Attitude Dynamics of a Spacecraft with Variable Structure at Presence of Harmonic Perturbations

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Abstract: - Attitude dynamics of a spacecraft (SC) with variable structure (inertia-mass parameters variation) is examined. Equations of the motion of the SC are obtained on the base of Hamiltonian formalism in Serret-Andoyer variables. These equations can be used for analysis and synthesis of conditions of the SC attitude motion on active legs of orbital trajectories. Analytical and numerical modeling of the SC motion is realized. Existence of the SC chaotic modes of motion is demonstrated with the help of Melnikov method and Poincaré sections. Also attitude motion of a dual-spin spacecraft (DSSC) is considered at presence of small internal harmonic torque between DSSC coaxial bodies.

Key-Words: - Spacecraft; Dual-Spin Spacecraft; Variable Structure; Hamiltonian; Phase Portrait; Serret-Andoyer Variables; Numerical Modeling; Section of Poincaré; Chaos; Melnikov Function.

1 Introduction

Analysis and synthesis of rigid bodies systems' and spacecraft's attitude motion remains one of the central problems of the theoretical and applied mechanics, including the space flight dynamics [1-24].

Many authors [2-24] in various formulations examine attitude motion of the rigid bodies systems, spacecraft (SC), gyrostat-satellites, and also dual-spin spacecraft (DSSC) with constant and variable structure.

In the papers [3-5] the DSSC attitude motion modes with the spacecraft longitudinal axis tilting are analyzed. Investigation of the tilting modes [3-5] was conducted on the base of direct integration of motion equations and numerical experiments.

Works [6-11, 16, 19] are devoted to the study of perturbed motion of the rigid bodies and SC, including the action of small harmonic disturbances, variability of the inertia-mass parameters, and also damping effect, e.g. viscous (aerodynamic) drag [8]. In [12, 20] the task of the gyrostat's motion considered in cases of unperturbed motion. Papers [13, 14] reveal some aspects of angular motion of gyrostats with changing structure.

In [15, 17, 18] the DSSC with variable structure was considered; also the attitude motion's evolutions were investigated with the help of full mathematical models for systems with variable structure. Also in [15, 17] was the qualitative method developed — this method based on phase trajectories curvature evaluating and can be applied to analysis and synthesis of the attitude motion modes of the SC (DSSC) with variable inertia-mass parameters.

In this paper the angular (attitude) motion of the SC with variable structure is considered on the base of mathematical model in Hamiltonian form. The variability of structure simulates real processes of mass-inertia parameters variation in the SC (these processes may be connected, for example, with the action of rocket engines and with small elastic vibrations). Also in this paper realization possibility of the SC tilting motion like implementation of chaotic modes is investigated. Chaotization of motion arises by reason of heteroclinic separatrix-orbits splitting in the phase space at presence of small harmonic perturbations.

So, in the paper the attitude dynamics of the two particular SC types is studied in the cases of free motion realization without acting of the external forces.

2 The Mathematical Model

One of the most effective modeling techniques in dynamics is the Hamiltonian formalism [1, 21-24]. The mathematical model of the SC attitude motion with variable structure also can be derived with the help of the Hamiltonian formalism.

To construct the mathematical model of the attitude motion of SC with variable structure we can use canonical equations based on the well-known tractate [1]:

$$\dot{q}_\sigma = \frac{\partial H}{\partial p_\sigma},\ \dot{p}_\sigma = -\frac{\partial H}{\partial q_\sigma} + P_\sigma + Q_\sigma, \tag{1}$$

where q_σ, p_σ — are canonical coordinates and conjugate moments; P_σ — the generalized reactive force referred to the coordinate q_σ (this force act due to the variation of mass); Q_σ — the generalized external force; H — the system Hamiltonian function.

Let us write a general form of the Hamiltonian corresponding to the attitude motion of a free SC without any potential energy function:

$$H = T = \frac{1}{2}\vec{\omega}\cdot\hat{\mathrm{I}}\cdot\vec{\omega}, \tag{2}$$

where $\vec{\omega} = (p, q, r)^T$ — the angular velocity vector; $\hat{\mathrm{I}}$ — the tensor of inertia.

