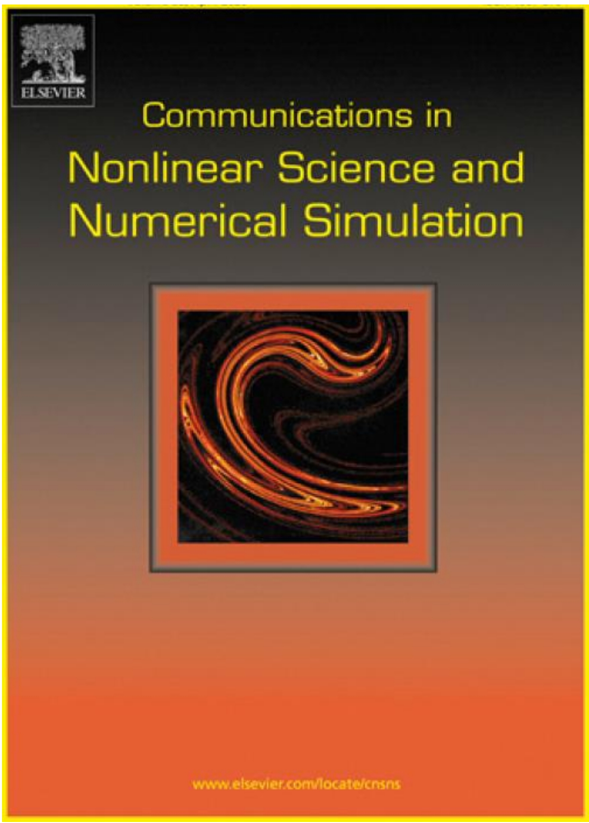

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Schemes of Momentum/Reaction Wheels Unloading by Mechanical Restructuring and Gearbox for Spacecraft

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A new principle and linked schemes of reaction/momentum wheels unloading are proposed and constructed. This principle is based on the instantaneous restructuring and changing the mechanical system itself, when additional internal degrees of freedom become available. These releasing additional degrees of freedom correspond to additional rotors, which take "after unfreezing" their own opportunity to rotate. At the connection of additional "unfrozen" rotors to the main wheels by gear type, these rotors become involved in the rotation with opposite direction relatively to the main wheels rotation, and it instantaneously provides nulling the relative angular momentum. Then electric motors of additional rotors and wheels can start the process of the rotational energy recovery, which decelerates and unloads angular velocities of all rotors and wheels. The proposed principle of releasing additional degrees of freedom and reconfiguring internal mechanical multi-rotor subsystem allow to immediately change values and directions of the relative angular momentum. This changeable multi-rotor subsystem represents the multifunctional attitude control system, which can be figuratively called as a "gearbox". As a part of the gearbox a special mechanical device is proposed, which represents a "gravitational damper" to unloading the total absolute angular momentum of spacecraft with the help of external gravitational torques. Activation of this devise is also fulfilled by releasing new degrees of freedom. Corresponding mechanical and mathematical models are build, and numerical modeling is provided to confirm the principle and all dynamical properties.

1. Introduction

THE construction of new methods and schemes of the attitude control of spacecraft (SC) and satellites was and still remains one of the most important problems of the attitude dynamics. In this framework, a very useful SC equipment for the attitude control is mechanical subsystems with rotors (wheels) placed on their own independent electric motors which can change the angular velocity of the rotors rotation, that change the distribution of the SC angular momentum between the SC main body and rotors. By this way, it is possible to initiate the rotation of the main body of the SC, or simply rotate it on the predefined angle in the inertial space with the help of the corresponding change of rotors angular velocities. Such rotors subsystems usually divided on the "momentum" and the "reaction" wheels.

As it is well known, momentum wheels typically represent the rotating wheels with a large, fixed angular momentum to provide overall stability of the SC attitude. The momentum wheels' angular velocity is gradually increased to absorb some external disturbances, which change the complete angular momentum of the SC. In addition, the well-known mechanical system to control the SC attitude motion is the reaction wheels. The reaction wheels are the rotors with small angular momentum. Attitude control systems use usually at least three independent reaction wheels, placed in three general axis of the SC. The reaction wheels begin to rotate (or to change the angular velocity) in cases when the SC must change its attitude or absorb external disturbance torques.

At using the momentum wheels or reaction wheels, the cases of a saturation of angular velocity of wheels are possible, when the wheels speed cannot to be increased more due to limits of the engines capacity, or other limits. Moreover, the angular momentum always is accumulated during the disturbances absorption – this accumulated angular momentum usually is transferred to wheels by the additional twisting. Therefore, it is always important to find new schemes to unloading the angular momentum of the wheels, and also to find new possibility to recover the accumulated kinetic energy of rotors rotation for reuse it in SC systems. This task and linked with it scientific and technical problems attracted attention of many scientists [1-32] and constructors [33-43] throughout history of space flight missions. Usually such schemes use the interaction of the SC with the external forces fields, e.g. with the gravity and magnet fields of planets, or use the aerodynamic interacting with planets' atmospheres, or torques from jet-propulsion engines. The magnetic systems to unloading the momentum/reaction wheels are investigated and designed, e.g., in works and patents [1, 3, 4-10, 14, 17, 19, 20, 24, 26, 29-31, 34, 39]. The gravity gradient action in framework of wheels unloading is described in [2, 7, 9, 15, 27, 32]. The propulsion systems application is considered in [5, 11, 28, 36, 38, 42, 43], including the schemes of low thrust, ion- and electro-engines using [16, 18, 21, 38, 40]. The solar pressure and aerodynamic schemes also are used to wheels unloading [7, 9, 15, 22, 32, 37].

In this article, a new principle and linked schemes of reaction/momentum wheels unloading are proposed and constructed. This principle is based on the instantaneous restructuring and changing the mechanical system itself. It implies the instantaneous change of SC mechanical structure (Fig.1) when additional internal degrees of freedom become available. These additional degrees of freedom correspond to additional rotors (identical to the main wheels), which take "after unfreezing" their own opportunity to rotate and to create their own angular momentums (Fig.2-a). If such additional "unfrozen" rotors become instantaneously connected to the main wheels by gear type (Fig.2-b), then these rotors become involved in the rotation with opposite direction relatively to the main wheels rotation, which instantaneously provides nulling the relative angular momentum of paired main wheels and "unfrozen rotors". After "unfrozen rotors" connections, the electric motors of all rotors and wheels can start the process of the energy recovery, when all electric motors switch to regimes of electric generators. This process, firstly, decelerates and unloads relative angular velocities of all rotors and wheels, and, secondly, transforms the rotational energy into electricity. To use the indicated principle it is possible build the multi-rotor mechanical subsystem allowing realization of different variants of unfreezing and connecting auxiliary rotors to unload the relative angular momentum of all wheels and to recover a part of their rotational energy. This multi-rotor subsystem can be figuratively called as a "gearbox" for spacecraft. The dynamics of spacecraft with the gearbox should be modelled in consideration of nonholonomic constraints formation [53-59].

So, the instantaneously nullification of the relative angular momentum can be realized by the connecting main wheels and "unfrozen rotors". This process translates the relative angular momentum of wheels to the main body of the spacecraft, and the main body starts its rotation, receiving the total value of the absolute angular momentum of the system. To discard the total absolute angular momentum of spacecraft (from the main body) in this paper the special mechanism is proposed in the framework of the gearbox construction. This mechanism is activated also on the principle of releasing additional degrees of freedom, and it implies the start of the angular motion of an internal rigid body with three different moments of inertia in a spherical cavity with a viscous liquid, placed into the main body of SC. The external central gravity field will acts on the internal body and will try to rotate it into attitude position, which is typical for gravitational stabilization [49-52]. The corresponding internal rotation of this body in the liquid will in its turn create the liquid friction, which will act on the main body. This internal rotation with friction is the cause for dissipation of the rotational energy of the main body. As the result it unloads the total absolute angular momentum of the spacecraft main body by the mediated interaction (through the friction) with the external gravity field. Such mechanical devices with releasing relative rotations of the internal body in viscous liquids can be called as "gravitational dampers" [60]. If we place the gravitational damper into the gearbox, then we obtain the quite effective equipment to control the attitude motion of spacecraft, and to unload the total absolute angular momentum of the spacecraft.

Thus, in this paper the mechanical constructing and mathematical modeling of spacecraft and their attitude dynamics with the gearbox are provided. In order to orient the reader, it is possible briefly indicate the structure of the article.

In **the paragraph 2** the general idea and main mechanical schemes are considered. There are possibilities of multi-rotor mechanical systems restructuring are identified, and corresponding implementations of additional "unfrozen" rotors releasing with their following connecting to main wheels, including schemes of "neutral pairs" of rotors.

In **the paragraph 3** modeling of multi-rotors spacecraft attitude dynamics is the fulfilled. There are presented all of important mathematical models and methods to study dynamics. At the begin, the mathematical model of the attitude dynamics in main Euler's form is described. Further in purposes of analysis of kinematical nonholonomic constraints in the dynamics with rotors connections, the mathematical approach to the nonholonomic system modeling is presented. Based on the nonholonomic approach, the dynamics is modeled and main aspects of this motion were indicated. After consideration of the Euler's mathematical model, the very important block of Hamiltonian models is followed, which includes the construction of Hamiltonian equations without and with kinematical constraints between the connected rotors. From this Hamiltonian model the important fact was observed and proved: the attitude motion of multi-rotor spacecraft with all connected compensated pairs of rotors will be equivalent to the attitude motion of free rigid body with corresponding inertia moments. This fact allows to substantially easily consider/simulate/synthesize the attitude dynamics of spacecraft on regimes with unloaded relative angular momentums of rotors, and to provide the unloading of the total angular momentum of the compete spacecraft using the external forces (e.g. at the active gravitational damper).

In **the paragraph 4**, some technical aspects of the multi-rotors systems application in the framework of the gearbox design are presented. The devise for the total absolute angular momentum unloading using the gravitational field is described – this devise can be called as the "gravitational damper". New schemes of the spacecraft attitude control with formations of compensated pairs of rotors, with orthogonal nonholonomic connections of rotors, and with the gravitational damper are proposed.

At the end, in **the conclusion** paragraph the main advantages of the proposed gearbox scheme are highlighted.

2. Main Mechanical Models

2.1. The General Idea of Main Wheels Unloading by SC Mechanical System Restructuring

Let us consider the attitude dynamics of a SC with main momentum/reaction wheels, which accumulate the angular momentum ($\mathbf{K}_x$, $\mathbf{K}_y$, $\mathbf{K}_z$) along all three main axes $\{x, y, z\}$, connected to the main body of SC (Fig.1), and develop new schemes of the wheels unloading.

First of all, it is needed to present the main idea of restructuring the mechanical system of the SC (Fig.1). Let us assume that the SC contains internal additional rotors, which are rigidly connected to the main body and cannot move relatively it within fulfilling the main program of SC attitude motion. In such "locked state" these additional rotors, per se, refer to the main SC rigid body, and can be considered as immovable elements of the main rigid construction of SC. When a necessity arises to unload the saturated momentum/reaction wheels, then additional rotors are released (Fig.2-a) and brought into contact with the wheels by gear type (Fig.2-b). At these connections pairs of wheel-rotor are built, which have zero values of angular momentums relative to the main body of SC, due to opposite directions and identical values of relative angular velocities of corresponded wheels and rotors, i.e. indicated additional rotors compensate the relative angular momentum of main wheels. By this reason, it is possible to call such rotors as the "rotors-compensators" (or the "compensating rotors"), and the pairs of wheels & rotors-compensators – as "compensated pairs".

This compensation of rotors translates their relative angular momentum to the main body of the spacecraft. Due to the conservation of the total absolute angular momentum, the main body must immediately start an absolute rotation to save the constant value of the total absolute angular momentum $\mathbf{K}=[\mathbf{K}_x, \mathbf{K}_y, \mathbf{K}_z]^T$ after compensation of the rotors. To unload the total absolute angular momentum of the spacecraft (from the main body) it is needed to use some special equipment, which interacts with external forces fields. For this purposes, in the section 4.5 an unloading mechanism is proposed, which uses the external gravity field, therefore, it can be figuratively called as "gravitational damper".

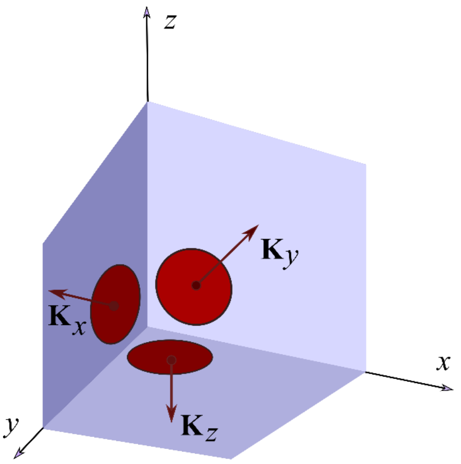


Fig. 1 The initial mechanical structure of SC with saturated wheels

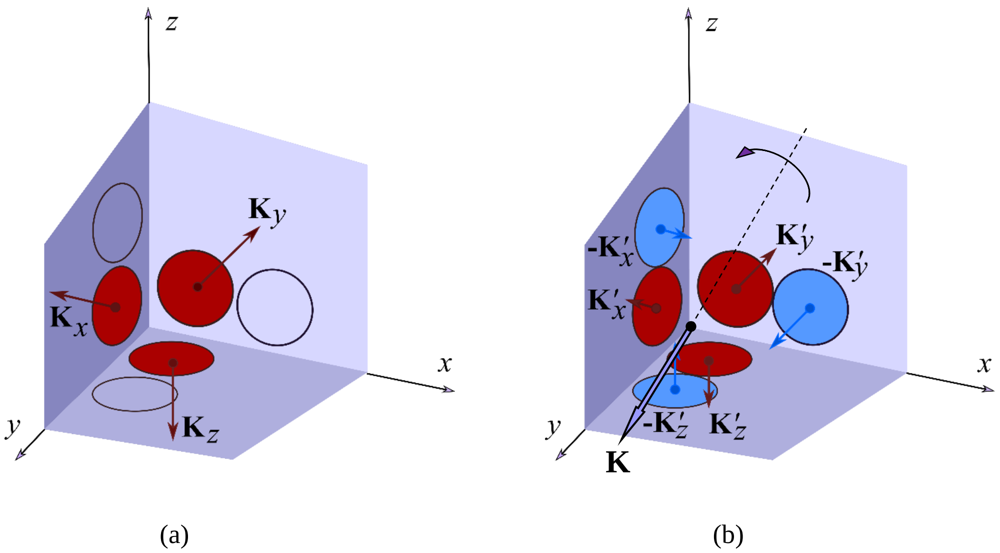


Fig. 2 Restructuring the SC mechanical system:

(a) - with the main wheels and the "unfrozen" free rotors;

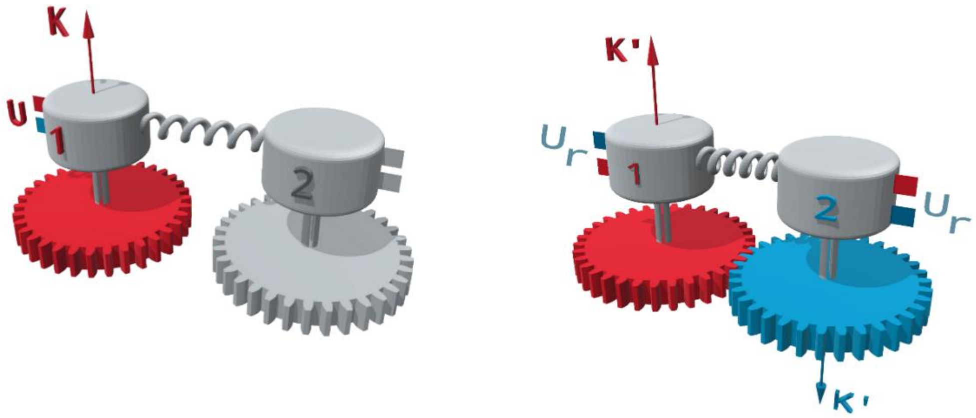
(b) – with the main wheels (red) and the connected "unfrozen" rotors (blue)

Since the rotors-compensators are installed on electric motors, they can work as generators and can produce the electricity and recover the kinetic energy of the rotation. So, at the activation of the compensated pairs the SC will realize the natural angular motion like simple rigid body, since the relative angular momentums of compensated pairs are equal to zero (the compensated pairs produce no gyroscopic torques). In this "rigid-body-regime" the SC can start the procedure of the rotors rotational energy recovery, and, also the procedure of the unloading the total absolute angular momentum at the initiated interaction with external forces fields.

After the recovery of the rotational energy rotors and after unloading the total absolute angular momentum from the main body of the spacecraft by the gravitational damper, the separation of the rotors-compensators from the main wheels realizes, the corresponding degrees of freedom become closed, and auxiliary rotors become rigidly fixed to the main body, and the SC goes on to its normal operation with controlling the attitude by the main momentum/reaction wheels.

2.2. Possible Schemes of Rotors Connection

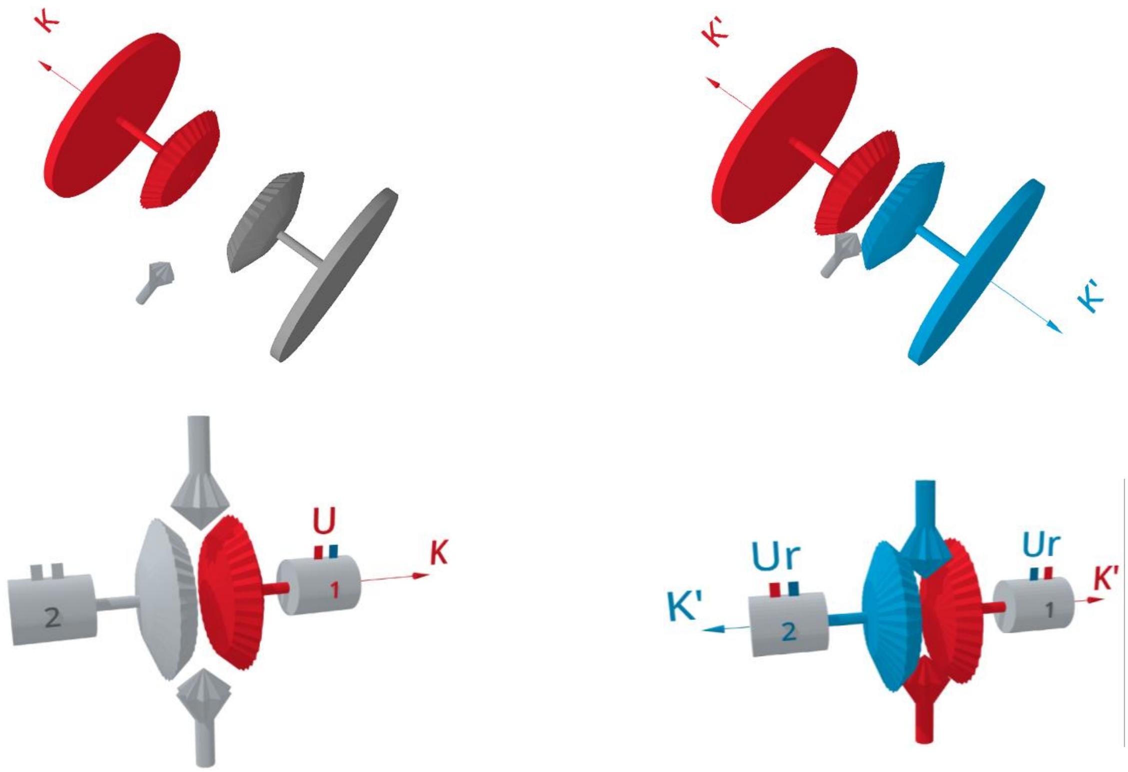
The considered above wheels relative angular momentum unloading process implies the connection of the main wheel with the rotor-compensator. To understand this very important procedure it is possible to give a short description of the mechanical structure of formation of compensated pairs. Let us consider the main wheel and rotors-compensators as mechanical gears. In the simplest case the rotor-compensator can be used, which is fully similar to the main wheel (including the electric motor). The compensated pair is formed by a direct contact between the main wheel and the rotor-compensator according to the following algorithm. At the step of the usual operation of the main wheel (Fig.3-a), it has the relative angular momentum **K** under the action of the electric motor (#1) with corresponding voltage (U) usage or freely, and the rotor-compensator is rigidly fixed relative the main body of SC. At the step of the wheel saturation, the rotor-compensator is released and brought into the connection with the main wheel (Fig.3-b), and then the main wheel and the rotor-compensator fulfill the opposite rotations with equal magnitudes of relative angular velocities, but with zero value of the total relative angular momentum. In this case, electric motors of the main wheel and the rotor-compensator play role of electric generators, create corresponding voltage (U_r) for energy recovery, and produce the internal torques, which are opposite by direction and equal by magnitude. Due to both motors are mounted onto main body of SC, the total internal torque is equal to zero, and, therefore, it does not affect the SC attitude in any way.



