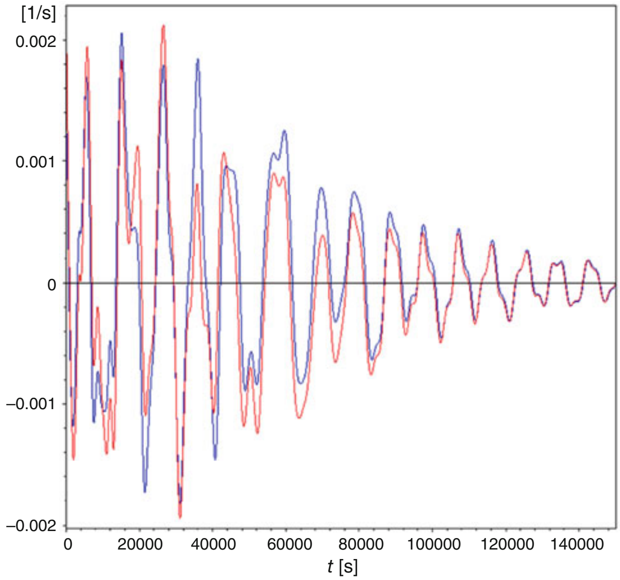



**Fig. 3** The time-evolution of the angular velocities components $p$ (blue) and $p'$ (red) of the main body and the damper-body

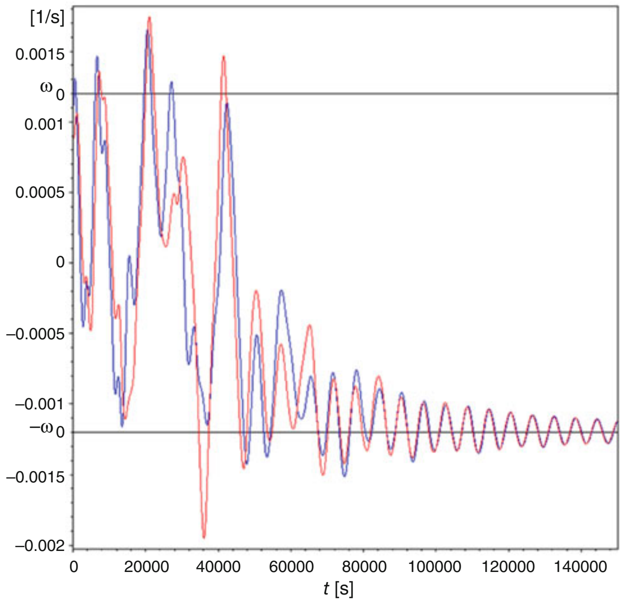



**Fig. 4** The time-evolution of the angular velocities components $q$ (blue) and $q'$ (red) of the main body and the damper-body

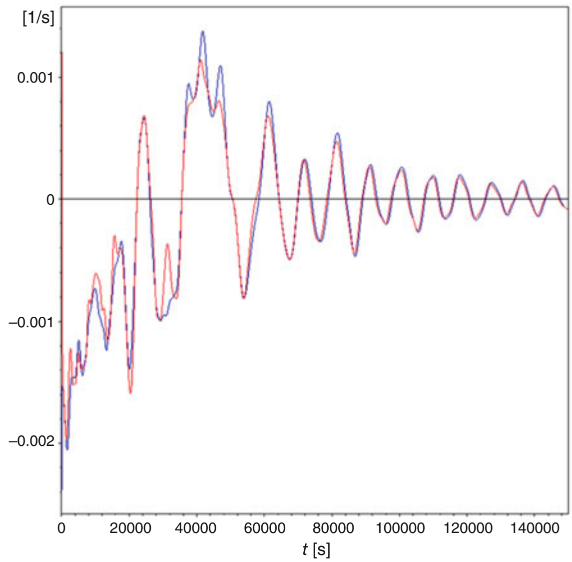



**Fig. 5** The time-evolution of the angular velocities components $r$ (blue) and $r'$ (red) of the main body and the damper-body