The generalized reactive force has the form [1]:

$$P_\sigma = \sum_{\nu=1}^{n}\frac{dm_\nu}{dt}\vec{u}_\nu\frac{\partial\vec{v}_\nu}{\partial\dot{q}_\sigma}, \tag{3}$$

where m_ν — the mass of ν-th rejected particle; $\vec{u}_\nu$ — the absolute velocity of the rejected particle; $\vec{v}_\nu$ — the vector of the relative motion velocity of the body point, which reject particle.

Assumption 1. The rejected particle is a material point which is separated from the main body. The particle receives the relative velocity at the time-moment of its separation from the body. Further interaction of the rejected particle and the main body is not available [1].

For the rigid body point we have the following absolute velocity of the angular motion about fixed point

$$\vec{v}_\nu = \vec{\omega}\times\vec{\rho}_\nu, \tag{4}$$

where $\vec{\rho}_\nu$ — the position vector of the point with number ν.

The reference frame used by the authors is located at the body point which coincides with the initial position of the SC mass center.

The absolute velocity of the rejected particle is

$$\vec{u}_\nu = \vec{v}_\nu + \vec{V}_{r\nu}, \tag{5}$$

where $\vec{V}_{r\nu}$ — the velocity of the rejected particle relative to the rigid body.

Expression (3) takes the form:

$$P_\sigma = P_{\sigma 1} + P_{\sigma 2}, \tag{6}$$

where

$$P_{\sigma 1} = \sum_{\nu=1}^{n}\frac{dm_\nu}{dt}\vec{v}_\nu\frac{\partial\vec{v}_\nu}{\partial\dot{q}_\sigma}, \tag{7}$$

$$P_{\sigma 2} = \sum_{\nu=1}^{n}\frac{dm_\nu}{dt}\vec{V}_{r\nu}\frac{\partial\vec{v}_\nu}{\partial\dot{q}_\sigma}. \tag{8}$$

We can calculate derivative $\frac{\partial\vec{v}_\nu}{\partial\dot{q}_\sigma}$ in (8) with the help of (4):

$$\frac{\partial\vec{v}_\nu}{\partial\dot{q}_\sigma} = \frac{\partial}{\partial\dot{q}_\sigma}\left(\vec{\omega}\times\vec{\rho}_\nu\right) = \frac{\partial\vec{\omega}}{\partial\dot{q}_\sigma}\times\vec{\rho}_\nu + \vec{\omega}\times\frac{\partial\vec{\rho}_\nu}{\partial\dot{q}_\sigma}. \tag{9}$$

Positions of points in rigid body (SC) does not depend on the generalized velocity, then $\vec{\omega}\times\frac{\partial\vec{\rho}_\nu}{\partial\dot{q}_\sigma} = 0$. Hence

$$\frac{\partial\vec{v}_\nu}{\partial\dot{q}_\sigma} = \frac{\partial}{\partial\dot{q}_\sigma}\left(\vec{\omega}\times\vec{\rho}_\nu\right) = \frac{\partial\vec{\omega}}{\partial\dot{q}_\sigma}\times\vec{\rho}_\nu \tag{10}$$

and

$$P_{\sigma 2} = \sum_{\nu=1}^{n}\frac{dm_\nu}{dt}\vec{V}_{r\nu}\cdot\left(\frac{\partial\vec{\omega}}{\partial\dot{q}_\sigma}\times\vec{\rho}_\nu\right). \tag{11}$$

We can rewrite the generalized reactive force:

$$\begin{aligned} P_{\sigma 1} &= \sum_{\nu=1}^{n}\frac{dm_\nu}{dt}\vec{v}_\nu\left(\frac{\partial\vec{\omega}}{\partial\dot{q}_\sigma}\times\vec{\rho}_\nu\right) = \\ &= \sum_{\nu=1}^{n}\frac{dm_\nu}{dt}\left(\vec{\omega}\times\vec{\rho}_\nu\right)\left(\frac{\partial\vec{\omega}}{\partial\dot{q}_\sigma}\times\vec{\rho}_\nu\right), \end{aligned} \tag{12}$$

where

$$\begin{aligned} &\left(\vec{\omega}\times\vec{\rho}_\nu\right)\left(\frac{\partial\vec{\omega}}{\partial\dot{q}_\sigma}\times\vec{\rho}_\nu\right) = \\ &= \vec{\omega}\cdot\left(\frac{\partial\vec{\omega}}{\partial\dot{q}_\sigma}\cdot\vec{\rho}_\nu^{\,2} - \vec{\rho}_\nu\left(\vec{\rho}_\nu\cdot\frac{\partial\vec{\omega}}{\partial\dot{q}_\sigma}\right)\right). \end{aligned} \tag{13}$$