(a) (b)

Fig. 3 The planar form of the passive/active rotor-compensator:
(a) – the usual operation of the main wheel (red)
(b) – the unloading main wheel (red) by the connected rotor-compensator (blue)

In addition, an axial form of the compensated pair can be constructed with the help of a small auxiliary intermediate gear (Fig.4). The axial shape of the compensated pair, as well as the depicted planar form (Fig.3), also can be applied in the framework of unloading momentum/reaction wheels.



(a) (b)

Fig. 4 The axial form of the passive/active compensated pair:

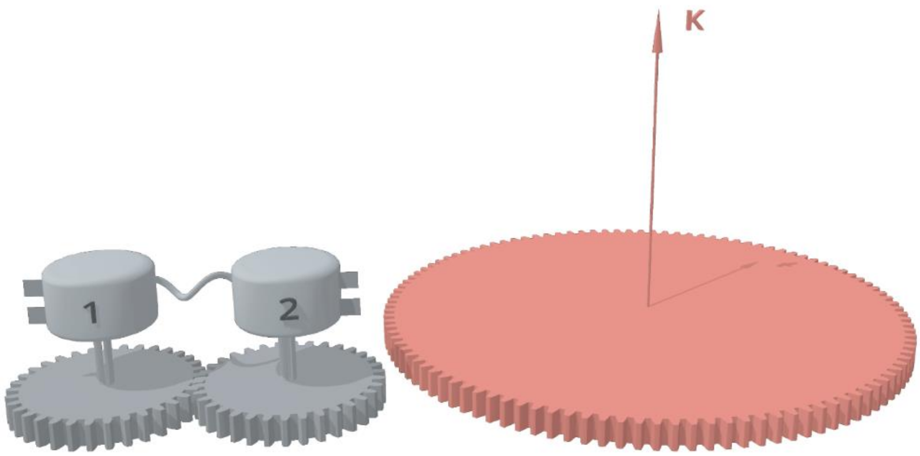
(a) – the usual operation of the main wheel (red)

(b) – the unloading main wheel (red) by the connected rotor-compensator (blue)

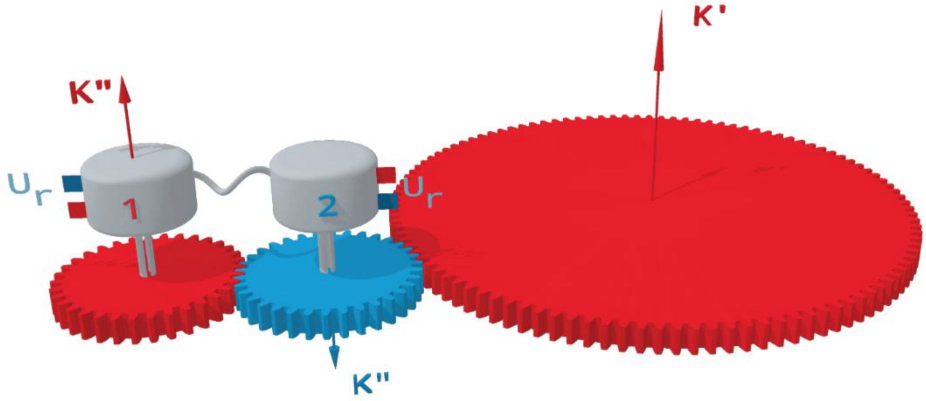
2.3. Neutral pairs of rotors

The property of the above-described compensated pair of rotors to have the zero value of the angular (relative) momentum can be applied to the desaturation of the freed momentum/reaction wheel.

The freed wheel here is the ideal model of the wheel, which was disconnected from its own electric motor, and now it fulfills the free rotation in “ideal” bearings without friction, i.e. it do not interact with the main body of the spacecraft through the creation of internal torques. Then to decelerate and to unload such freed wheels, it is enough to connect to them the compensated pair of rotors (Fig.5). In these cases, such auxiliary compensated pairs of rotors can be called as “neutral pairs”, that is more appropriate.



(a)



(b)

Fig. 5 Using the compensated pair to unloading the freed wheel:

(a) – the saturated freed wheel (pink) and inactive neutral pair (grey)

(b) –unloading freed wheel (red) by the connected neutral pair

The neutral pairs in active phase accumulate the energy in generators regimes. This energy recovery process decelerates the angular velocity of rotors, and it results in the freed wheel unloading. The angular velocity of the main freed wheel can by decreased slowly and carefully down to any desirable level, and after decreasing the velocity the neutral pair disconnects from the wheel. The important property of the neutral pair is the zero value of the relative angular momentum, and, therefore, the neutral pair do not create the gyroscopic torques and do not affect the attitude dynamics of main body of the spacecraft. In other words, the rotors of the neutral pair do not change the qualitative properties of the attitude motion; they simply add the inertial masses to the main body (as if they were motionless relative the main body). In the same time, the direct connection of the neutral pair to the saturated wheel will immediately absorb some part of the kinetic energy of the relative rotation, and transmits through oneself the part of the angular momentum of the wheel to the main body. The unloading of the total absolute angular momentum from the main body is fulfilled with the help of external torques (e.g., with the help of the “gravitational damper”, which is described in the section 4.5). After this immediate unloading the relative angular momentum (or a part of them), the neutral pair can be disconnected from the wheel, and can recover the part of the rotational energy by the pair of electrical engines. This approach of the relative angular momentum unloading and energy recovery can be in demand in the framework of the nanosatellites construction, when the simple/simplest equipment is preferable; and the neutral pair of rotors is exactly this case.

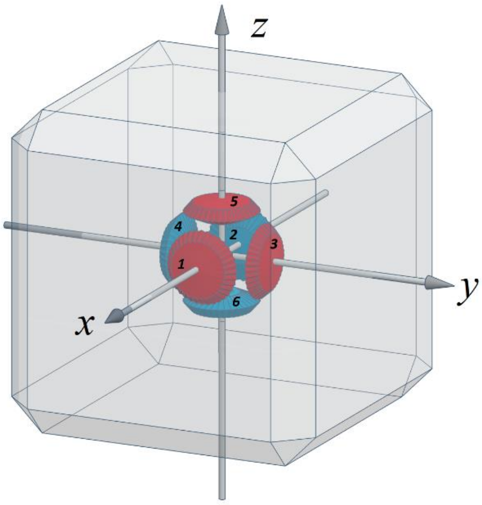
3. Modeling the Attitude Dynamics of SC at Compensated Rotors Formation

3.1. The Mathematical Model of the Attitude Dynamics

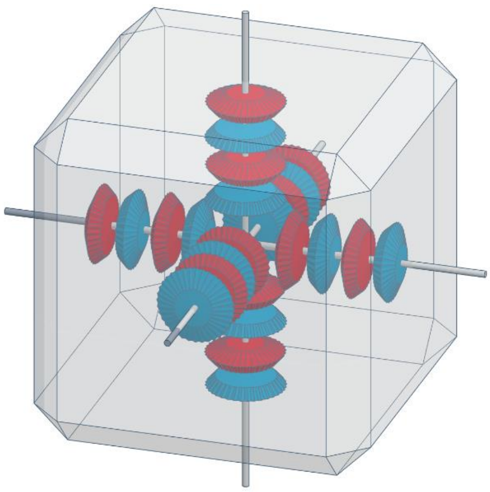
Let us build the attitude dynamics model of a SC, which continuously describes the SC angular motion during all three stage of the main wheels operation: the motion with saturated wheels, the motion at the process of wheels unloading by the formation of compensated pairs, and, the motion after decoupling of the compensated pairs of rotors.

Let us in our modeling consider the external torque-free attitude dynamics of SC; i.e. in this research we investigate the angular motion of SC at the internal redistributions of the relative angular momentum between the rotors and the main body at the constant total absolute angular momentum of the system.

This mathematical model can be constructed both for planar forms of compensated pairs (fig.3) and for their axial forms (fig.4). In this paper, we consider the mechanical and mathematical models of axial forms of wheels and their compensated pairs formation.



(a)



(b)

Fig. 6 The mechanical structure of the spacecraft with axial wheels and their compensated pairs:
(a) – the single-laeyr form of axial compensated pairs (the pairs of rotors ## {1 and 2}, {3, 4}, {5, 6})
(b) – the multi-layer form of axial compensated pairs

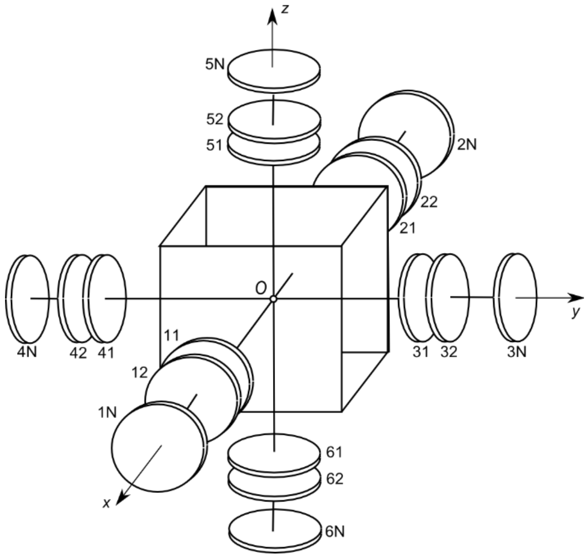


Fig. 7 The mechanical structure of the multi-layer multi-rotor spacecraft

The corresponded mechanical model is presented at the figure (fig.6-a), where the spacecraft collects the three main wheels (red wheels ##1, 3, 5) with corresponded axial rotors (blue rotors ##2, 4, 6) forming the compensated pairs along the main axes of the coordinates frame *Oxyz*, connected to the main body of SC. Also by analogy, the multi-layer scheme can be constructed (fig.6-b) for cases of large quantity of wheels - it generalizes the constructional scheme on the multirotor case [44] which allows to fulfill complex series of angular reorientations, using conjugated rotors spin-up and captures [48].

This system contains N layers with rotors on the six general directions coinciding with the principle axes of the main body. We also assume that each layer contains the equal rotors, but the rotors in the different layers are different. For the angular momentum of the generalized 6N-rotors-system written in the *Oxyz*, connected with the main body, we have

$$\mathbf{K} = \mathbf{K}_m + \mathbf{K}_r; \quad \mathbf{K}_m = \begin{bmatrix} Ap \\ Bq \\ Cr \end{bmatrix}; \quad \mathbf{K}_r = \sum_{l=1}^{N} I_l \begin{bmatrix} \sigma_{1l} + \sigma_{2l} \\ \sigma_{3l} + \sigma_{4l} \\ \sigma_{5l} + \sigma_{6l} \end{bmatrix}; \tag{1}$$

Here $\mathbf{K}_m$ is the angular momentum of the system with fixed (relative the main body) rotors; $\mathbf{K}_r$ is the relative angular momentum of rotors; σ_{kl} is the relative angular velocity of the *kl*-th rotor (relatively the main body); $A = \tilde{A} + 4\bar{J} + 2\bar{I}$, $B = \tilde{B} + 4\bar{J} + 2\bar{I}$, $C = \tilde{C} + 4\bar{J} + 2\bar{I}$; $\bar{J} = \sum_{l=1}^{N} J_l$; $\bar{I} = \sum_{l=1}^{N} I_l$; $\{\tilde{A}, \tilde{B}, \tilde{C}\}$ - are the main inertia moments of the main body; I_l and J_l are the longitudinal and the equatorial inertia moments of the *l*-layer-rotor relatively the point *O*. The main vector equations of the system motion in the coordinates' frame *Oxyz* is the following:

$$\frac{d\mathbf{K}}{dt} + \boldsymbol{\omega} \times \mathbf{K} = \mathbf{M}^e \tag{2}$$

The equation (2) can be written in the following scalar form

$$\begin{cases} A\dot{p}+\sum_{l=1}^{N}I_l\left\{\dot{\sigma}_{1l}+\dot{\sigma}_{2l}\right\}+\left(C-B\right)qr+\left[q\sum_{l=1}^{N}I_l\left\{\sigma_{5l}+\sigma_{6l}\right\}-r\sum_{l=1}^{N}I_l\left\{\sigma_{3l}+\sigma_{4l}\right\}\right]=M_x^e \\ B\dot{q}+\sum_{l=1}^{N}I_l\left\{\dot{\sigma}_{3l}+\dot{\sigma}_{4l}\right\}+\left(A-C\right)pr+\left[r\sum_{l=1}^{N}I_l\left\{\sigma_{1l}+\sigma_{2l}\right\}-p\sum_{l=1}^{N}I_l\left\{\sigma_{5l}+\sigma_{6l}\right\}\right]=M_y^e \\ C\dot{r}+\sum_{l=1}^{N}I_l\left\{\dot{\sigma}_{5l}+\dot{\sigma}_{6l}\right\}+\left(B-A\right)qp+\left[p\sum_{l=1}^{N}I_l\left\{\sigma_{3l}+\sigma_{4l}\right\}-q\sum_{l=1}^{N}I_l\left\{\sigma_{1l}+\sigma_{2l}\right\}\right]=M_z^e \end{cases} \tag{3}$$

The relative rotation equations of the main wheels are $\left(l=1..N\right)$:

$$\begin{cases} I_l\left(\dot{p}+\dot{\sigma}_{1l}\right)=M_{1l}^i+M_{1lx}^e; \\ I_l\left(\dot{q}+\dot{\sigma}_{3l}\right)=M_{3l}^i+M_{3ly}^e; \\ I_l\left(\dot{r}+\dot{\sigma}_{5l}\right)=M_{5l}^i+M_{5lz}^e; \end{cases} \tag{4}$$

$$\begin{cases} I_l\left(\dot{p}+\dot{\sigma}_{2l}\right)=M_{2l}^i+M_{2x}^e; \\ I_l\left(\dot{q}+\dot{\sigma}_{4l}\right)=M_{4l}^i+M_{4ly}^e; \\ I_l\left(\dot{r}+\dot{\sigma}_{6l}\right)=M_{6l}^i+M_{6lz}^e \end{cases} \tag{5}$$

where M_{jl}^i are internal torques acting on the *j*-th rotor from the side of the main body {*j*=1,2,…6}, and $M_{jl\{x,y,z\}}^e$ - external torques. Equations (3), (4) and (5) completely describe the natural dynamics of the system, when the rotors motion corresponds to fully independent degrees of freedom.

On the motion intervals where rotors (## 2*l*, 4*l*, 6*l*) are fixed (relative the main body) we must simply change the equations (5) on the following kinematical constraints:

$$\begin{cases} \dot{\sigma}_{2l}=0;\ \ \sigma_{2l}=0; \\ \dot{\sigma}_{4l}=0;\ \ \sigma_{4l}=0; \\ \dot{\sigma}_{6l}=0;\ \ \sigma_{6l}=0. \end{cases} \tag{6}$$

The immediate nullification of the relative angular momentum is realized by the restructuring of the mechanical system, when in its internal structure the rotor-compensator releases, and this freed rotor gets connection with the main wheel. This mechanical process can be completely formulated by the following kinematical constraints:

$$\sigma_{2l}=-\sigma_{1l}; \tag{7}$$

$$\sigma_{4l}=-\sigma_{3l}; \tag{8}$$

$$\sigma_{6l}=-\sigma_{5l} \tag{9}$$

where the constraint (7) is fulfilled after forming the compensated pair of rotors {1*l* and 2*l*}; (8) – after forming the compensated pair of rotors {3*l* and 4*l*}; (9) – after forming the compensated pair of rotors {5*l* and 6*l*}.

So, the complete mathematical model of the spacecraft attitude motion is divided on two stage with corresponding constraints:

1. The attitude motion of spacecraft on the interval with saturated main wheels is described by the equation system (3) at conditions (6) with *N* systems (4);

2. The attitude motion of spacecraft after forming compensated pairs of rotors is described by the equation system (3) at constraints (7)-(9).

In the purpose of better understanding the principle of the compensated pairs formation let us to investigate the single-layer (N=1) scheme (fig.6-a), and therefore, we do not indicate the corresponding index (l) of the layer number. The kinematical constraint describing the rotation of connected rotors in opposite directions has the shape:

$$\sigma_{i+1} \equiv -\sigma_i, \qquad \{i = 1,3,5\} \tag{10}$$

where σ_i – is the angular velocity of the rotor #i relative the main body. At the constrains (10) the nullification of sum of their relative angular velocities and accelerations takes place:

$$(\sigma_i + \sigma_{i+1}) \to 0; \quad (\dot{\sigma}_i + \dot{\sigma}_{i+1}) \to 0\,. \tag{11}$$

As can we see, the nonholonomic constraints between angular velocities of different rotors are actual. In this purpose the dynamics of nonholonomic systems should be considered, therefore, in the next section we shortly describe the main method of the nonholonomic systems modeling, which allows to write correct equations to describe the relative motion of the rotors after forming the compensated pairs.