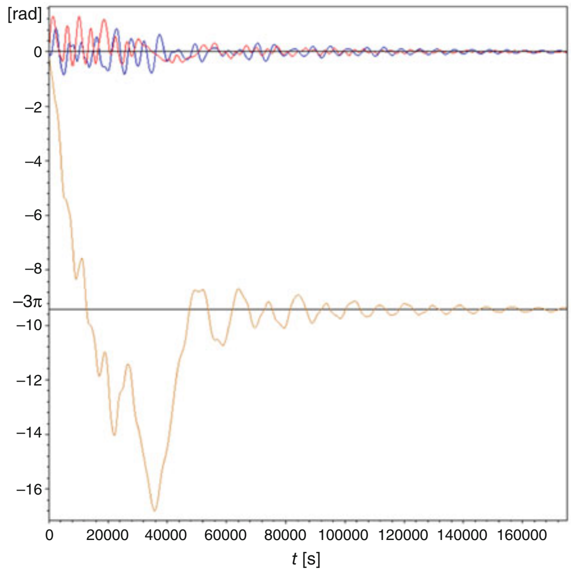



**Fig. 6** The time-evolution of spatial angles $\theta_1$ (red), $\theta_2$ (blue), $\theta_3$ (gold) of the attitude of the main body: the gravitational orientation is achieved (the main body is placed along the orbital axes)

## 4 Conclusions

The scheme of the satellite angular momentum unloading basing on the internal gravitational damper in the spherical cavity with viscous liquid was proposed. This scheme uses the external gravitational field to change the attitude of the internal damper-body relative the main body of the satellite, and to create, therefore,

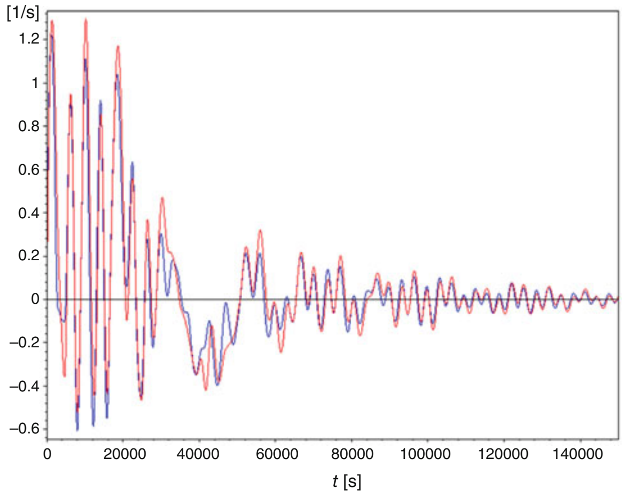



**Fig. 7** The time-evolution of angles $\theta_1$ (red) and $\psi_1$ (blue) of the attitude of the main body and the damper-body

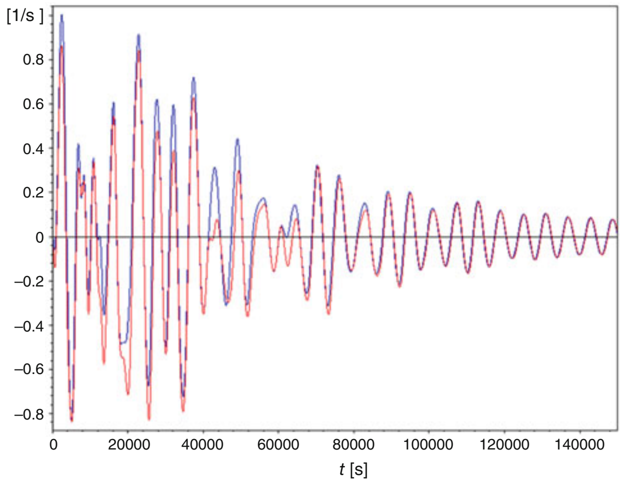



**Fig. 8** The time-evolution of angles $\theta_2$ (red) and $\psi_2$ (blue) of the attitude of the main body and the damper-body

dissipative torque of bodies' interaction due to the internal viscous friction, which unloads the angular momentum of the system.

The mathematical model of the attitude motion of satellites relative the orbital coordinates frame at the action of gravitational torques was constructed.