Then $P_{\sigma 1}$ has the shape:

$$P_{\sigma 1} = \sum_{\nu=1}^{n}\frac{dm_\nu}{dt}\vec{\omega}\cdot\left(\frac{\partial\vec{\omega}}{\partial\dot{q}_\sigma}\cdot\vec{\rho}_\nu^{\,2} - \vec{\rho}_\nu\left(\vec{\rho}_\nu\cdot\frac{\partial\vec{\omega}}{\partial\dot{q}_\sigma}\right)\right). \tag{14}$$

It is possible to reduce the term in (14):

$$\begin{aligned} &\vec{\omega}\cdot\left(\frac{\partial\vec{\omega}}{\partial\dot{q}_\sigma}\vec{\rho}_\nu^{\,2} - \vec{\rho}_\nu\left(\vec{\rho}_\nu\cdot\frac{\partial\vec{\omega}}{\partial\dot{q}_\sigma}\right)\right) = \\ &= p\frac{\partial p}{\partial\dot{q}_\sigma}\left(y_\nu^2 + z_\nu^2\right) + q\frac{\partial q}{\partial\dot{q}_\sigma}\left(x_\nu^2 + z_\nu^2\right) + \\ &+ r\frac{\partial r}{\partial\dot{q}_\sigma}\left(z_\nu^2 + y_\nu^2\right) - \left(q\frac{\partial p}{\partial\dot{q}_\sigma} + p\frac{\partial q}{\partial\dot{q}_\sigma}\right)x_\nu y_\nu - \\ &- \left(r\frac{\partial p}{\partial\dot{q}_\sigma} + p\frac{\partial r}{\partial\dot{q}_\sigma}\right)x_\nu z_\nu - \left(r\frac{\partial q}{\partial\dot{q}_\sigma} + q\frac{\partial r}{\partial\dot{q}_\sigma}\right)y_\nu z_\nu, \end{aligned} \tag{15}$$

where $\vec{\rho}_\nu = \left(x_\nu, y_\nu, z_\nu\right)$.

After substitution of the term (15) into expression (14) we can rewrite (12):

$$P_{\sigma 1}=\sum_{\nu=1}^{n}\frac{dm_\nu}{dt}\vec{\omega}\cdot\left(\frac{\partial\vec{\omega}}{\partial\dot{q}_\sigma}\cdot\vec{\rho}_\nu^{\,2}-\vec{\rho}_\nu\left(\vec{\rho}_\nu\cdot\frac{\partial\vec{\omega}}{\partial\dot{q}_\sigma}\right)\right)=$$

$$\begin{aligned}
&=p\frac{\partial p}{\partial\dot{q}_\sigma}\sum_{\nu=1}^{n}\frac{dm_\nu}{dt}\left(y_\nu^2+z_\nu^2\right)+\\
&+q\frac{\partial q}{\partial\dot{q}_\sigma}\sum_{\nu=1}^{n}\frac{dm_\nu}{dt}\left(x_\nu^2+z_\nu^2\right)+\\
&+r\frac{\partial r}{\partial\dot{q}_\sigma}\sum_{\nu=1}^{n}\frac{dm_\nu}{dt}\left(z_\nu^2+y_\nu^2\right)-\\
&-\left(q\frac{\partial p}{\partial\dot{q}_\sigma}+p\frac{\partial q}{\partial\dot{q}_\sigma}\right)\sum_{\nu=1}^{n}\frac{dm_\nu}{dt}x_\nu y_\nu-\\
&-\left(r\frac{\partial p}{\partial\dot{q}_\sigma}+p\frac{\partial r}{\partial\dot{q}_\sigma}\right)\sum_{\nu=1}^{n}\frac{dm_\nu}{dt}x_\nu z_\nu-\\
&-\left(r\frac{\partial q}{\partial\dot{q}_\sigma}+q\frac{\partial r}{\partial\dot{q}_\sigma}\right)\sum_{\nu=1}^{n}\frac{dm_\nu}{dt}y_\nu z_\nu.
\end{aligned}\tag{16}$$