3.2. The general approach to the nonholonomic system dynamics investigation

Let us shortly indicate the general methodology of the nonholonomic systems description [53-59]. Assume that we have s nonholonomic constraints in the following form:

$$\begin{aligned} &\sum_{j=1}^{m} b_{\beta j}(q_1,...,q_m,t)\dot{q}_j + b_\beta(q_1,...,q_m,t) = 0, \\ &\beta = 1,2,...,s \end{aligned} \tag{12}$$

Here q_j – are coordinates corresponded to the system's degrees of freedom (m – is the number of degrees of freedom). The general equations of the system dynamics are (T is the kinetic energy):

$$\sum_{j=1}^{m}\left(\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_j} - \frac{\partial T}{\partial q_j} - Q_j\right)\delta q_j = 0 \tag{13}$$

where variations δq_j now are not independent; they are connected (due to (12)) by the s independent expressions:

$$\sum_{j=1}^{m} b_{\beta j}(q_1,...,q_m,t)\delta q_j = 0 \tag{14}$$

To build the dynamical equations for nonholonomic system, it is possible to subtract from equations (13) expressions (14) multiplied by indefinite Lagrangian multipliers λ_β:

$$\sum_{j=1}^{m}\left(\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_j} - \frac{\partial T}{\partial q_j} - Q_j - \sum_{\beta=1}^{s}\lambda_\beta b_{\beta j}(q_1,...,q_m,t)\right)\delta q_j = 0 \tag{15}$$

Taking in mind that from (14) we can obtain *n* (where *n*=*m*-*s*) dependences for variations δq_{n+k} (where *k*=1..*s*) expressed through variations $\delta q_1,...,\delta q_n$; and then variations $\delta q_1,...,\delta q_n$ can be considered as independent main variations.

Let us now choice the multipliers λ_β such way that variations $\delta q_{n+1},...,\delta q_m$ will be equal to zeros. Therefore, in (15) we will see only independent variations $\delta q_1,...,\delta q_n$, and consequently their corresponded coefficients in brackets will equal to zeros, that allows to write the following dynamical equations:

$$\begin{aligned} &\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_j}-\frac{\partial T}{\partial q_j}-Q_j-\sum_{\beta=1}^{s}\lambda_\beta b_{\beta j}\left(q_1,...,q_m,t\right); \\ &j=1,...,m \end{aligned} \tag{16}$$

And we must add to equations (16) the constraints (12). From (16) and (12) it is possible to completely find all of the coordinates q_j and multipliers λ_β. So, the nonholonomic system will be fully described in terms of its coordinates.

3.3. The dynamics of the nonholonomic multi-rotor rigid bodies system

Taking in the mind the expressions (7)-(10) it is possible to write following nonholonomic/kinematical constraints, which include the relative rotors angular velocities:

$$b_{WW}\sigma_W + b_{WV}\sigma_V = 0 \tag{17}$$

In (17) constant coefficients b_{WW} and b_{WV} define the gear ratio between the rotor #*W* and the rotor #*V*. In our case we firstly consider the direct connections between opposite wheels to form the compensated pair of rotors:

$$b_{11}\sigma_1 + b_{12}\sigma_2 = 0;\;\; b_{33}\sigma_3 + b_{34}\sigma_4 = 0;\; b_{55}\sigma_5 + b_{56}\sigma_6 = 0 \tag{18}$$

where all coefficients $b_{ij} = 1$. From (18) by the way of differentiation the expressions follow:

$$b_{11}\dot{\sigma}_1 + b_{12}\dot{\sigma}_2 = 0;\;\; b_{33}\dot{\sigma}_3 + b_{34}\dot{\sigma}_4 = 0;\; b_{55}\dot{\sigma}_5 + b_{56}\dot{\sigma}_6 = 0 \tag{19}$$

With the help of theoretical explanations (12)-(16) it is possible to write the nonholonomic dynamical equations (16) for rotors relative rotations at the activation of constraints (17):

$$\begin{cases} I\left(\dot{p}+\dot{\sigma}_1\right) = M_1^i + M_{1x}^e + b_{11}\lambda_{12}; \\ I\left(\dot{q}+\dot{\sigma}_3\right) = M_3^i + M_{3y}^e + b_{33}\lambda_{34}; \\ I\left(\dot{r}+\dot{\sigma}_5\right) = M_5^i + M_{5z}^e + b_{55}\lambda_{56}; \end{cases} \tag{20}$$

$$\begin{cases} I\left(\dot{p}+\dot{\sigma}_2\right) = M_2^i + M_{2x}^e + b_{12}\lambda_{12}; \\ I\left(\dot{q}+\dot{\sigma}_4\right) = M_4^i + M_{4y}^e + b_{34}\lambda_{34}; \\ I\left(\dot{r}+\dot{\sigma}_6\right) = M_6^i + M_{6z}^e + b_{56}\lambda_{56} \end{cases} \tag{21}$$

where λ_{ij} are the indefinite Lagrangian multipliers. After formal resolution of systems (20), (21) and (19) we will have exact expressions for the Lagrangian multipliers:

$$\begin{cases} \lambda_{12} = \dfrac{I\dot{p}\left(b_{11}+b_{12}\right)-b_{11}\left[M_1^i+M_{1x}^e\right]-b_{12}\left[M_2^i+M_{2x}^e\right]}{b_{11}^2+b_{12}^2}\mathrm{H}\left(t,a,b\right); \\ \lambda_{34} = \dfrac{I\dot{q}\left(b_{33}+b_{34}\right)-b_{33}\left[M_3^i+M_{3y}^e\right]-b_{34}\left[M_4^i+M_{4y}^e\right]}{b_{33}^2+b_{34}^2}\mathrm{H}\left(t,a,b\right); \\ \lambda_{56} = \dfrac{I\dot{r}\left(b_{55}+b_{56}\right)-b_{55}\left[M_5^i+M_{5z}^e\right]-b_{56}\left[M_6^i+M_{6z}^e\right]}{b_{55}^2+b_{56}^2}\mathrm{H}\left(t,a,b\right); \end{cases} \tag{22}$$

$$\mathrm{H}\left(t,a,b\right) = H\left(t-a\right)-H\left(t-b\right)$$

Thus, the nonholonomic dynamics fully described by the equations (3), (20), (21) and (22). The function H(t,a,b) in (22) formally "turns on" the constraints (18) at the time-moment $t=a$ and "turns off" at the moment $t=b$, where usual Heaviside's function is used:

$$H\left(t\right) = \begin{cases} 1, & if \quad t \geq 0 \\ 0, & if \quad t < 0 \end{cases}$$

For the numerical illustration we present the simple case of the direct connection. Let us numerically investigate the one layer system attitude dynamics with direct connections of rotors $\left(b_{ij}=1,\ \forall i,j\right)$ without any external torques $\left(M_x^e=M_y^e=M_z^e=M_{kx}^e=M_{ky}^e=M_{kz}^e=0\right)$ through the following phases of the motion:

1. The initial torque-free motion with movable wheels ## 1, 3, 5 and with the fixed (relative the main body) wheels ## 2, 4, 6 on time-interval $t \in \left(-\infty, t_{\text{previous}}\right)$.
2. The spin-up of wheels ## 1, 3, 5 on time-interval $t \in \left[t_{\text{previous}}, t_{\text{spinup}}\right]$ by the internal torques $M_1^i,\ M_3^i,\ M_5^i$ at fixed wheels ## 2, 4, 6.
3. The second torque-free motion phase on time-interval $t \in \left(t_{\text{spinup}}, t_{\text{constrain}}\right)$ with movable wheels ## 1, 3, 5 at fixed wheels ## 2, 4, 6.
4. The motion phase with activated constraints (17) on the time-interval $t \in \left[t_{\text{constrain}}, t_{\text{free}}\right)$.
5. The motion phase of the natural (holonomic) dynamics on the time-interval $t \in \left[t_{\text{free}}, t_{\text{final}}\right)$ with freely rotating wheels (## 1-6) after disconnection of the constraints (17).
6. The final motion phase of the natural (holonomic) dynamics on the time-interval $t \in \left[t_{\text{final}}, +\infty\right)$ with freely rotating wheels ## 1, 3, 5 at fixed wheels ## 2, 4, 6.

Here it is important to note some aspects of modeling the multi-phases dynamics. So, the wheels spin-up process described by the internal torque with the following stepwise constant form:

$$M_k^i = m_k\,\mathrm{H}\left(t, t_{\text{previous}}, t_{\text{spinup}}\right); \quad m_k = \text{const}; \quad k=1,3,5 \tag{23}$$

The fixation of wheels (## 2, 4, 6) relative the main body can be modelled by the internal torques corresponding to creation of a large and exponential fast friction in the following shape:

$$M_k^i = -\nu\sigma_k\left[1 - H\left(t - t_{\text{constrain}}\right) + H\left(t - t_{\text{final}}\right)\right]; \quad \nu = \text{const} >> 1; \quad k = 2,4,6 \tag{24}$$

At the time-moment of "switching-on" the constraints (18), the values of angular velocities of the main body and wheels change due to some impact between the gears at the formation of compensated pairs; there is the angular momentum and kinetic energy redistributions occur. If consider the perfectly elastic collision impact without the loses of energy, it is possible to recalculate the values from the conditions of conservation of complete magnitudes of the angular momentums components and kinetic energy corresponding to directions of rotations of rotors:

$$\begin{cases} \begin{cases} K_x\left(t_{\text{constrain}} - 0\right) = K_x\left(t_{\text{constrain}} + 0\right) = K_x\left(t_0\right); \\ T_x\left(t_{\text{constrain}} - 0\right) = T_x\left(t_{\text{constrain}} + 0\right) = T_x\left(t_0\right); \end{cases} \\ \begin{cases} K_y\left(t_{\text{constrain}} - 0\right) = K_y\left(t_{\text{constrain}} + 0\right) = K_y\left(t_0\right); \\ T_y\left(t_{\text{constrain}} - 0\right) = T_y\left(t_{\text{constrain}} + 0\right) = T_y\left(t_0\right); \end{cases} \\ \begin{cases} K_z\left(t_{\text{constrain}} - 0\right) = K_z\left(t_{\text{constrain}} + 0\right) = K_z\left(t_0\right); \\ T_z\left(t_{\text{constrain}} - 0\right) = T_z\left(t_{\text{constrain}} + 0\right) = T_z\left(t_0\right) \end{cases} \end{cases} \tag{25}$$

where designations $\left(t_{\text{constrain}} - 0\right)$ and $\left(t_{\text{constrain}} + 0\right)$ indicate the time-moments "infinitely close to the left" and "infinitely close to the right" from the time-moment $t_{\text{constrain}}$ of "switching-on" the constraints (18). In other words, the initial values of "axial components" of the angular momentum and the kinetic energy define the value of angular velocities of the main body and rotors. For example, to calculate the values corresponding to axis *x* we have the following algebraic equations (with understanding the rotors constraints: $\sigma_2\left(t_{\text{constrain}} - 0\right) = 0;\ \sigma_2\left(t_{\text{constrain}} + 0\right) = -\sigma_2\left(t_{\text{constrain}} + 0\right)$, and with addition of initial value $K_x\left(t_0\right)$):

$$\begin{cases} K_x\left(t_{\text{constrain}} - 0\right) = Ap\left(t_{\text{constrain}} - 0\right) + I\sigma_1\left(t_{\text{constrain}} - 0\right) = K_x\left(t_0\right); \\ K_x\left(t_{\text{constrain}} + 0\right) = Ap\left(t_{\text{constrain}} + 0\right) = K_x\left(t_0\right); \\ T_x\left(t_{\text{constrain}} - 0\right) = \frac{1}{2}I\left(p\left(t_{\text{constrain}} - 0\right) + \sigma_1\left(t_{\text{constrain}} - 0\right)\right)^2 + \\ + \frac{1}{2}\left(A - 2I\right)p^2\left(t_{\text{constrain}} - 0\right) + \frac{1}{2}Ip^2\left(t_{\text{constrain}} - 0\right) = T_x\left(t_0\right); \\ T_x\left(t_{\text{constrain}} + 0\right) = \frac{1}{2}\left(A - 2I\right)p^2\left(t_{\text{constrain}} + 0\right) + \\ + \frac{1}{2}I\left(p\left(t_{\text{constrain}} + 0\right) + \sigma_1\left(t_{\text{constrain}} + 0\right)\right)^2 + \frac{1}{2}I\left(p\left(t_{\text{constrain}} + 0\right) + \left(-\sigma_1\left(t_{\text{constrain}} - 0\right)\right)\right)^2 = T_x\left(t_0\right); \end{cases} \tag{26}$$

So, from (26) at the predefined (initial) values $K_x\left(t_0\right)$ and $T_x\left(t_0\right)$, the magnitudes $\left\{p\left(t_{\text{constrain}} - 0\right),\ p\left(t_{\text{constrain}} + 0\right),\ \sigma_1\left(t_{\text{constrain}} - 0\right),\ \sigma_1\left(t_{\text{constrain}} + 0\right)\right\}$ can be calculated as solutions of algebraic systems. Similar calculations following from the conservation of values $\left\{K_y\left(t_0\right), T_y\left(t_0\right)\right\}$ and $\left\{K_z\left(t_0\right), T_z\left(t_0\right)\right\}$ allow to find the values $\left\{q\left(t_{\text{constrain}} - 0\right),\ q\left(t_{\text{constrain}} + 0\right),\ \sigma_3\left(t_{\text{constrain}} - 0\right),\ \sigma_3\left(t_{\text{constrain}} + 0\right)\right\}$ and

$\{r(t_{\text{constrain}}-0),\ r(t_{\text{constrain}}+0),\ \sigma_5(t_{\text{constrain}}-0),\ \sigma_5(t_{\text{constrain}}+0)\}$.

The numerical modeling results are presented at the fig.8. The fragments (fig.8-a,b) correspond to the case when the compensated pairs are formed along all connected axes (x, y, z) by the synchronous initiation all of the constraints $\{\lambda_{12},\ \lambda_{34},\ \lambda_{56}\}$; and the fragments (fig.8-c,d) correspond to only one compensated pair formation along axis x by the constraint λ_{12}.The initial conditions and parameters for simulation are presented at the table 1.

Table 1.

Fig.8	t_{previous}	t_{spinup}	$t_{\text{constraint}}$	t_{free}	t_{final}	$p(t^*)$	$q(t^*)$	$r(t^*)$	$\sigma_1(t^*)$	$\sigma_3(t^*)$	$\sigma_5(t^*)$	$p(t^*)$	$q(t^*)$	$r(t^*)$	$\sigma_1(t^*)$	$\sigma_3(t^*)$	$\sigma_5(t^*)$
	[s]					[rad/s] ($t^*= t_{\text{constraint}}$ - 0)						[rad/s] ($t^*= t_{\text{constraint}}$ + 0)					
(a), (b)	-250	-200	0	200	300	-0.19	0.19	0.35	18.46	1.42	8.5	-0.05	0.2	0.4	13	1	6
(c), (d)	-550	-500	0	200	200	-0.16	0.2	0.4	18.44	1	6	-0.05	0.2	0.4	13	1	6

Fig.8	A	B	C	I	$K_x(t_0)$	$K_y(t_0)$	$K_z(t_0)$	$T_x(t_0)$	$T_y(t_0)$	$T_z(t_0)$	$m_{1,2,3}$	v
	[kg*m2]				[kg*m^2/s]			[kg*m^2/s^2]			[N*m]	[N*m*s]
(a), (b)	5	6	7	0.038	-0.25	1.2	2.8	6.47	0.16	1.94	0.1	500
(c), (d)	5	6	7	0.03	-0.25	N\A	N\A	5.11	N\A	N\A	0.1	1000

N\A-not applying

In the first case (fig.8-a,b) is shown step-wise process of transition from high-frequency large values of the angular velocity components to oscillations with reduced amplitudes. Moreover, the final motion in this case corresponds to dynamics into co-called heteroclinic region (where the area of separatrix is achieved) – it is quite important from the point of view of passage to chaotic regimes, or intentionally initiating chaos [45]. The second case (fig.8-c,d) shows the possible reducing the angular velocity of the main body down to the smallest values with the help of only one compensated pair formation, that shows applicability of the suggested dynamical algorithm of change the angular velocity of the main body at the synchronous process of unloading wheels (we see the fivefold/eightfold reducing of magnitudes of relative angular velocities of wheels).

It is important to note, that the conservation of the total absolute angular momentum of the system is numerically checked at the modeling (fig.8-f) during all phases of the angular motion with/without and before/after formation/decoupling of compensated pairs of rotors.

To qualitative understanding the considered processes in the complete phase space, it is worth to use the canonical coordinates and corresponding Hamiltonian equations, and, therefore, in the next sections we proceed to the canonical Serret-Andoyer-Deprit canonical variables, as it was descripted in details in [44].

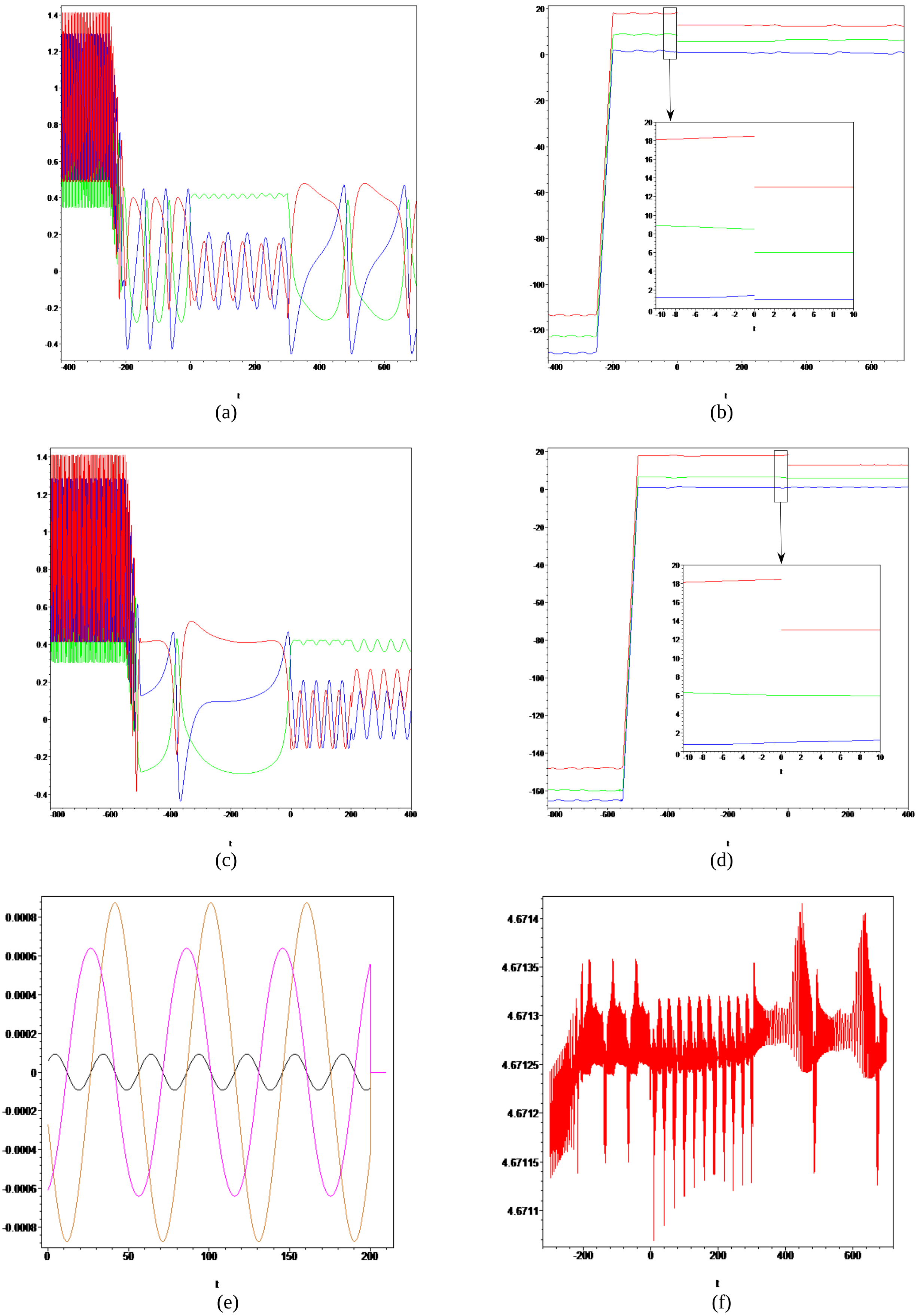


Fig. 8 Modeling results of the spacecraft motion during all stages of wheels functioning:
(a), (c): $p(t)$ – red, $q(t)$ – blue, $r(t)$ – green; (b), (d): $\sigma_1(t)$ – red, $\sigma_3(t)$ – blue, $\sigma_5(t)$ – green
(e): $\lambda_{12}(t)$ – magenta, $\lambda_{34}(t)$ – gold, $\lambda_{56}(t)$ – black (for the case (a));
(f): The conservation of complete angular momentum of the system $|\mathbf{K}(t)|$ during all phases in the case (a)

3.4. Hamiltonian Form of Attitude Dynamics Equations of Spacecraft with saturated main wheels

The important part of research of proposed step-wise dynamics corresponds to study of the canonical phase space of the system before and after forming compensated pairs. In this connection the Hamiltonian formalism should be used. The more appropriate canonical variables for multi-rotor mechanical system attitude dynamics investigation are the well-known Serret-Andoyer-Deprit canonical variables [e.g. 44-47]. With the help of these variables, the angular motion of the main body in the inertial coordinates frame *OXYZ* (fig.9) is described by the angles $(\varphi_3, \varphi_2, l)$ of the rotations around the axis *OZ*, around the direction of the angular momentum, and about the axis Oz (Fig.2). The canonical Serret-Andoyer-Deprit momentums can be defined by the following manner, basing on the vector of the angular momentum of complete system **K**:

$$L = \frac{\partial \mathrm{T}}{\partial \dot{l}} = \mathbf{K}\cdot\mathbf{k}, \quad G = \frac{\partial \mathrm{T}}{\partial \dot{\varphi}_2} = \mathbf{K}\cdot\mathbf{s} = K, \quad H = \frac{\partial \mathrm{T}}{\partial \dot{\varphi}_3} = \mathbf{K}\cdot\mathbf{k}', \quad \Delta_{ij} = \frac{\partial \mathrm{T}}{\partial \dot{\delta}_{ij}} = \frac{\partial \mathrm{T}}{\partial \sigma_{ij}} \tag{27}$$

where δ_{ij} describes the relative rotation angle of the *ij*-rotor $\left(\dot{\delta}_{ij} = \sigma_{ij}\right)$.