The numerical modeling was provided, that confirms the main suggested principle of the angular momentum unloading. As we can see from the modeling results,

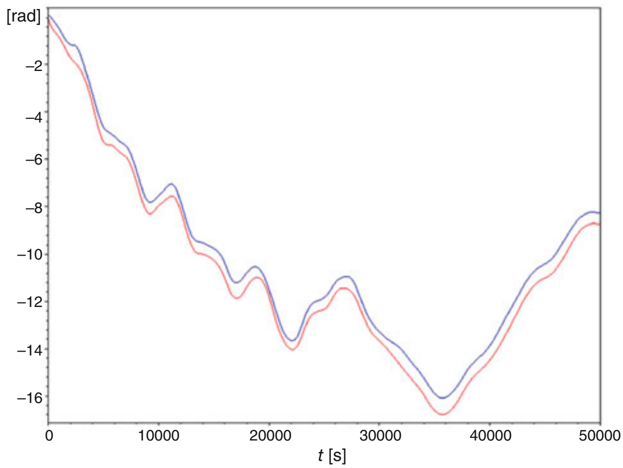



**Fig. 9** The time-evolution of spatial angles $\theta_3$ (red) and $\psi_3$ (blue) of the attitude of the main body and the damper-body: the angles have identical dynamics; they will coincide to 150,000 s

the process of the initial angular momentum unloading took about 150,000 seconds for parameters from Table 1.

Due to its simplicity, the studied scheme can be applied in cases of nanosatellites.

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## References

1. A.V. Doroshin, Attitude control of spider-type multiple-rotor rigid bodies systems. Proceedings of the world congress on engineering 2009, London, U.K. Vol. II, pp. 1544–1549 (2009)
2. A.V. Doroshin, Homoclinic solutions and motion chaotization in attitude dynamics of a multi-spin spacecraft. Commun. Nonlinear Sci. Numer. Simul. **19**(7), 2528–2552 (2014)
3. L.K. Davis, “Motion damper” U.S. Patent No. 3,399,317. Washington, DC: U.S. Patent and Trademark Office (1968)
4. A.E. Sabroff, Advanced spacecraft stabilization and control techniques. J. Spacecr. Rocket. **5**(12), 1377–1393 (1968)
5. Y. Mashtakov, S. Tkachev, M. Ovchinnikov, Use of external torques for desaturation of reaction wheels. J. Guid. Control Dynam. **41**(8), 1663–1674 (2018)
6. D. Tong, Spacecraft momentum dumping using gravity gradient. J. Spacecr. Rocket. **35**(5), 714–717 (1998)
7. A. Skullestad, Modeling and control of a gravity gradient stabilised satellite. Model. Identif. Control **20**(1), 3–26 (1999)
8. B.K. Powell, Gravity-gradient momentum management. J. Spacecr. Rocket. **9**(6), 385–386 (1972)
9. S. Kedare, S. Ulrich, Formulation of torque-optimal guidance trajectories for a CubeSat with degraded reaction wheels. In AIAA guidance, navigation, and control conference, p. 0088 (2016)

10. T. Burns, H. Flashner, Adaptive control applied to momentum unloading utilizing the low earth orbital environment. Guidance, Navigation and Control Conference, 391–401 (1989)
11. T.F. Burns, H. Flashner, Adaptive control applied to momentum unloading using the low earth orbital environment. J. Guid. Control Dynam. **15**(2), 325–333 (1992)
12. Y.W. Jan, J.C. Chiou, Unloading law for a LEO spacecraft with two-gimbals solar array. Acta Astronaut. **51**(12), 843–854 (2002)
13. A.V. Bogachev, E.A. Vorob'eva, N.E. Zubov, E.A. Mikrin, M.S. Misrikhanov, V.N. Ryabchenko, S.N. Timakov, Unloading angular momentum for inertial actuators of a spacecraft in the pitch channel. J. Comput. Syst. Sci. Int. **50**(3), 483–490 (2011)
14. V.M. Morozov, V.I. Kalenova, Satellite control using magnetic moments: Controllability and stabilization algorithms. Cosm. Res. **58**, 158–166 (2020). https://doi.org/10.1134/S0010952520030041