Assumption 2. Let us consider the SC as a rigid body with constant distances between all its internal points $\left(\dot{\vec{\rho}}_\nu=0\right)$. Then the following expression takes place:

$$\forall\,\nu:\ \frac{d\vec{\rho}_\nu}{dt}=0\ \Rightarrow\sum_{\nu=1}^{n}m_\nu\frac{d\vec{\rho}_\nu}{dt}=0,$$

hence

$$\sum_{\nu=1}^{n}\left(\vec{\rho}_\nu\frac{dm_\nu}{dt}\right)=\frac{d}{dt}\left(M\vec{\rho}_C\right),\tag{17}$$

where $M=M(t)$ is the SC mass; $\vec{\rho}_C=\vec{\rho}_C(t)$ — the position vector of the SC mass center.

Some terms in expression (16) reducing gives us:

$$\begin{aligned}
&\sum_{\nu=1}^{n}\frac{dm_\nu}{dt}x_\nu y_\nu=\sum_{\nu=1}^{n}\frac{d}{dt}\left(m_\nu x_\nu y_\nu\right)=\\
&=\frac{d}{dt}\sum_{\nu=1}^{n}m_\nu x_\nu y_\nu=\dot{I}_{xy}\\
&\sum_{\nu=1}^{n}\frac{dm_\nu}{dt}x_\nu z_\nu=\dot{I}_{xz}\\
&\sum_{\nu=1}^{n}\frac{dm_\nu}{dt}y_\nu z_\nu=\dot{I}_{yz}
\end{aligned}\tag{18}$$

and

$$\begin{aligned}
&\sum_{\nu=1}^{n}\frac{dm_\nu}{dt}\left(y_\nu^2+z_\nu^2\right)=\sum_{\nu=1}^{n}\frac{d}{dt}\left(m_\nu\left(y_\nu^2+z_\nu^2\right)\right)=\\
&=\frac{d}{dt}\sum_{\nu=1}^{n}m_\nu\left(y_\nu^2+z_\nu^2\right)=\dot{I}_{xx}\\
&\sum_{\nu=1}^{n}\frac{dm_\nu}{dt}\left(x_\nu^2+z_\nu^2\right)=\dot{I}_{yy}\\
&\sum_{\nu=1}^{n}\frac{dm_\nu}{dt}\left(x_\nu^2+y_\nu^2\right)=\dot{I}_{zz}.
\end{aligned}\tag{19}$$

Taking into account (18), expression (16) can be rewritten as follows:

$$\begin{aligned}
P_{\sigma 1}&=p\frac{\partial p}{\partial\dot{q}_\sigma}\dot{I}_{xx}+q\frac{\partial q}{\partial\dot{q}_\sigma}\dot{I}_{yy}+r\frac{\partial r}{\partial\dot{q}_\sigma}\dot{I}_{zz}-\\
&-\left(q\frac{\partial p}{\partial\dot{q}_\sigma}+p\frac{\partial q}{\partial\dot{q}_\sigma}\right)\dot{I}_{xy}-\left(r\frac{\partial p}{\partial\dot{q}_\sigma}+p\frac{\partial r}{\partial\dot{q}_\sigma}\right)\dot{I}_{xz}-\\
&-\left(r\frac{\partial q}{\partial\dot{q}_\sigma}+q\frac{\partial r}{\partial\dot{q}_\sigma}\right)\dot{I}_{yz}
\end{aligned}$$

or in the matrix form:

$$P_{\sigma 1}=\vec{\omega}^T\dot{I}\frac{\partial\vec{\omega}}{\partial\dot{q}_\sigma}.\tag{20}$$

As it is well known, the tensor of inertia of any rigid body can have diagonal form in the body reference frame whose axes coincide with the principal axes of the body. Let us use the principal axes as the main reference frame connected to the body. Then we have general diagonal tensor of inertia $\left(I_{xy}=I_{yz}=I_{xz}\equiv 0\right)$, hence

$$P_{\sigma 1}=p\frac{\partial p}{\partial\dot{q}_\sigma}\dot{A}+q\frac{\partial q}{\partial\dot{q}_\sigma}\dot{B}+r\frac{\partial r}{\partial\dot{q}_\sigma}\dot{C},\tag{21}$$

where $A=I_{xx}$, $B=I_{yy}$, $C=I_{zz}$; $\vec{\omega}=\left(p,q,r\right)^T$ — are the components of the angular velocity $\vec{\omega}$.