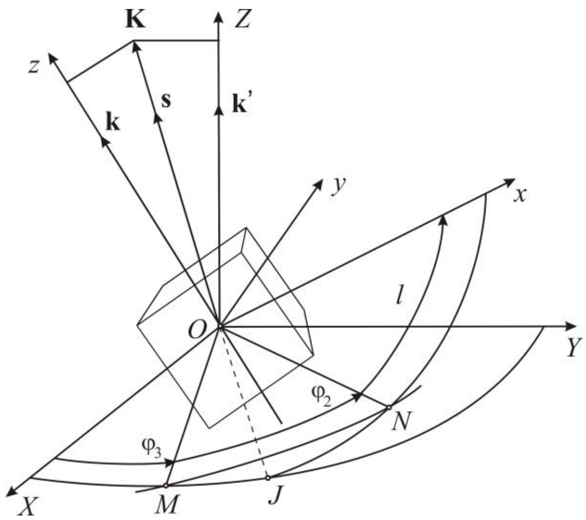


Fig. 9 The main body of the spacecraft and Serret-Andoyer-Deprit canonical coordinates

Let us to build the Hamiltonian form of the motion equations. The components of the system angular momentum can be expressed as the functions of the Serret-Andoyer-Deprit variables [44] for the SC with rotating main wheels (## 1*j*, 3*j*, 5*j*) at the fixed paird rotors (## 2*j*, 4*j*, 6*j*):

$$\begin{aligned} K_x &= \sqrt{G^2 - L^2}\sin l = Ap + \sum_{j=1}^{N} I_j \sigma_{1j} \\ K_y &= \sqrt{G^2 - L^2}\cos l = Bq + \sum_{j=1}^{N} I_j \sigma_{3j} \\ K_z &= L = Cr + \sum_{j=1}^{N} I_j \sigma_{5j}, \qquad (L \le G) \end{aligned} \tag{28}$$

The angular momentums of main wheels have the form:

$$\Delta_{1j} = I_j\left(p+\sigma_{1j}\right); \quad \Delta_{3j} = I_j\left(q+\sigma_{3j}\right); \quad \Delta_{5j} = I_j\left(r+\sigma_{5j}\right); \tag{29}$$

From (29) the expressions follow

$$\sum_{j=1}^{N} I_j\sigma_{1j} = \sum_{j=1}^{N}\Delta_{1j} - p\sum_{j=1}^{N} I_j \tag{30}$$

$$\sum_{j=1}^{N} I_j\sigma_{3j} = \sum_{j=1}^{N}\Delta_{3j} - q\sum_{j=1}^{N} I_j \tag{31}$$

$$\sum_{j=1}^{N} I_j\sigma_{5j} = \sum_{j=1}^{N}\Delta_{5j} - r\sum_{j=1}^{N} I_j \tag{32}$$

In view of (6) from the expressions (29)-(32) the components of the main body angular velocity follow:

$$p = \frac{1}{\hat{A}}\left[\sqrt{G^2-L^2}\sin l - \sum_{j=1}^{N}\Delta_{1j}\right] \tag{33}$$

$$q = \frac{1}{\hat{B}}\left[\sqrt{G^2-L^2}\cos l - \sum_{j=1}^{N}\Delta_{3j}\right] \tag{34}$$

$$r = \frac{1}{\hat{C}}\left[L - \sum_{j=1}^{N}\Delta_{5j}\right] \tag{35}$$

where $\hat{A} = A-\bar{I}$; $\hat{B} = B-\bar{I}$; $\hat{C} = C-\bar{I}$.

Also we can rewrite the system's kinetic energy and the angular momentum’s components:

$$\begin{cases} T = T_0 + \sum_{j=1}^{N} T_j; \quad 2T_0 = \tilde{A}p^2 + \tilde{B}q^2 + \tilde{C}r^2; \quad T_j = \sum_{i=1}^{6} T_{ij}; \\ 2T_{1j} = J_j\left(q^2+r^2\right) + \frac{\Delta_{1j}^2}{I_j}; \quad 2T_{3j} = J_j\left(p^2+r^2\right) + \frac{\Delta_{3j}^2}{I_j}; \quad 2T_{5j} = J_j\left(p^2+q^2\right) + \frac{\Delta_{5j}^2}{I_j}; \\ 2T_{2j} = J_j\left(q^2+r^2\right) + I_j p^2; \; 2T_{4j} = J_j\left(p^2+r^2\right) + I_j q^2; \quad 2T_{6j} = J_j\left(p^2+q^2\right) + I_j r^2; \\ 2T_j = \left[4J_j + I_j\right]\left(p^2+q^2+r^2\right) + \frac{1}{I_j}\left[\Delta_{1j}^2 + \Delta_{3j}^2 + \Delta_{5j}^2\right]; \\ 2\sum_{j=1}^{N} T_j = \left(p^2+q^2+r^2\right)\sum_{j=1}^{N}\left[4J_j + I_j\right] + \sum_{j=1}^{N}\frac{1}{I_j}\left[\Delta_{1j}^2 + \Delta_{3j}^2 + \Delta_{5j}^2\right]; \end{cases} \tag{36}$$

$$2T = \hat{A}p^2 + \hat{B}q^2 + \hat{C}r^2 + 2T_R; \qquad 2T_R = \sum_{j=1}^{N}\frac{1}{I_j}\left[\Delta_{1j}^2 + \Delta_{3j}^2 + \Delta_{5j}^2\right]; \tag{37}$$

$$\begin{cases} K_x = \hat{A}p + \sum_{j=1}^{N}\Delta_{1j} = \hat{A}p + D_1; \\ K_y = \hat{B}q + \sum_{j=1}^{N}\Delta_{3j} = \hat{B}q + D_3; \\ K_z = \hat{C}r + \sum_{j=1}^{N}\Delta_{5j} = \hat{C}r + D_5; \end{cases} \tag{38}$$

$$K^2 = \left(\hat{A}p + D_1\right)^2 + \left(\hat{B}q + D_3\right)^2 + \left(\hat{C}r + D_5\right)^2, \tag{39}$$

where D_k are the following axial summarized angular momentums of the rotors:

$$D_1=\sum_{j=1}^{N}\Delta_{1j},\quad D_3=\sum_{j=1}^{N}\Delta_{3j},\quad D_5=\sum_{j=1}^{N}\Delta_{5j} \tag{40}$$

Taking into account the expressions (40), (30)-(32), the main dynamical equations (3) for spacecraft with saturated main wheels are rewritten in the unbalanced-gyrostat-form:

$$\begin{cases}\hat{A}\dot{p}+\dot{D}_1+\left(\hat{C}-\hat{B}\right)qr+\left[qD_5-rD_3\right]=M_x^e\\ \hat{B}\dot{q}+\dot{D}_3+\left(\hat{A}-\hat{C}\right)rp+\left[rD_1-pD_5\right]=M_y^e\\ \hat{C}\dot{r}+\dot{D}_5+\left(\hat{B}-\hat{A}\right)pq+\left[pD_3-qD_1\right]=M_z^e\end{cases} \tag{41}$$

From the system (4) we obtain the equations for summarized rotors angular momentums (40):

$$\dot{D}_1=M_1^i+M_1^e;\quad \dot{D}_3=M_3^i+M_3^e;\quad \dot{D}_5=M_5^i+M_5^e, \tag{42}$$

where $M_1^i=\sum_{l=1}^{N}M_{1l}^i,\;\; M_3^i=\sum_{l=1}^{N}M_{3l}^i,\;\; M_5^i=\sum_{l=1}^{N}M_{5l}^i,$

Let us consider the dynamics of SC already with saturated wheels and, therefore, without action of any internal and external torques, and then:

$$D_i=\mathrm{const}_i;\quad \Delta_j=\mathrm{const}_j \tag{43}$$

With the help of (33)-(35) we express the kinetic energy (37) as the function of the Serret-Andoyer-Deprit variables:

$$\begin{aligned}T=\frac{G^2-L^2}{2}\left[\frac{\sin^2 l}{\hat{A}}+\frac{\cos^2 l}{\hat{B}}\right]+\frac{1}{2\hat{C}}\left(L-D_5\right)^2-\sqrt{G^2-L^2}\left\{\frac{D_1\sin l}{\hat{A}}+\frac{D_3\cos l}{\hat{B}}\right\}+\\ +\frac{D_1^2}{2\hat{A}}+\frac{D_3^2}{2\hat{B}}+\frac{1}{2}\sum_{j=1}^{N}\frac{1}{I_j}\left[\Delta_{1j}^2+\Delta_{3j}^2+\Delta_{5j}^2\right]\end{aligned} \tag{44}$$

The Hamiltonian and canonical equations have the form (P – is the system potential energy, which equals to zero in our torque-free case):

$$\begin{gathered}\mathscr{H}=T+P\left(l,\varphi_2,\varphi_3,\delta_{ij}\right);\\ \dot{x}=\frac{\partial\mathscr{H}}{\partial X};\quad \dot{X}=-\frac{\partial\mathscr{H}}{\partial x};\\ x=\left\{l,\varphi_2,\varphi_3,\delta_{ij}\right\},\;\; X=\left\{L,G,H,\Delta_{ij}\right\}\end{gathered} \tag{45}$$

In the considered case, when the system is torque-free and the main wheels are already saturated, the Hamiltonian equations for main positional coordinates $\{l, L\}$ take the following form, since other coordinates are cyclic $\left(G=\mathrm{const},\; H=\mathrm{const},\; \Delta_{ij}=\mathrm{const}\right)$:

$$\begin{cases}\dot{l}=L\left[\frac{1}{\hat{C}}-\frac{\sin^2 l}{\hat{A}}-\frac{\cos^2 l}{\hat{B}}\right]+\frac{L}{\sqrt{G^2-L^2}}\left\{\frac{D_1\sin l}{\hat{A}}+\frac{D_3\cos l}{\hat{B}}\right\}-\frac{D_5}{\hat{C}};\\ \dot{L}=\left(G^2-L^2\right)\left(\frac{1}{\hat{B}}-\frac{1}{\hat{A}}\right)\sin l\cos l+\sqrt{G^2-L^2}\left\{\frac{D_1\cos l}{\hat{A}}-\frac{D_3\sin l}{\hat{B}}\right\}\end{cases} \tag{46}$$

So, the equations (46) define the motion of the spacecraft at the stage, when the main wheels are saturated.

3.5. Hamiltonian Form of Attitude Dynamics Equations of Spacecraft with compensated pairs of rotors

Now we intend to write the equations for the stage of the motion after compensated pairs of rotors forming. Let us consider the simple case of synchronous formations ($t_{12}=t_{34}=t_{56}=t_{comp}$) of all compensated pairs of rotors on all axes in all layers (i.e. the all wheels already are compensated). The components of the system angular momentum can be expressed as the functions of the Serret-Andoyer-Deprit variables, taking into consideration the conditions (10) of the compensated pairs of rotors formation $\left(\sigma_{2j}=-\sigma_{1j},\ \sigma_{4j}=-\sigma_{3j},\ \sigma_{6j}=-\sigma_{5j}\right)$:

$$\begin{cases} K_x=\sqrt{G^2-L^2}\sin l = Ap+\sum\limits_{j=1}^{N} I_j\left(\sigma_{1j}+\sigma_{2j}\right)=Ap \\ K_y=\sqrt{G^2-L^2}\cos l = Bq+\sum\limits_{j=1}^{N} I_j\left(\sigma_{3j}+\sigma_{4j}\right)=Bq \\ K_z=L=Cr+\sum\limits_{j=1}^{N} I_j\left(\sigma_{5j}+\sigma_{6j}\right)=Cr \end{cases} \tag{47}$$

The canonical momentums for the relative motion of the *j*-layer-rotors are:

$$\begin{cases} \Delta_{1j}=I_j\left(p+\sigma_{1j}\right); \quad \Delta_{2j}=I_j\left(p+\sigma_{2j}\right) \\ \Delta_{3j}=I_j\left(q+\sigma_{3j}\right); \quad \Delta_{4j}=I_j\left(q+\sigma_{4j}\right) \\ \Delta_{5j}=I_j\left(r+\sigma_{5j}\right); \quad \Delta_{6j}=I_j\left(r+\sigma_{6j}\right) \end{cases} \tag{48}$$

From (48), (29) and from (47) the expressions follow:

$$\sum_{j=1}^{N} I_j\left(\sigma_{1j}+\sigma_{2j}\right)=\sum_{j=1}^{N}\left(\Delta_{1j}+\Delta_{2j}\right)-2p\sum_{j=1}^{N} I_j=0; \qquad p=\frac{\sum\limits_{j=1}^{N}\left(\Delta_{1j}+\Delta_{2j}\right)}{2\sum\limits_{j=1}^{N} I_j}=\frac{1}{A}\sqrt{G^2-L^2}\sin l \tag{49}$$

$$\sum_{j=1}^{N} I_j\left(\sigma_{3j}+\sigma_{4j}\right)=\sum_{j=1}^{N}\left(\Delta_{3j}+\Delta_{4j}\right)-2q\sum_{j=1}^{N} I_j=0; \qquad q=\frac{\sum\limits_{j=1}^{N}\left(\Delta_{3j}+\Delta_{4j}\right)}{2\sum\limits_{j=1}^{N} I_j}=\frac{1}{B}\sqrt{G^2-L^2}\cos l \tag{50}$$

$$\sum_{j=1}^{N} I_j\left(\sigma_{5j}+\sigma_{6j}\right)=\sum_{j=1}^{N}\left(\Delta_{5j}+\Delta_{6j}\right)-2r\sum_{j=1}^{N} I_j=0; \qquad r=\frac{\sum\limits_{j=1}^{N}\left(\Delta_{5j}+\Delta_{6j}\right)}{2\sum\limits_{j=1}^{N} I_j}=\frac{L}{C} \tag{51}$$

Then at the constraints (10) the kinetic energy takes the form:

$$\begin{cases} T=T_0+\sum\limits_{j=1}^{N} T_j; \quad 2T_0=\tilde{A}p^2+\tilde{B}q^2+\tilde{C}r^2; \quad T_j=\sum\limits_{i=1}^{6} T_{ij}; \\ 2T_{1j}=J_j\left(q^2+r^2\right)+I_j\left(p+\sigma_{1j}\right)^2; \quad 2T_{3j}=J_j\left(p^2+r^2\right)+\left(q+\sigma_{3j}\right)^2; \quad 2T_{5j}=J_j\left(p^2+q^2\right)+\left(r+\sigma_{5j}\right)^2; \\ 2T_{2j}=J_j\left(q^2+r^2\right)+I_j\left(p+\sigma_{2j}\right)^2; \quad 2T_{4j}=J_j\left(p^2+r^2\right)+\left(q+\sigma_{4j}\right)^2; \quad 2T_{6j}=J_j\left(p^2+q^2\right)+\left(r+\sigma_{6j}\right)^2; \\ 2T_j=\left[4J_j+2I_j\right]\left(p^2+q^2+r^2\right)+I_j\left(\sigma_{1j}^2+\sigma_{2j}^2+\sigma_{3j}^2+\sigma_{4j}^2+\sigma_{5j}^2+\sigma_{6j}^2\right); \end{cases} \tag{52}$$

$$2T=\left(\tilde{A}+4\bar{J}+2\bar{I}\right)p^2+\left(\tilde{B}+4\bar{J}+2\bar{I}\right)q^2+\left(\tilde{C}+4\bar{J}+2\bar{I}\right)r^2+2\sum_{j=1}^{N} I_j\left(\sigma_{1j}^2+\sigma_{2j}^2+\sigma_{3j}^2+\sigma_{4j}^2+\sigma_{5j}^2+\sigma_{6j}^2\right)$$

In the considered case the canonical impulses for angles of relative rotations are:

$$\Delta_{kj} = \frac{\partial \mathrm{T}}{\partial \dot{\delta}_{kj}} = \frac{\partial \mathrm{T}}{\partial \sigma_{kj}} = I_j \sigma_{kj}$$

and therefore, the following expression of the kinetic energy of the system with all compensated pairs of rotors (52) in terms of the Serret-Andoyer-Deprit variables (47) will have the form:

$$T = \frac{1}{2}\left[\left(G^2 - L^2\right)\left(\frac{\sin^2 l}{A} + \frac{\cos^2 l}{B}\right) + \frac{L^2}{C}\right] + \frac{1}{2}\sum_{j=1}^{N}\sum_{k=1}^{6}\frac{\Delta_{kj}^2}{I_j}; \tag{53}$$

If we consider the motion without any potential torques $\left(P\left(l, \varphi_2, \varphi_3, \delta_{ij}\right) = 0\right)$ then the Hamiltonian and the dynamics of the considered system will correspond to a simple torque-free rigid body with inertia moments $\{A, B, C\}$:

$$\mathscr{H} = \frac{1}{2}\left[\left(G^2 - L^2\right)\left(\frac{\sin^2 l}{A} + \frac{\cos^2 l}{B}\right) + \frac{L^2}{C}\right] + \frac{1}{2}\sum_{j=1}^{N}\sum_{k=1}^{6}\frac{\Delta_{kj}^2}{I_j}; \tag{54}$$

Basing on the Hamiltonian (54), it is possible to see, that the canonical equations for main positional coordinates $\{l, L\}$ and the corresponded phase portrait of the system, as well as the system's dynamics will have the form, equal to the case of a torque-free rigid body motion (fig.10). So, by this reasons the compensated pairs of rotors do not affect the dynamics of the main body of the spacecraft. Therefore, the compensated pairs' rotational energy recovery does not influence on the dynamics of the main body spacecraft.

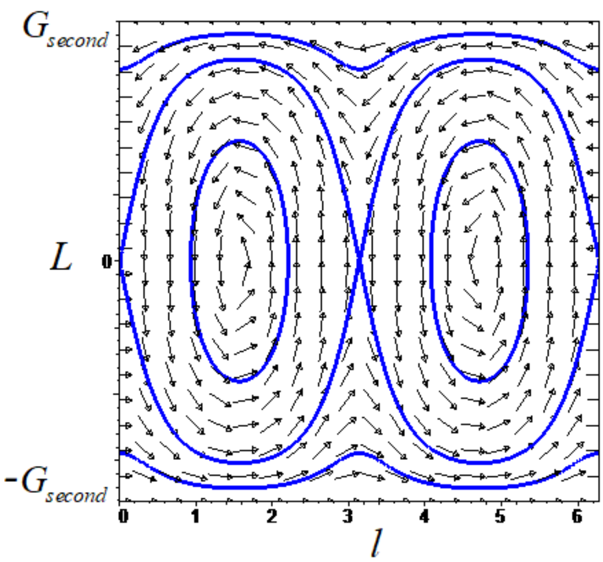


Fig. 10 The standard phase portrait {*l*, *L*} of attitude dynamics of spacecraft after compensated pairs formation which is equal to the phase portrait of a free rigid body

From the different shapes of the Hamiltonians (54) and (44) we can conclude, that the phase portraits of the motion on two stages (the first stage is the motion with saturated main wheels, and the second one is the stage after compensated pairs of rotors forming) will be different. The phase spaces of motion on these two stages are presented at the fig.11, where the left column of portraits describes the motion on the first stage, and the right column corresponds to pictures of “jumps” of one concrete phase trajectory (red) to the motion on the second stage (blue). The immediate change of the phase portraits forms occurs at the time-moment t_{comp} of the compensated pairs formation. If we imagine one single phase trajectory of the system, which belong to both phase spaces, the we need to recalculate its phase point, which is final for the first stage, and initial for the second stage from the conditions of the continuity of the angular velocity components of the main body of spacecraft $\{p, q, r\}$. Therefore, the values of coordinates of phase point at the time t_{comp} can be recalculated with the help of expressions (33)-(35) and (49)-(51):

$$\begin{cases} p\left(t_{comp}\right) = \dfrac{1}{\hat{A}}\left[\sqrt{G_{first}^2 - L_{first}^2}\sin l_{first} - D_1\right] = \dfrac{1}{A}\sqrt{G_{second}^2 - L_{second}^2}\sin l_{second}; \\ q\left(t_{comp}\right) = \dfrac{1}{\hat{B}}\left[\sqrt{G_{first}^2 - L_{first}^2}\cos l_{first} - D_3\right] = \dfrac{1}{B}\sqrt{G_{second}^2 - L_{second}^2}\cos l_{second}; \\ r\left(t_{comp}\right) = \dfrac{1}{\hat{C}}\left[L_{first} - D_5\right] = \dfrac{1}{C}L_{second}; \\ G_{first}^2 = \left(\hat{A}p\left(t_{comp}\right) + D_1\right)^2 + \left(\hat{B}q\left(t_{comp}\right) + D_3\right)^2 + \left(\hat{C}r\left(t_{comp}\right) + D_5\right)^2; \\ G_{second}^2 = \left[Ap\left(t_{comp}\right)\right]^2 + \left[Bq\left(t_{comp}\right)\right]^2 + \left[Cr\left(t_{comp}\right)\right]^2 \end{cases} \tag{55}$$

where the low index {“first” or “second”} shows the correspondence to the first or to the second stage of the spacecraft motion. From (55) the values for coordinates of the phase point follow:

$$\begin{cases} L_{first}\left(t_{comp}\right) = \hat{C}\left[r\left(t_{comp}\right) + D_5\right]; \qquad L_{second}\left(t_{comp}\right) = Cr\left(t_{comp}\right); \\ l_{first}\left(t_{comp}\right) = \arcsin\dfrac{\hat{A}p\left(t_{comp}\right) + D_1}{\sqrt{G_{first}^2 - L_{first}^2}}; \quad l_{second}\left(t_{comp}\right) = \arccos\dfrac{Bq\left(t_{comp}\right)}{\sqrt{G_{second}^2 - L_{second}^2}} \end{cases} \tag{56}$$

The values (56) define the final conditions at the first stage and the initial conditions at the second stage of the attitude motion of the spacecraft. So, we present at the right column of fig.11 the “interconnected” phase trajectory of complete phase trajectory, started in the one type of the phase space (the red trajectory) and jumped into the new phase space (the blue trajectory) at the time t_{comp}. The corresponded numerical parameters for calculations are described by the table 2.

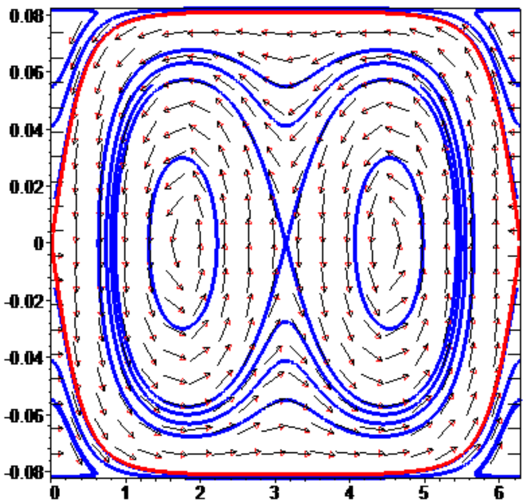
From the numerical modeling it is ossible to see, firstly, possibility of jumping dynamicas from one separatrix trajectory to another (fig.11-b, d), secondly, the jump from the rest (the equilibrium point) onto separatrix (fig.11-f), thirdly, jumps from the regime of the direct precession to the back precession (fig.11-h, j), and, finally, jump from complex dynamical regime with large oscilations of nutation angle to the regime of permanent rotation relative equatorial axis x (fig.11-l).

The obtained mathematical models in Hamiltonian form and corresponding phase portraits are very informative from the mechanical point of view and can be used in the framework of an extended analysis of the attitude dynamics of spacecraft with saturated wheels and after their unloading at the action of some additional factors and perturbations, including study of complex dynamical regimes and chaos.

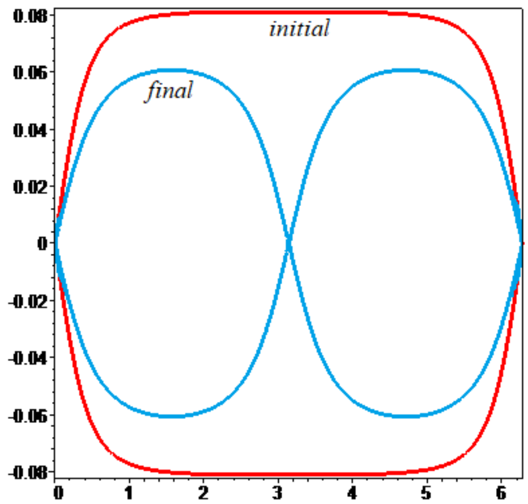
Table 2.

Fig.11	$\tilde{A}$	$\tilde{B}$	$\tilde{C}$	$\bar{J}$	$\bar{I}$	$p(t_{comp})$	$q(t_{comp})$	$r(t_{comp})$	D_1	D_3	D_5
	[kg*m^2]					[rad/s]			[kg*m^2/s]		
(a), (b)	4	7	10	0.05	0.01	0.01	0.00	0.00	0	0.0101	0
(c), (d)	4	7	10	0.05	0.01	0.01	0.00	0.00	0	0.201	0
(e), (f)	5	6	7	0.05	0.01	0.00	0.10	0.00	0.301	0	0
(g), (h)	3	6	8	0.05	0.01	0.01	0.02	-0.05	0.0501	0.3002	0.5995
(i), (j)	4	7	10	0.05	0.01	0.00	0.01	-0.05	0.01	0.1001	0.0595
(k), (l)	5	6	7	0.05	0.01	0.09	0.00	0.00	0.06	0.07	0.09

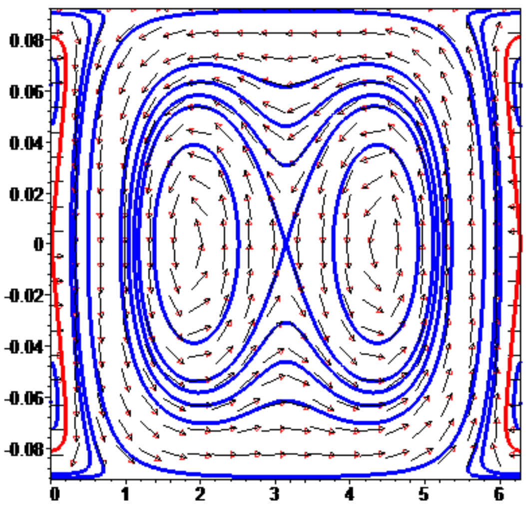
As it was indicated above, the system's dynamics with compensated pairs of all rotors will be equal to the case of a torque-free rigid body motion. This proven dynamical aspect, among other things, can be used at the design of equipment/methods to unloading the total angular momentum of complete spacecraft by the interaction with the external forces/torques. For example, when the need for the spacecraft angular momentum discharge arises, the mechanical system can be restructured into the form with compensated wheels, and in this regime the spacecraft will realize the "simple and well-known" case of rigid body attitude motion, and in this regime can be used the simple form of the angular momentum unloading without gyroscopic effects of rotated wheels (i.e. without the complex attitude dynamics of mechanical systems of gyrostats class). This is very important dynamical result.



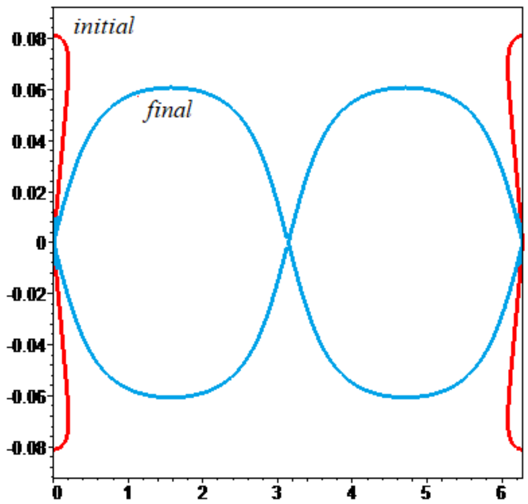
(a)



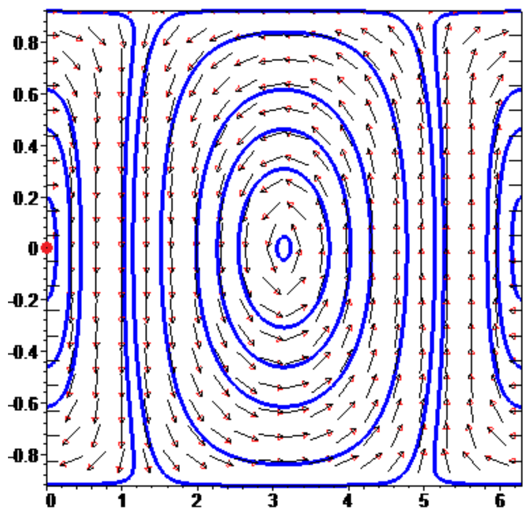
(b)



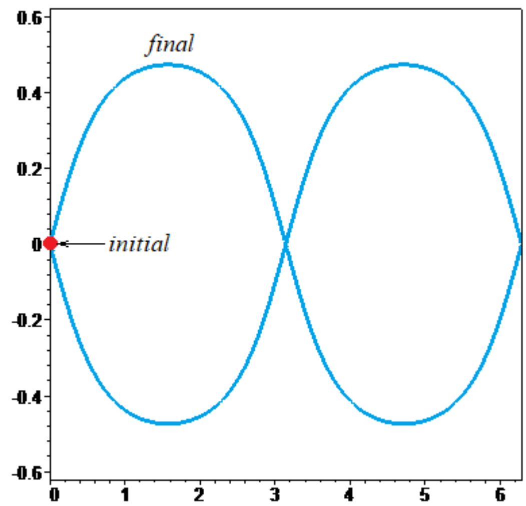
(c)



(d)



(e)



(f)

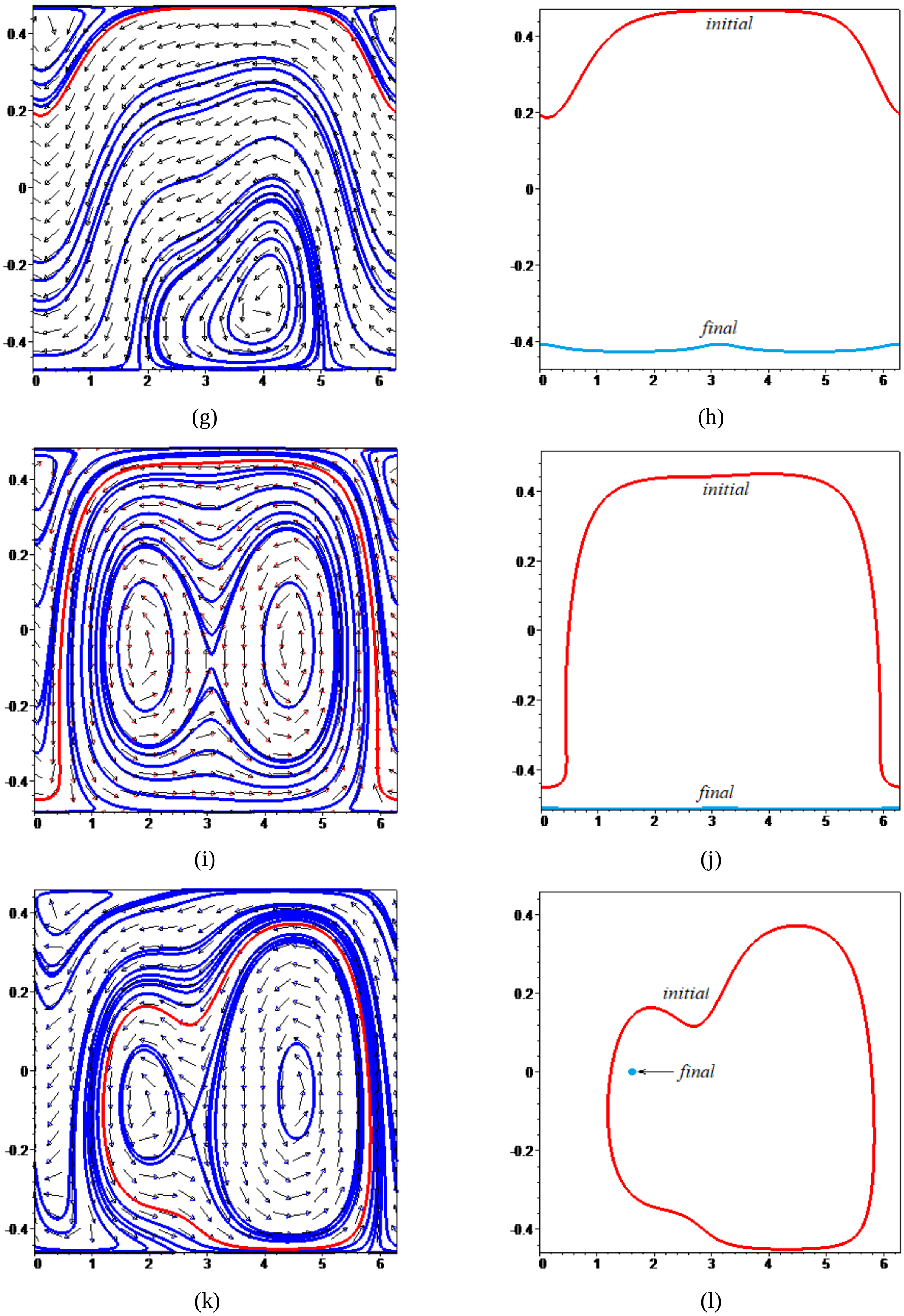


Fig. 11 Phase portraits {*l*, *L*} of attitude dynamics of spacecraft:

left column – phase portrait SC with saturated main wheels; right column – the single phase trajectory changing after compensated pairs forming (the jump from red to blue trajectory at the time-point t_{comp})

4. The Gearbox for Spacecraft, New Schemes of Attitude Control and Instantaneous Spatial Reorientations of Multi-Rotor Spacecraft

In addition to above described schemes of rotors subsystems formation to wheels unloading, it is possible to suggest another direction of this idea to advised attitude control of multi-rotor spacecraft. The presented below schemes of combinations of freed/compensated/captured rotors allow to construct new principles of attitude control realization. In the framework of attitude control, the freed/compensated/captured rotors can form different subsystems with their connections/unlocking/capture during the angular motion of the main body of spacecraft. Therefore, the described below spacecraft's internal multi-rotor subsystems will play the role of actuators of the attitude control system. These multi-rotor subsystems can be formed on the base of an integrated multi-rotor kernel, which allows to constitute any predefined combinations and unifications of rotors. By this reasons, we can call this multi-rotor kernel as the "gearbox for spacecraft", which control its attitude dynamics. So, in the next sections of the paper we present necessary details of forming the multi-rotor elements of the multifunctional gearbox for spacecraft attitude control.

4.1. Conjugate Rotors Spin-Up And Their Captures

Let us briefly describe the main aspects of using conjugate rotors in the framework of the multi-rotor spacecraft construction, and corresponding operations with them at the fulfill the conjugated spin-up and subsequent captures, that was explained in [48]. We can call as the conjugate rotors the rotors, which are placed on the one common axis and which spin under the action of spin-up torques with equal but opposite by the sign magnitudes. At the spin-up of these conjugate rotors their summarized relative (to the main body of spacecraft) angular momentum has the zero-value during whole time-interval of their relative rotation. Thus, the identical rotors rotating with equal angular velocities in opposite directions on the identical electric motors produced the identical torques, as it is presented at the figure (fig.12), will constitute the system of two conjugate rotors.