Assumption 3. Assume that all particles are rejected in the same direction (the rocket engine of the SC has the single direction of trust). Then the relative velocity (jet-vector) for all rejected points in the body frame is:

$$\forall\,\nu:\ \ V_{r\nu}=V=\left(V_1,V_2,V_3\right)^T.$$

In this case we can write:

$$\begin{aligned}
P_{\sigma 2}&=\sum_{\nu=1}^{n}\frac{dm_\nu}{dt}\vec{V}_{r\nu}\cdot\left(\frac{\partial\vec{\omega}}{\partial\dot{q}_\sigma}\times\vec{\rho}_\nu\right)=\\
&=\vec{V}\cdot\sum_{\nu=1}^{n}\frac{dm_\nu}{dt}\left(\frac{\partial\vec{\omega}}{\partial\dot{q}_\sigma}\times\vec{\rho}_\nu\right).
\end{aligned}\tag{22}$$

After transformation we have:

$$\begin{aligned}
P_{\sigma 2}&=\vec{V}\cdot\sum_{\nu=1}^{n}\frac{dm_\nu}{dt}\left(\frac{\partial\vec{\omega}}{\partial\dot{q}_\sigma}\times\vec{\rho}_\nu\right)=\\
&=\vec{V}\cdot\sum_{\nu=1}^{n}\left(\frac{\partial\vec{\omega}}{\partial\dot{q}_\sigma}\times\frac{dm_\nu}{dt}\vec{\rho}_\nu\right)=\\
&=\vec{V}\cdot\frac{\partial\vec{\omega}}{\partial\dot{q}_\sigma}\times\left(\sum_{\nu=1}^{n}\frac{dm_\nu}{dt}\vec{\rho}_\nu\right).
\end{aligned}\tag{23}$$

With the help of (17) we get:

$$
\begin{aligned}
P_{\sigma 2} &= \vec{V} \cdot \frac{\partial \vec{\omega}}{\partial \dot{q}_\sigma} \times \frac{d}{dt}\left(M \vec{\rho}_C\right) = \\
&= \left[\vec{V}, \frac{\partial \vec{\omega}}{\partial \dot{q}_\sigma}, \frac{d}{dt}\left(M \vec{\rho}_C\right)\right],
\end{aligned}
\tag{24}
$$

where $\left[\vec{V}, \frac{\partial \vec{\omega}}{\partial \dot{q}_\sigma}, \frac{d}{dt}\left(M \vec{\rho}_C\right)\right]$ is mixed product.

Then we obtain new form of (6):

$$
P_\sigma = \vec{\omega}^T \hat{I} \frac{\partial \vec{\omega}}{\partial \dot{q}_\sigma} + \left[\vec{V}, \frac{\partial \vec{\omega}}{\partial \dot{q}_\sigma}, \frac{d}{dt}\left(M \vec{\rho}_C\right)\right]. \tag{25}
$$

Finally, after substitution (25) into (1) we can write motion equations of the SC with variable structure:

$$
\begin{aligned}
\dot{q}_\sigma &= \frac{\partial H}{\partial p_\sigma}, \\
\dot{p}_\sigma &= -\frac{\partial H}{\partial q_\sigma} + \vec{\omega}^T \hat{I} \frac{\partial \vec{\omega}}{\partial \dot{q}_\sigma} + \left[\vec{V}, \frac{\partial \vec{\omega}}{\partial \dot{q}_\sigma}, \frac{d}{dt}\left(M \vec{\rho}_C\right)\right].
\end{aligned}
\tag{26}
$$

Assumption 4. Assume that the center of mass remains at the same place in the body volume during process of the SC's structure changing:

$$
\frac{d}{dt}\left(M \vec{\rho}_C\right) = \dot{M} \vec{\rho}_C .
$$

Then (25) become:

$$
\begin{aligned}
P_\sigma &= \vec{\omega}^T \hat{I} \frac{\partial \vec{\omega}}{\partial \dot{q}_\sigma} + \dot{M} \cdot \vec{V} \cdot \frac{\partial \vec{\omega}}{\partial \dot{q}_\sigma} \times \vec{\rho}_C = \\
&= \vec{\omega}^T \hat{I} \frac{\partial \vec{\omega}}{\partial \dot{q}_\sigma} + \dot{M}\left[\vec{V}, \frac{\partial \vec{\omega}}{\partial \dot{q}_\sigma}, \vec{\rho}_C\right].
\end{aligned}
\tag{27}
$$