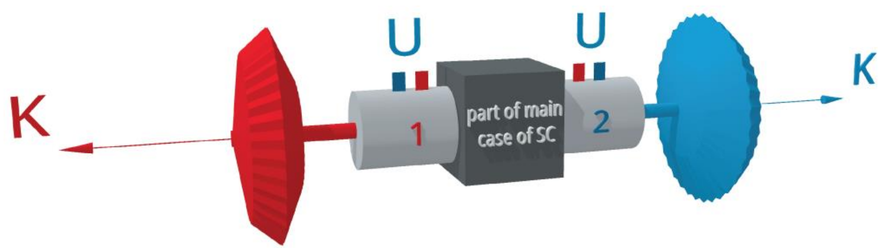
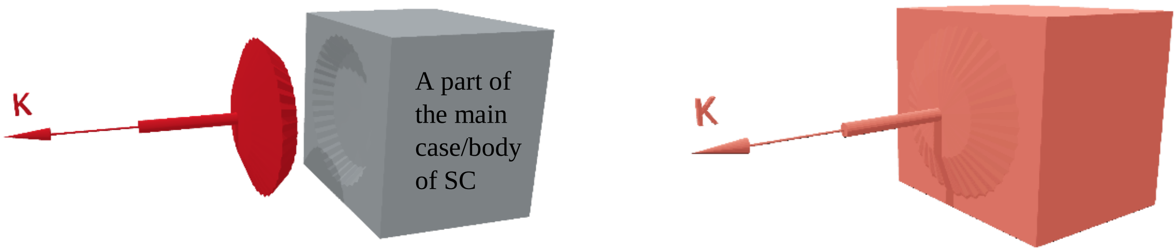


Fig. 12 The conjugate rotors on the identical opposite electric motors

On the one hand, the conjugate rotors represent the neutral pair of rotors, but, on the other hand, in contradistinction to the neutral pair the system of the conjugate rotors implies opportunity of the immediate termination of relative rotation of one of the rotors transferring the whole rotor's relative angular momentum to the main body of the spacecraft. This immediate (fast-acting) transfer of the angular momentum of the rotor is fulfilled through the internal interaction between the rotor and the main body by the direct contact (by the large friction), and therefore such mechanical process we can call as the “mechanical capture”, or simply, as the “capture”. At the capture, the main body immediately change its angular velocity that allows to influence on the attitude dynamics of the spacecraft, and to control it. The simplest scheme of the rotor capture is depicted at the figure (fig.13), where the gear-type connection is schematically present. Here we can note that the synchronous captures of the conjugate rotors do not change the dynamics, because their summarized relative angular momentum is equal to zero. Moreover, the two separated in the time captures of two conjugate rotors give to the main body the additional angular momentum on the time-interval between the captures. This stepwise increase in value of angular momentum of the main body can be used, for example, to initiate the turn of spacecraft.



(a) (b)

Fig. 13 The capture of the rotor with the immediate transfer the relative angular momentum to the spacecraft main body: the freely rotating rotor (a) and the rotor captured by the main body (b)

The conjugate rotors can be used for immediate change of the main body angular momentum with the help of one of conjugate rotors capturing (the main body receives the relative angular momentum of the captured rotor). Here it is worth to note that the second conjugate rotor remains in the free relative rotation, and if it will be captured, then the angular momentum of the main body will return to its starting value before the capture of the first conjugate rotor. Also in cases of captures implementations the phase portrait of the spacecraft will instantly change its type by analogy with “phase jumps” like at the fig.11. So, it is quite effective and fast-acting technique for attitude control implementation.

4.2. The orthogonal connection of wheels

In addition to scheme of direct connection of rotors described in section 3.3, it is important consider the scheme with the orthogonal connections (fig.14). In this case remains actual all aspects of the indicated above nonholonomic dynamics and all principle of the motion mathematical modeling, including rules of constraints writing (12)-(17).

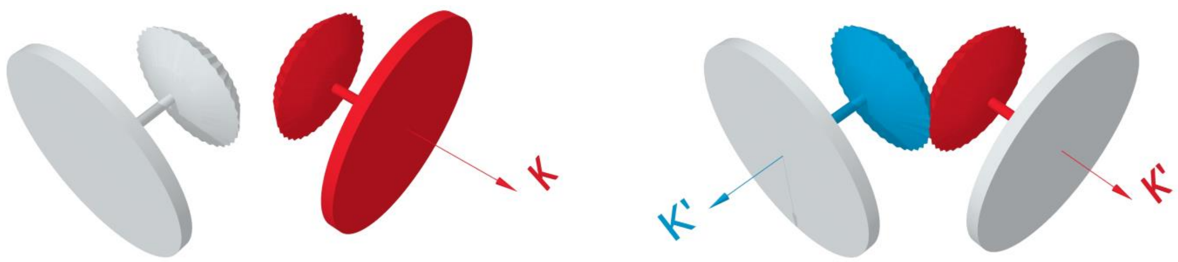


Fig. 14 The orthogonal connection of two wheels with the formation of nonholonomic constraint

Let us consider as an example [59] the system nonholonomic dynamics at the orthogonal connection of wheels ## 3 and 5, and let the gears have a non-integer ratio $(\sigma_5 = n_{53}\sigma_3)$. The corresponding coefficients and the constraint (17) will be following:

$$b_{55} = 1; \;\; b_{53} = -n_{53}; \quad \sigma_5 - n_{53}\sigma_3 = 0 \tag{57}$$

Then, with the help (16) (17) and (57), we can write the equations for wheels relative rotation:

$$\begin{cases} I\left(\dot{p}+\dot{\sigma}_1\right) = M_1^i; \\ I\left(\dot{q}+\dot{\sigma}_3\right) = M_3^i - n_{53}\lambda_{53}; \\ I\left(\dot{r}+\dot{\sigma}_5\right) = M_5^i + \lambda_{53}; \end{cases} \quad \begin{cases} I\left(\dot{p}+\dot{\sigma}_2\right) = M_2^i; \\ I\left(\dot{q}+\dot{\sigma}_4\right) = M_4^i; \\ I\left(\dot{r}+\dot{\sigma}_6\right) = M_6^i. \end{cases} \tag{58}$$

From (57) the expression can be obtained by the way of differentiation:

$$\dot{\sigma}_5 - n_{53}\dot{\sigma}_3 = 0 \tag{59}$$

The Lagrangian multiplier λ_{53} for the constraint between rotors ##3 and 5 can be found from (59) and (58):

$$\lambda_{53} = -\frac{1}{n_{53}^2+1}\left(n_{35}\left[I\dot{q} - M_3^i\right] - \left[I\dot{r} - M_5^i\right]\right)\mathrm{H}\left(t, t_{\text{constraint}}, t_{\text{free}}\right), \tag{60}$$

where function $\mathrm{H}\left(t, t_{\text{constraint}}, t_{\text{free}}\right)$ "switches on" the action of the constraint during the time-interval $t \in \left[t_{\text{constraint}}, t_{\text{free}}\right]$. In the result, we have the complete equations system {(3), (58), (60)} to model the nonholonomic dynamics. The rotors ## 2, 4, 6 we consider as fixed relative the main body $(\sigma_2 \equiv \sigma_4 \equiv \sigma_6 \equiv 0)$, and to provide relative immovability of "frozen" rotors in (61) we can formally select such internal torques (or formally "enable" large fast exponential friction like (24)):

$$M_2^i = I\dot{p}; \qquad M_4^i = I\dot{q}; \qquad M_6^i = I\dot{r} \tag{61}$$

As it was indicated above in the section 3.3, at the time-point of the nonholonomic constraint initiation the recalculation of the motion parameters should be fulfilled from the ideal conditions of conservation of corresponding components of the angular momentum and the kinetic energy (we as previously do not take into account the losses of energy due to impact of gears):

$$
\left\{
\begin{array}{l}
\left\{
\begin{array}{l}
Bq\left(t_{\text{constraint}}-0\right)+I\sigma_3\left(t_{\text{constraint}}-0\right)=K_y\left(0\right);\\
Cr\left(t_{\text{constraint}}-0\right)+I\sigma_5\left(t_{\text{constraint}}-0\right)=K_z\left(0\right);\\
\dfrac{1}{2}\left(B-I\right)q^2\left(t_{\text{constraint}}-0\right)+\dfrac{I}{2}\left(q\left(t_{\text{constraint}}-0\right)+\sigma_3\left(t_{\text{constraint}}-0\right)\right)^2=T_y\left(0\right);\\
\dfrac{1}{2}\left(C-I\right)r^2\left(t_{\text{constraint}}-0\right)+\dfrac{I}{2}\left(r\left(t_{\text{constraint}}-0\right)+\sigma_5\left(t_{\text{constraint}}-0\right)\right)^2=T_z\left(0\right);
\end{array}
\right.\\
\\
\left\{
\begin{array}{l}
Bq\left(t_{\text{constraint}}+0\right)+I\sigma_3\left(t_{\text{constraint}}+0\right)=K_y\left(0\right);\\
Cr\left(t_{\text{constraint}}+0\right)+In_{53}\sigma_3\left(t_{\text{constraint}}+0\right)=K_z\left(0\right);\\
\dfrac{1}{2}\left(B-I\right)q^2\left(t_{\text{constraint}}+0\right)+\dfrac{I}{2}\left(q\left(t_{\text{constraint}}+0\right)+\sigma_3\left(t_{\text{constraint}}+0\right)\right)^2=T_y\left(0\right);\\
\dfrac{1}{2}\left(C-I\right)r^2\left(t_{\text{constraint}}+0\right)+\dfrac{I}{2}\left(r\left(t_{\text{constraint}}+0\right)+n_{53}\sigma_3\left(t_{\text{constraint}}+0\right)\right)^2=T_z\left(0\right)
\end{array}
\right.
\end{array}
\right.
\tag{62}
$$

So, the modelling results are presented below at the figures (fig.15, 16), which are implemented for parameters from the table 3.

Table 3.

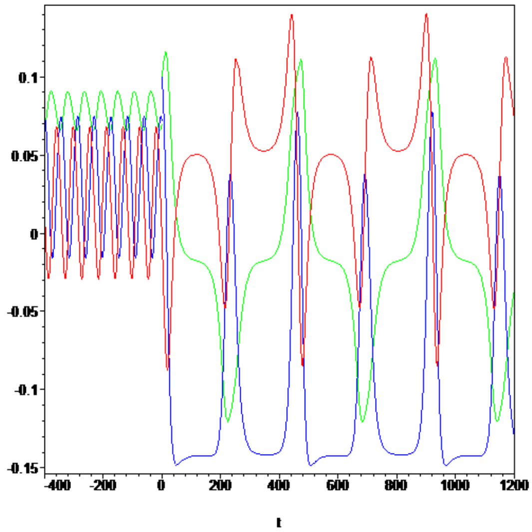
Fig.	$t_{\text{constraint}}$	t_{free}	$p(t^*)$	$q(t^*)$	$r(t^*)$	$\sigma_1(t^*)$	$\sigma_3(t^*)$	$\sigma_5(t^*)$	$p(t^*)$	$q(t^*)$	$r(t^*)$	$\sigma_1(t^*)$	$\sigma_3(t^*)$	$\sigma_5(t^*)$
	[s]		[rad/s] ($t^*= t_{\text{constraint}}$ - 0)						[rad/s] ($t^*= t_{\text{constraint}}$ + 0)					
15	0	50	0	0.07	0.07	0.15	0.1	0.15	0	0.1	0.1	0.15	-0.1	-0.15
16	0	500	0	0.07	0.07	0.15	0.1	0.15	0	0.1	0.1	0.15	-0.1	-0.15

Fig.8	A	B	C	I	$K_x(t_0)$	$K_y(t_0)$	$K_z(t_0)$	$T_x(t_0)$	$T_y(t_0)$	$T_z(t_0)$	n_{53}
	[kg*m2]				[kg*m^2/s]			[kg*m^2/s^2]			[1]
15, 16	5	7	10	1	0.15	0.6	0.85	0.01	0.03	0.05	1.5

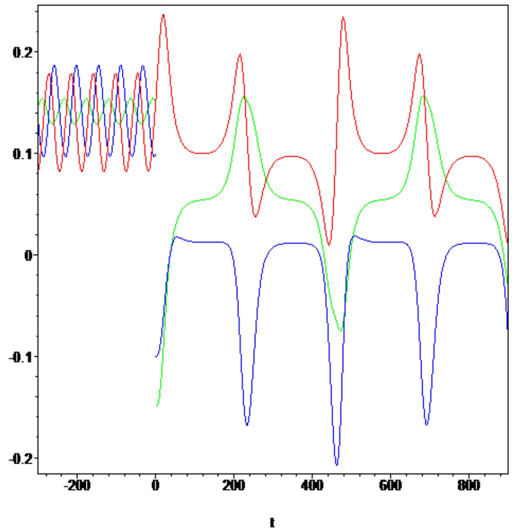
As can we see (fig.15,16), the essential redistribution of components of angular velocities and angular momentums takes place before and after initiation of the orthogonal connection of rotors ## 3 and 5 with the formation of the constraint λ_{53}. Also the change of the position of body is fulfilled – this is confirmed by values of angles θ_i between the directions of the immovable vector of the system angular momentum and the body axes:

$$
\theta_x=\arccos\frac{K_x}{K};\ \theta_y=\arccos\frac{K_y}{K};\ \theta_z=\arccos\frac{K_z}{K}
\tag{63}
$$

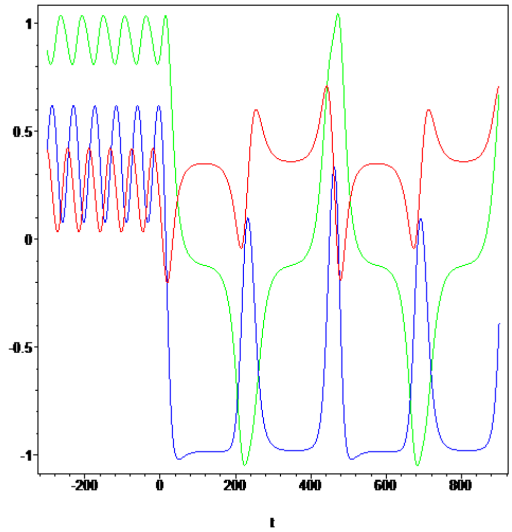
The modeling among other things shows that the time t_{free} of the constraint (57) disabling affects the magnitude of angles (63). It allows to select this time-moment to obtain the desired reorientation of the main body: the case fig.15-d corresponds to increasing the average value of θ_y, and the case fig.15-d – to increasing the average value of θ_z. So, the orthogonal connection of wheels can be applied to immediate changing components of the relative angular momentum and the attitude position of the main body.



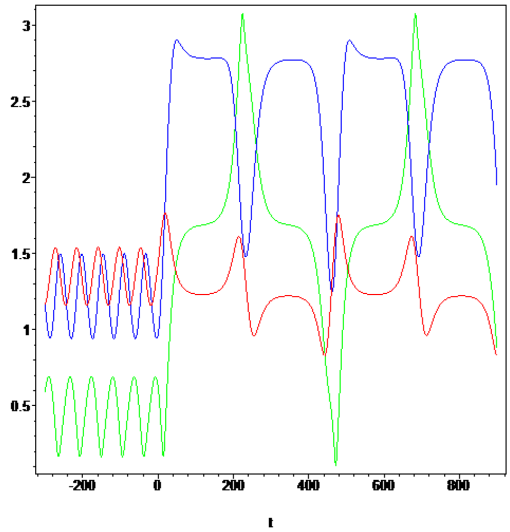
(a): $p(t)$ – red, $q(t)$ – blue, $r(t)$ – green



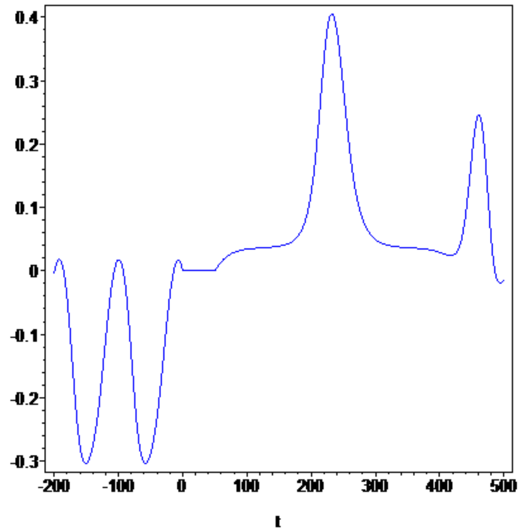
(b): $\sigma_1(t)$ – red, $\sigma_3(t)$ – blue, $\sigma_5(t)$ – green



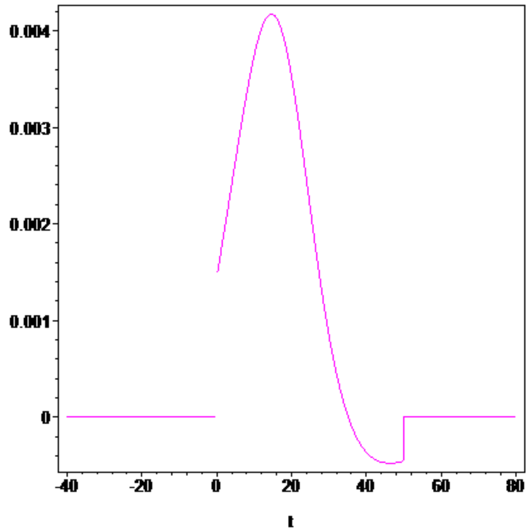
(c): $K_x(t)$ – red, $K_y\,(t)$ – blue, $K_z\,(t)$ – green