The motion equations take the form:

$$
\begin{aligned}
\dot{q}_\sigma &= \frac{\partial H}{\partial p_\sigma}, \\
\dot{p}_\sigma &= -\frac{\partial H}{\partial q_\sigma} + \vec{\omega}^T \hat{I} \frac{\partial \vec{\omega}}{\partial \dot{q}_\sigma} + \dot{M}\left[\vec{V}, \frac{\partial \vec{\omega}}{\partial \dot{q}_\sigma}, \vec{\rho}_C\right].
\end{aligned}
\tag{28}
$$

3 Canonical equations of the SC motion in the Serret-Andoyer variables

For examination of the SC motion we can use the Serret-Andoyer variables [21-24], which connected with the angular moment $\mathbf{K}$ of the SC (fig. 1):

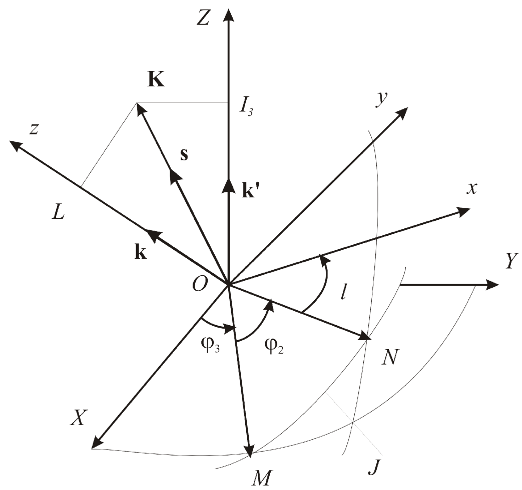

Figure 1 — The Serret-Andoyer variables

$$
\begin{gathered}
L = \frac{\partial \mathrm{T}}{\partial \dot{l}} = \mathbf{K} \cdot \mathbf{k}, \; G = \frac{\partial \mathrm{T}}{\partial \dot{\varphi}_2} = \mathbf{K} \cdot \mathbf{s} = \left|\mathbf{K}\right| = K, \\
I_3 = \frac{\partial \mathrm{T}}{\partial \dot{\varphi}_3} = \mathbf{K} \cdot \mathbf{k}', \quad L \le G, \\
K_x = Ap = \sqrt{G^2 - L^2} \sin l, \\
K_y = Bq = \sqrt{G^2 - L^2} \cos l, \\
K_z = Cr = L.
\end{gathered}
\tag{29}
$$

Assume that the body inertia tensor remains diagonal form during the mass modification process. Let us consider an asymmetric triaxial SC:

$$
\hat{I} = diag\left(A(t), B(t), C(t)\right), \quad A > B > C \; \forall t .
$$

Then the SC angular velocity components in the Serret-Andoyer variables have the from [24]:

$$
\begin{gathered}
p = \frac{1}{A}\sqrt{G^2 - L^2} \sin l, \quad q = \frac{1}{B}\sqrt{G^2 - L^2} \cos l, \\
r = \frac{L}{C}.
\end{gathered}
\tag{30}
$$

In considered case the system Hamiltonian can be written [21-24]:

$$
\begin{gathered}
H = \frac{G^2 - L^2}{2}\left(\frac{\sin^2 l}{A} + \frac{\cos^2 l}{B}\right) + \frac{L^2}{2C}, \\
\dot{\varphi}_2 = \frac{\partial H}{\partial G}, \; \dot{l} = \frac{\partial H}{\partial L}, \dot{G} = -\frac{\partial H}{\partial \varphi_2}, \dot{L} = -\frac{\partial H}{\partial l}.
\end{gathered}
\tag{31}
$$