(d): $\theta_x(t)$ – red, $\theta_y(t)$ – blue, $\theta_z(t)$ – green

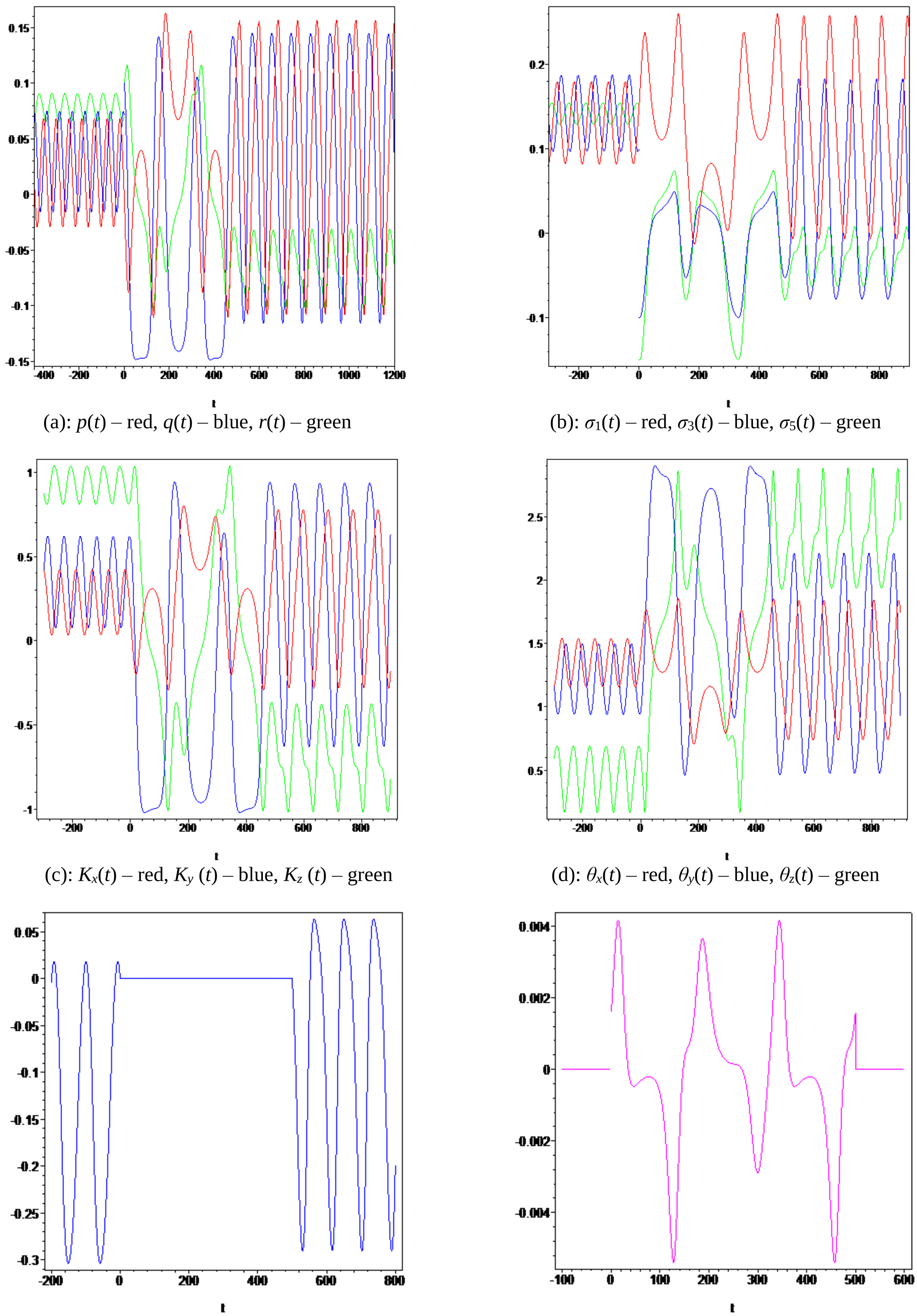


(e): the fulfilment of the constraint (57)



(f): the value of $\lambda_{53}(t)$

Fig. 15 The numerical modeling of the dynamics at the orthogonal connection of wheels – case 1



(a): $p(t)$ – red, $q(t)$ – blue, $r(t)$ – green

(b): $\sigma_1(t)$ – red, $\sigma_3(t)$ – blue, $\sigma_5(t)$ – green

(c): $K_x(t)$ – red, $K_y(t)$ – blue, $K_z(t)$ – green

(d): $\theta_x(t)$ – red, $\theta_y(t)$ – blue, $\theta_z(t)$ – green

(e): the fulfilment of the constraint (57)

(f): the value of $\lambda_{53}(t)$

Fig. 16 The numerical modeling of the dynamics at the orthogonal connection of wheels – case 2

4.3. Variants of forming and using the compensated pair of rotors to change the relative angular momentum

The idea of the formation of the neutral pair, schematically described in the second paragraph, allows to build the element of the multi-rotor kernel to change and to control the value of the relative angular momentum of the wheel. Let us consider the formation of the neutral pair by the way of releasing (fig.17-a) and connecting two auxiliary rotors (fig.17-b), which can work in three different variants. All of them should be calculated from the conditions of conservations of the total angular momentum and the energy, like it was fulfilled above at the consideration of some concrete examples.

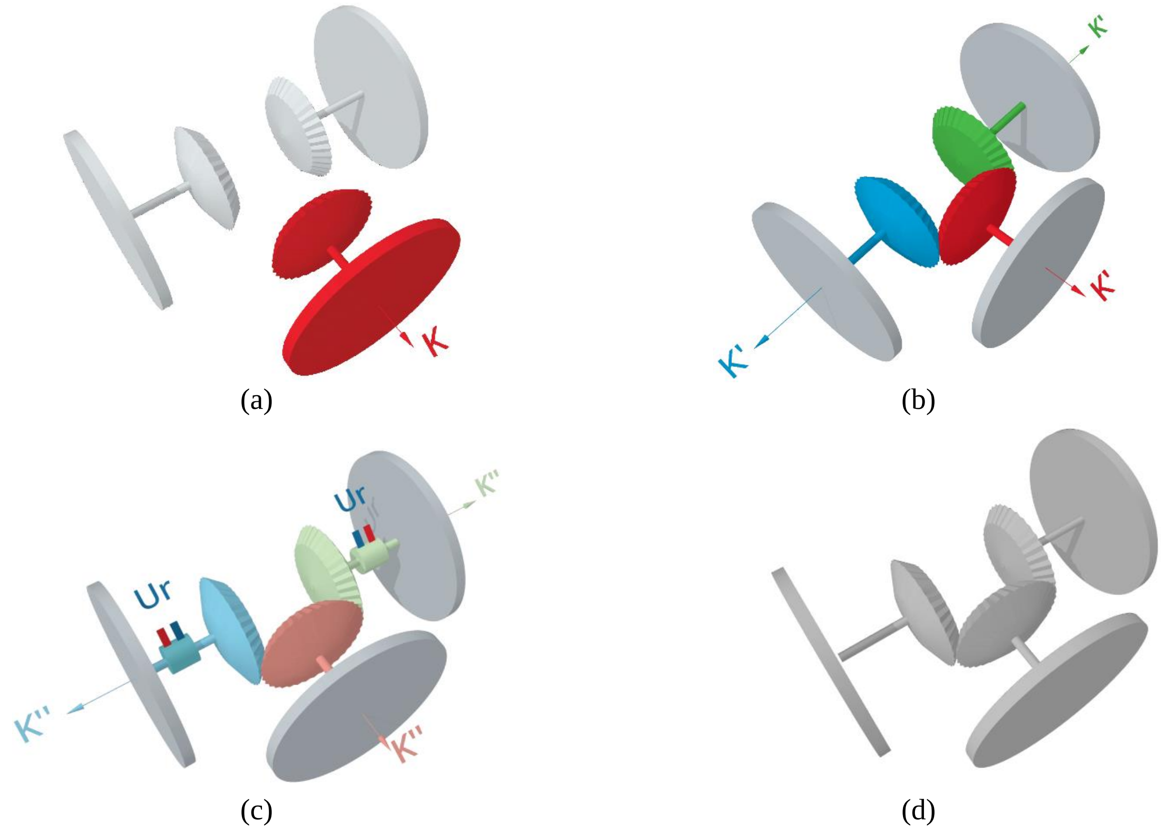


Fig. 17 The variants of forming and using the neutral pairs and the rotational energy recovery

The first variant corresponds to the simple connection of freed pair of auxiliary rotors (fig.17-b) to reduce the relative angular momentum of the wheel from the initial value *K* to the value *K'*. The second variant (fig.17-c) implies the procedure of the rotational energy recovery, when the auxiliary rotors interacts with the main body through identical electric motors working in the regime of generators, that creates in each electric motor corresponding internal torque and decelerates the angular velocities of all rotors (we can note here that the inverse process is possible, when the all rotors start from the relative rest and increase their angular velocities by the spin-up internal torques). The third variant (fig.17-d) corresponds to connection of wheel to fixed immovable auxiliary rotors, and then the direct capture will be realized (in substance, this corresponds to direct attachment to the main rigid body by analogy with the scheme depicted at fig.13), and the relative angular momentum of the wheel will be immediately transferred to the spacecraft main body.

4.4. Changing the direction of relative angular momentum of the wheel through forming two compensated pairs

The change of the relative angular momentum also is possible through the formation of the neutral pairs by the following stepwise way. At the first step the group of three connected auxiliary rotors is released (fig.18-a), which is connected to the active wheel involving into the rotation all of rotors (fig.18-b). In fact at the second step the two neutral pairs are formed (they also are compensated pairs), and therefore summarized relative angular momentum of this rotors equals to zero. At the third step, all of rotors disconnect reciprocal mechanical joining (fig.18-c), and after it, at the last fourth step the new auxiliary rotor is added (fig.18-d) to connect with three rotating rotors and to compensate their common relative angular momentum (fig.18-e). As the result, the relative angular momentum of wheels will be turned on $\pi/2$ (undoubtedly, some change in the value of the relative angular momentum will take place). Inside the neutral group of four connected rotors the recovery of rotational energy can be implemented through the all four identical rotor's electric motors activation in the generators regimes.

So, this change of the relative angular momentum provides, in fact, a change in the spatial position of the spacecraft in the absolute space, since the total moment is preserved (but now it is differently distributed between the internal rotors and the main body). That is, when the relative angular momentum changes, the spacecraft must turn in the space and must initiate the corresponding rotation. This is the new and promising possibility for the new type of the attitude control system developing.

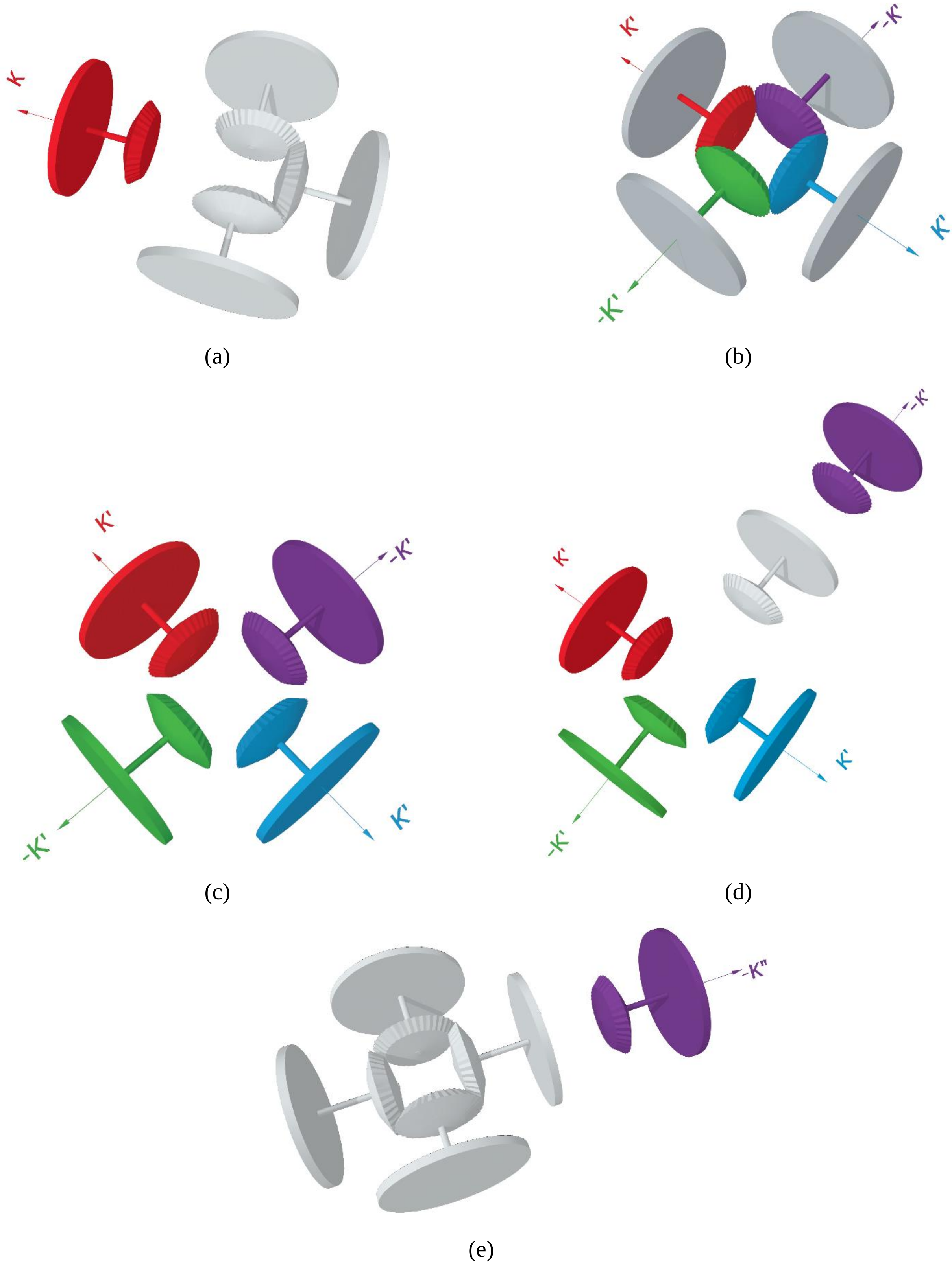


Fig. 18   Changing the relative angular momentum onto orthogonal direction

4.5. New Possibilities in Attitude Control.

Gearbox for Spacecraft. In the proposed above sections the different schemes of mechanical reconfigurations were presented and described. These schemes of reconfigurations allow to realize different operations with the spacecraft relative angular momentum created by the changeable subsystem of internal rotors. In the framework of this internal subsystem, the rotors can, firstly, be released with arising corresponded degrees of freedom, secondly, be captured by the main body of spacecraft with blocking their degrees of freedom, thirdly, they can form various connections with each other (e.g. by gear-type) and can become disconnected. On the base of this idea, the special multi-rotor kernel can be constructed, which allows to fulfill all indicted above operations with internal rotors. Due to the specifics of the functional of this subsystem, it could be called as “the gearbox for spacecraft”. The internal construction of the gearbox does not discussed in the present paper, because this is not a scientific, but an engineering problem. Moreover, all internal elements of the gearbox and all conceptions of their use already were described above. However, we can note the possibility of using the 3D multi-layer multi-rotor structure (fig.6-b), where all of rotor have independent electric motors and various types of gear-connections between each other to realizing the logic of their subsystems functioning.

The proposed gearbox will be able to perform all operations with internal rotors to unload relative angular momentum of momentum/reaction wheels, to recover the stored rotational energy of wheels, to spin-up and to capture conjugate rotors, and, finally to change the value and/or the direction of relative angular momentum by any necessary way, that, in its turn, allows to create any necessary gyroscopic torques to rotate spacecraft (41):

$$M_x^{control} = -\dot{D}_1 - \left[qD_5 - rD_3\right]; \quad M_y^{control} = -\dot{D}_3 - \left[rD_1 - pD_5\right]; \quad M_z^{control} = -\dot{D}_5 - \left[pD_3 - qD_1\right].$$

At the end of this paragraph we should underline, that the considered above schemes of changing dynamical properties of multi-rotor spacecraft by the mechanical restructuring were linked only with the relative angular momentum internal redistribution and unloading. But the total problem of the angular momentum unloading implies its complete discharge from spacecraft to external space environment. To unload the total absolute angular momentum of spacecraft it is needed to use the interaction with external environments and corresponding forces/torques. In this purpose it possible to suggest the simple mechanical device, which can be placed inside the gearbox.

The gravitational damper. So, let us place into the center of our gearbox an immovable sphere with viscous liquid (fig.19-a), in which the second internal sphere (fig.19-b) can rotate in the liquid, creating the friction. Inside the internal sphere the rigid body with three different inertia moments is rigidly fixed (fig.19-c). In other words, this mechanical construction allows to the internal rigid body together with the internal sphere to rotate in the resistance liquid relative external sphere. This relative rotation with friction dissipate the rotational energy of the main body and it discharges the system total absolute angular momentum. The internal body with the three different inertia moments starts its rotation under the action of the external gravitational field of a planet. This relative rotation of the internal sphere can be allowed or prohibited by corresponding mechanical locks, which release or lock three degrees of freedom of relative rotation of the internal body under the action of gravity torques. At the releasing these three degrees of freedom the mechanical restructuring is realized and the corresponding processes of the rotational energy dissipation and the angular momentum unloading are started. This mechanical device uses the external forces of gravity and, therefore, can be called as the internal gravitational damper [60]. We can note, it is more preferable to use the gravitational damper to unload the angular momentum when the all rotors of the gearbox are “frozen” or connected in compensated pairs, when the spacecraft attitude dynamics is equivalent to the dynamics of simple rigid body (as it was shown in this paper). Then the discharge of the angular momentum by the gravitational damper is fulfilled directly from the main body of the spacecraft. The gravitational damper can be added into the gearbox construction as one of the main parts. Detailed study of the attitude dynamics of the spacecraft with the gravitational damper it is an important scientific topic for further research.

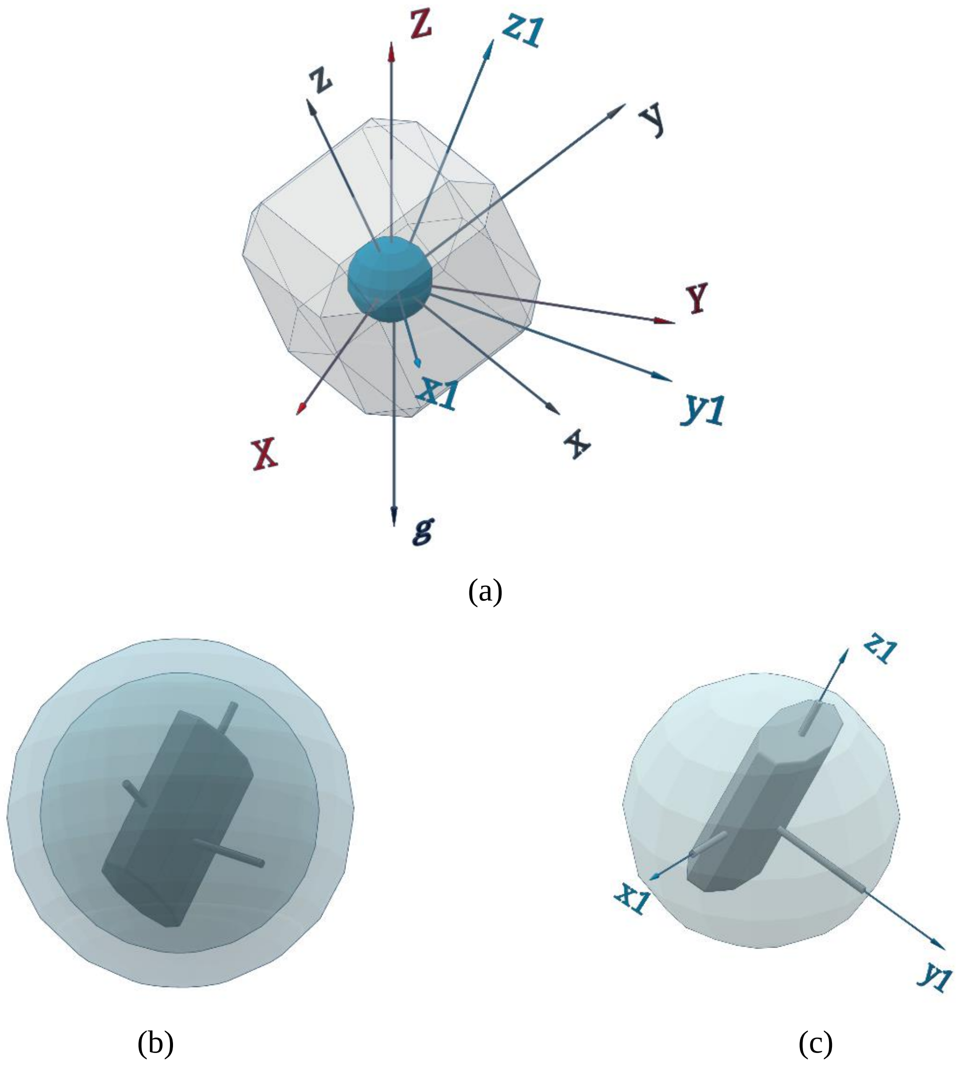


Fig. 19 The internal gravitational damper to unloading complete angular momentum of spacecraft

Indicating the alternative scheme of liquid-filled dampers for energy dissipation in space technology, we can mention such systems, like viscous “magnetic dampers” [61], and also “fluid rings” [62-65].

The viscous magnetic damper works on the principle of liquid friction creation by the movable internal magnetic element, which interact with external magnetic fields.

The “fluid rings” represents a device is pumping liquid through ring tubes using a pump. Fluid rings create a circulation of the liquid mass in the ring tube, and this creates a gyroscopic effect that can be used to control the spacecraft angular motion (the “liquid rotors” are rotated). Also, moving liquid masses in fluid rings can resist to spacecraft angular motion, i.e. it is also possible to create the internal liquid friction, which will change the relative angular momentum of the “liquid rotor” and will translate the part of the relative angular momentum to the main body. Fluid rings do not provide an interaction with any external forces fields and, therefore, it cannot unload the absolute angular momentum.

So, summarizing the indicated above aspects of the process of relative and/or total absolute angular momentum changing it is possible to conclude the following. New opportunities arise at the using the “gearboxes for spacecraft” as the new type of the attitude control systems and corresponding mechanical actuators, which can immediately redistribute the relative angular momentum of wheels and the main body, can immediately unload the rotors relative angular momentum, can unload the total absolute angular momentum by gravitational forces, and can change the phase portrait and dynamical behavior of spacecraft. All of this opportunities based on the principle of additional degrees of freedom releasing.

Like any technical system, the proposed “gearboxes for spacecraft” is not free from disadvantages. The main disadvantage of the proposed system is its certain redundancy (many degrees of freedom, many bodies, and many types of interbody connections). However, it represents the new method of redistribution of the spacecraft relative angular momentum with additional possibilities of transferring its part to “new” internal bodies with releasing “new” degree of freedom: then relative angular momentum can be transferred to “new” bodies on the longitudinal axes (relative the initial direction of the main rotor angular momentum), or to “new” bodies on orthogonal (to main rotor angular momentum) axes with large changes of the spacecraft angular attitude – this cannot be done by any currently available unloading scheme. In other words, on the one hand, the method is proposed for unloading the relative angular momentum of rotors/wheels, but on the other hand, it represents the new method for the angular motion control due to the resource of accumulated relative angular momentum. Perhaps, the last aspect is even more important.

5. Conclusion

The problem of unlading the relative angular momentum of reaction/momentum wheels is solved with the help of the new proposed principle of mechanical system restructuring. This principle suppose releasing new additional degrees of freedom and using them to initiate additional internal motion to redistribute the relative angular momentum between the system elements. According to the proposed principle, inside the main body of the spacecraft the additional rotors can be freed and connected to the main wheels by the gear type. Such connections constitute new rotors' mechanical subsystems instantly nullifying the relative angular momentum, and redistributing the rotational energy of main wheels between new rotors systems with the possibility of the recovery of electric energy consumed initially to spin-up of wheels.

The constructed principle also allows to build the multi-rotors mechanical subsystems with changeable quantity of internal degrees of freedom and different types and possibilities of rotors releasing/connecting/spin-up/disconnecting/capturing. Such multi-rotors mechanical subsystems can be used to realize different schemes of redistributing and changing the relative angular momentum of the spacecraft. It in its turn, it allows to change and to control not only the angular velocities of internal rotors, but also the angular attitude and velocity of the main body of the spacecraft. The multi-rotors subsystems were figuratively called in this paper as "gearboxes". The gearbox can have different types of its internal structure. To unloading the total absolute angular momentum in the framework of the spacecraft gearbox the "gravitational damper" is proposed. This damper interact with the external gravitational field, and through the liquid friction it dissipates the rotational energy and the total absolute angular momentum. So, the presented gearbox can be considered as the integrated form of multifunctional attitude control systems.

Summarizing all considered in this paper mechanical schemes and methods, the following advantages of the proposed gearbox can be highlighted.

Firstly, the formation of compensated pairs of rotors inside the gearbox can instantly bring the spacecraft into the "mono-body" dynamics at still rotating rotors during their kinetic energy recovery process. If we compensate all rotors of the spacecraft, then the influence of gyroscopic torques from rotating rotors will be nullified, and the whole relative angular momentum of rotors will be transferred to the main body (the whole total absolute angular momentum of the system in this case will be accumulated in the main body). It is very important property of the attitude dynamics, because in this regime we will have the simplest phase space/portrait (fig.10) of a simple rigid body, instead the gyrostat's phase spaces (fig.11, the left column). In other words, by forming compensated pairs, we instantly "turn on" the simple rigid-body-dynamics, when the simple attitude control can be carried out. It is also easy to go back into the "gyrostat mode" - we just need to disconnect compensated pairs, and to capture the auxiliary additional rotors. This rigid-body-dynamics allows to easy unload the total absolute angular momentum directly from the main body of the spacecraft by usual ways, including suggested mechanical scheme of the gravitational damper, or other actuators (thrusters, magnet coils, etc.).

Secondly, the proposed gearbox has ability to transfer the relative angular momentum of one rotating rotor to additional orthogonal rotors (and besides, with different gears ratios b_{ij}). In this case, the main body must undoubtedly fulfill an angular reorientation to provide the conservation of the total absolute angular momentum of the system. This allow to perform complex angular maneuvers that are not available for usual reaction wheels systems (e.g., it can be the instantly activation of precessions around transversal axes of spacecraft, or attitude reorientations with large angles).

Thirdly, in the frame of the gearbox different variants of rotors connection are available (planar, orthogonal, mixed schemes) with different gears ratios b_{ij} (to use a predefined part of the relative angular momentum of rotors), and also multiple variants of kinetic energy recovery are possible inside connected rotors.

Finally, all of these aspects characterize the proposed gearbox as the new multifunctional actuator for the spacecraft attitude control due to the resource of accumulated relative angular momentum of internal rotors/wheels.

Acknowledgments

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References

[1]. Rennie, R. G., & Chanowitz, H. H. (1964). A Magnetic Unloading System for an Ultra Stable Unmanned Spacecraft. *IEEE Transactions on Aerospace*, *2*(2), 420-427.

[2]. Sabroff, A. E. (1968). Advanced spacecraft stabilization and control techniques. *Journal of Spacecraft and Rockets*, *5*(12), 1377-1393.

[3]. Stakem, P. H. (1977). Passive magnetic momentum wheel unloading. *Journal of Spacecraft and Rockets*, *14*(6), 335-338.

[4]. Camillo, P. J., & Markley, F. L. (1980). Orbit-averaged behavior of magnetic control laws for momentum unloading. *Journal of Guidance and Control*, *3*(6), 563-568.

[5]. Sackheim, R. L., Fritz, D. E., & Mackli, H. (1980). Performance trends in spacecraft auxiliary propulsion systems. *Journal of spacecraft and rockets*, *17*(5), 390-395.

[6]. Ninomiya, K., Nakatani, I., Takahashi, T., & Maeda, K. (1984). Magnetic torquing scheme for attitude control of a wheel-stabilized astronomy satellite. *IFAC Proceedings Volumes*, *17*(2), 2915-2920.

[7]. Burns, T., & Flashner, H. (1989). Adaptive control applied to momentum unloading utilizing the low earth orbital environment. *Guidance, Navigation and Control Conference*, 391-401.

[8]. Flashner, H., & Burns, T. F. (1990). Spacecraft momentum unloading-The cell mapping approach. *Journal of Guidance, Control, and Dynamics*, *13*(1), 89-98.

[9]. Burns, T. F., & Flashner, H. (1992). Adaptive control applied to momentum unloading using the low earth orbital environment. *Journal of guidance, control, and dynamics*, *15*(2), 325-333.

[10]. Linder, D., Tolek, J., & Lombardo, J. (1992, January). Guidance, navigation, and control subsystem for the EOS-AM spacecraft. In *Guidance, Navigation and Control Conference* (p. 4336).

[11]. Brown, C. D. (1996). *Spacecraft propulsion*. American Institute of Aeronautics and Astronautics.

[12]. Choi, M. T., & Flashner, H. (2000, August). Neural-network-based spacecraft attitude control and momentum

management. In *AIAA Guidance, Navigation, and Control Conference and Exhibit* (p. 4455).

[13]. Tsiotras, P., Shen, H., & Hall, C. (2001). Satellite attitude control and power tracking with energy/momentum wheels. *Journal of Guidance, Control, and Dynamics*, *24*(1), 23-34.

[14]. Lovera, M. (2001). Optimal magnetic momentum control for inertially pointing spacecraft. *European Journal of Control*, *7*(1), 30-39.

[15]. Jan, Y. W., & Chiou, J. C. (2002). Unloading law for a LEO spacecraft with two-gimbals solar array. *Acta Astronautica*, *51*(12), 843-854.

[16]. Uesugi, K. T. (2003). Space engineering spacecraft (MUSES) program in ISAS featuring its latest mission "Hayabusa." *International Conference on Recent Advances in Space Technologies*, 2003. RAST '03. Proceedings Of.

[17]. Roberts, B., Kruk, J., Ake, T., Englar, T., Class, B., & Rovner, D. (2004, August). Three-axis attitude control with two reaction wheels and magnetic torquer bars. In *AIAA Guidance, Navigation, and Control Conference and Exhibit* (p. 5245).

[18]. Randolph, T., Dougherty, R., Oleson, S., Fiehler, D., & Dipprey, N. (2004). The Prometheus 1 spacecraft preliminary electric propulsion system design. In *41st AIAA/ASME/SAE/ASEE Joint Propulsion Conference & Exhibit* (p. 3889).

[19]. Giulietti, F., Quarta, A. A., & Tortora, P. (2006). Optimal control laws for momentum-wheel desaturation using magnetorquers. *Journal of guidance, control, and dynamics*, *29*(6), 1464-1468.

[20]. Kalender, S., & Flashner, H. (2006, August). A New Approach for Control of Periodically Time Varying Systems with Application to Spacecraft Momentum Unloading. In *AIAA Guidance, Navigation, and Control Conference and Exhibit* (p. 6355).

[21]. Kawaguchi, J. I., Kuninaka, H., Fujiwara, A., & Uesugi, T. (2006). MUSES-C, Its launch and early orbit operations. *Acta Astronautica*, *59*(8-11), 669-678.

[22]. Leary J.C. et al. (2007) The MESSENGER Spacecraft. In: Domingue D.L., Russell C.T. (eds) The Messenger Mission to Mercury. Springer, New York, NY.

[23]. Pittet, C., Despre, N., Tarbouriech, S., & Prieur, C. (2008). Nonlinear controller design for satellite reaction wheels unloading using anti-windup techniques. In *AIAA Guidance, Navigation and Control Conference and Exhibit* (p. 7009).

[24]. Lebedev, D. V. (2008). Momentum unloading excessive reaction-wheel system of a spacecraft. *Journal of Computer and Systems Sciences International*, *47*(4), 613-620.

[25]. Ismail, Z., & Varatharajoo, R. (2010). A study of reaction wheel configurations for a 3-axis satellite attitude control. *Advances in Space Research*, *45*(6), 750-759.

[26]. Kalender, S., & Flashner, H. (2011). Design of spacecraft momentum unloading using discrete-time formulation. *Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering*, *225*(3), 312-324.

[27]. Bogachev, A. V., Vorob'eva, E. A., Zubov, N. E., Mikrin, E. A., Misrikhanov, M. S., Ryabchenko, V. N., & Timakov, S. N. (2011). Unloading angular momentum for inertial actuators of a spacecraft in the pitch channel. *Journal of Computer and Systems Sciences International*, *50*(3), 483-490.

[28]. Acar, Y., & Horri, N. M. (2013). Optimal momentum unloading of reaction wheels in the presence of attitude control

errors. In *AIAA Guidance, Navigation, and Control (GNC) Conference* (p. 4962).

[29]. Guiggiani, A., Kolmanovsky, I., Patrinos, P., & Bemporad, A. (2015, July). Constrained model predictive control of spacecraft attitude with reaction wheels desaturation. In *2015 European Control Conference (ECC)* (pp. 1382-1387). IEEE.

[30]. Jayakody, H. S., Shi, L., Katupitiya, J., & Kinkaid, N. (2016). Robust Adaptive Coordination Controller for a Spacecraft Equipped with a Robotic Manipulator. Journal of Guidance, Control, and Dynamics, 39(12), 2699–2711.

[31]. Chocron, S., & Choukroun, D. (2019). Robust relative navigation for spacecraft rendezvous using differential drag. *Acta Astronautica*, *158*, 32-43.

[32]. Y. Mashtakov, S. Tkachev, M. Ovchinnikov. Use of External Torques for Desaturation of Reaction Wheels // Journal of Guidance, Control and Dynamics, 2018, Volume: 41, Issue: 8,

[33]. Pistiner, Josef Siegfried, and Ludwig Muhlfelder. "Spacecraft closed loop three-axis momentum unloading system." U.S. Patent No. 4,010,921. 8 Mar. 1977.

[34]. Linder, David M., Neil E. Goodzeit, and Marc Schwarzschild. "Spacecraft momentum unloading using controlled magnetic torques." U.S. Patent No. 5,123,617. 23 Jun. 1992.

[35]. Cohen, Walter J., Neil E. Goodzeit, and Michael A. Paluszek. "Spacecraft attitude control system with reaction wheel bearing protection." U.S. Patent No. 5,248,118. 28 Sep. 1993.

[36]. Tilley, Scott W., Tung Y. Liu, and John S. Higham. "Spacecraft attitude control and momentum unloading using gimballed and throttled thrusters." U.S. Patent No. 5,349,532. 20 Sep. 1994.

[37]. Decanini, Jorge H. "Spacecraft control by electrochromic devices." U.S. Patent No. 5,305,971. 26 Apr. 1994.

[38]. Price, Xen, et al. "Spacecraft attitude control system using low thrust thrusters." U.S. Patent No. 6,053,455. 25 Apr. 2000.

[39]. Sevaston, George E. "Method and apparatus for spacecraft wheel desaturation." U.S. Patent No. 6,254,036. 3 Jul. 2001.

[40]. Hennigan, Michael Ross. "Reaction wheel desaturation apparatus." U.S. Patent No. 6,523,785. 25 Feb. 2003.

[41]. Guyot, Francois. "Method and system for unloading the inertia wheels of a spacecraft." U.S. Patent No. 8,798,816. 5 Aug. 2014.

[42]. Woo, Byoungsam, Farid Ganji, and John Hutton Cooper. "Spacecraft momentum unload and station-keeping techniques." U.S. Patent 8,998,146, April 7, 2015.

[43]. Weiss, Avishai, Stefano Di Cairano, and Uros Kalabic. "Model Predictive control of spacecraft." U.S. Patent No. 9,874,879. 23 Jan. 2018.

[44]. Doroshin, A.V. (2014). Homoclinic solutions and motion chaotization in attitude dynamics of a multi-spin spacecraft. *Communications in Nonlinear Science and Numerical Simulation*, *19*(7), 2528-2552.

[45]. Doroshin, A.V. (2018). Chaos as the hub of systems dynamics. The part I – The attitude control of spacecraft by involving in the heteroclinic chaos. *Communications in Nonlinear Science and Numerical Simulation*. Volume 59, June 2018, Pages 47–66.

[46]. Doroshin, A.V. (2019). Regimes of Regular and Chaotic Motion of Gyrostats in the Central Gravity Field. *Communications in Nonlinear Science and Numerical Simulation*. Volume 69, Pages 416–431.

[47]. Aslanov V.S. (2017). Rigid Body Dynamics for Space Applications. Butterworth-Heinemann.

[48]. Doroshin, A.V. Attitude Control of Spider-type Multiple-rotor Rigid Bodies Systems (2009) Proceedings of the World Congress on Engineering 2009 Vol II WCE 2009, London, U.K., pp1544-1549.

[49]. Iñarrea, M., Lanchares, V., Pascual, A. I., & Elipe, A. (2017). Stability of the permanent rotations of an asymmetric gyrostat in a uniform Newtonian field. *Applied Mathematics and Computation*, 293, 404-415.

[50]. Gutnik S.A., Sarychev V.A. (2014) Dynamics of an axisymmetric gyrostat satellite. Equilibrium positions and their stability. *Journal of Applied Mathematics and Mechanics* 78, 249–257

[51]. Tikhonov A. A., & Tkhai V. N. (2016). Symmetric oscillations of charged gyrostat in weakly elliptical orbit with small inclination. *Nonlinear Dynamics*, 85(3), 1919-1927.

[52]. Vera J. A. (2013). The gyrostat with a fixed point in a Newtonian force field: Relative equilibria and stability. *Journal of Mathematical Analysis and Applications*, 401(2), 836-849.

[53]. Markeyev A.P. Dynamics of a body in contact with a solid surface. Moscow. Nauka. 1992. P. 336.

[54]. Kozlov V.V., On the integration theory of equations of nonholonomic mechanics, *Regular and Chaotic Dynamics*, 2002, 7(2), pp.161–176.

[55]. Borisov A.V., & Mamaev I.S. (2015). Symmetries and reduction in nonholonomic mechanics. *Regular and Chaotic Dynamics*, 20(5), pp.553-604.

[56]. Bloch A.M., Krishnaprasad P.S., Marsden J.E., and Murray R., Nonholonomic mechanical systems with symmetry, *Arch. Rational Mech. Anal*., 136, 21–99 (1996).

[57]. Bloch A.M., Marsden J.E., & Zenkov D.V. (2005). Nonholonomic dynamics. *Notices of the AMS*, 52(3), pp.320-329.

[58]. Kalenova V.I., Karapetjan A.V., Morozov V.M., & Salmina M.A. (2007). Nonholonomic mechanical systems and stabilization of motion. *Journal of Mathematical Sciences*, 146(3), pp.5877-5905.

[59]. Doroshin A.V. "Change of mechanical structures of spacecraft with variable quantity of degrees of freedom in purposes of reaction/momentum wheels unloading" (2020). *IOP Conf. Ser.: Mater. Sci. Eng*. 984 012006.

[60]. Doroshin A.V. "Unloading the angular momentum of spacecraft using internal gravitational dampers" (2021). NODYCON 2021: Second International Nonlinear Dynamics Conference. Sapienza University of Rome, p.222.

[61]. Davis L. (1964). A system for passive control of satellites through the viscous coupling of gravity gradient and magnetic fields. American Institute of Aeronautics and Astronautics Astrodynamics Guidance and Control Conference.

[62]. Maynard, R. S. (1988). Fluidic momentum controller. National Aeronautics and Space Administration Report. US Patent 4776541.

[63]. Lurie, B. J., Schier, J. A., & Iskenderian, T. C. (1991). Fluid-loop reaction system. National Aeronautics and Space Administration Report. US Patent 5026008.

[64]. Nobari N. A., Misra A. K. (2012). Attitude dynamics and control of satellites with fluid ring actuators. Journal of Guidance, Control, and Dynamics, 35(6), 1855-1864.

[65]. Nobari N. A., Misra A. K. (2014). A hybrid attitude controller consisting of electromagnetic torque rods and an active fluid ring. Acta Astronautica, 94(1), 470-479.