From (31) components of the vector $\vec{\omega}$ also can be expressed through the Serret-Andoyer variables:

$$
\begin{aligned}
p &= \frac{1}{A}\sqrt{\frac{\dot{\varphi}_2^2}{\alpha_G^2(l,t)} - \frac{\dot{l}^2}{\alpha_L^2(l,t)}} \sin l, \\
q &= \frac{1}{B}\sqrt{\frac{\dot{\varphi}_2^2}{\alpha_G^2(l,t)} - \frac{\dot{l}^2}{\alpha_L^2(l,t)}} \cos l, \\
r &= \frac{1}{C}\frac{\dot{l}}{\alpha_L(l,t)},
\end{aligned}
\tag{32}
$$

where

$$\alpha_L(l,t) = \frac{1}{C(t)} - \frac{\sin^2 l}{A(t)} - \frac{\cos^2 l}{B(t)},$$
$$\alpha_G(l,t) = \frac{\sin^2 l}{A(t)} + \frac{\cos^2 l}{B(t)} \tag{33}$$

With the help of (32), (33) we can write the following partial derivatives

$$\frac{\partial \vec{\omega}}{\partial \dot{l}} = \left(\frac{\partial p}{\partial \dot{l}}, \frac{\partial q}{\partial \dot{l}}, \frac{\partial r}{\partial \dot{l}}\right)^T = \begin{pmatrix} -\dfrac{\sin l}{A\sqrt{G^2-L^2}}\dfrac{L}{\alpha_L} \\ -\dfrac{\sin l}{A\sqrt{G^2-L^2}}\dfrac{L}{\alpha_L} \\ \dfrac{1}{C\alpha_L} \end{pmatrix}, \tag{34}$$

$$\frac{\partial \vec{\omega}}{\partial \dot{\varphi}_2} = \left(\frac{\partial p}{\partial \dot{\varphi}_2}, \frac{\partial q}{\partial \dot{\varphi}_2}, \frac{\partial r}{\partial \dot{\varphi}_2}\right)^T = \begin{pmatrix} \dfrac{\sin l}{A\sqrt{G^2-L^2}}\dfrac{G}{\alpha_G} \\ \dfrac{\cos l}{B\sqrt{G^2-L^2}}\dfrac{G}{\alpha_G} \\ 0 \end{pmatrix} \tag{35}$$

$$\frac{\partial \vec{\omega}}{\partial \dot{\varphi}_3} = \left(\frac{\partial p}{\partial \dot{\varphi}_3}, \frac{\partial q}{\partial \dot{\varphi}_3}, \frac{\partial r}{\partial \dot{\varphi}_3}\right)^T = (0,0,0)^T. \tag{36}$$

In this work we consider the case when $\rho_C = (0,0,0)^T$ during the inertia-mass changing (e.g. during propellant burning). It corresponds to symmetrical modes of solid propellant burning. Then from (25) follow:

$$P_l = \frac{L}{\alpha_L}\left(\frac{1}{C^2}\dot{C} - \frac{\sin^2 l}{A^2}\dot{A} - \frac{\cos^2 l}{B^2}\dot{B}\right), \tag{37}$$

$$P_{\varphi_2} = \frac{G}{\alpha_G}\left(\frac{\sin^2 l}{A^2}\dot{A} + \frac{\cos^2 l}{B^2}\dot{B}\right). \tag{38}$$

$$P_{\varphi_3} = 0 \tag{39}$$

Take into account expressions (1), (37), (38) and (39) we can obtain the free $(Q_s = 0)$ attitude motion equations of the SC with variable structure (mass-inertia parameters):

$$\begin{cases} \dot{l} = L\left(\dfrac{1}{C} - \dfrac{\sin^2 l}{A} - \dfrac{\cos^2 l}{B}\right), \\ \dot{L} = -\dfrac{1}{2}\left(G^2 - L^2\right)\left(\dfrac{1}{A} - \dfrac{1}{B}\right)\sin 2l + \\ +\dfrac{L}{\alpha_L}\left(\dfrac{1}{C^2}\dot{C} - \dfrac{\sin^2 l}{A^2}\dot{A} - \dfrac{\cos^2 l}{B^2}\dot{B}\right), \\ \dot{\varphi}_2 = G\left(\dfrac{\sin^2 l}{A} + \dfrac{\cos^2 l}{B}\right), \\ \dot{G} = \dfrac{G}{\alpha_G}\left(\dfrac{\sin^2 l}{A^2}\dot{A} + \dfrac{\cos^2 l}{B^2}\dot{B}\right), \\ \dot{\varphi}_3 = 0, \ \dot{I}_3 = 0. \end{cases} \tag{40}$$

It is needed to note, that the classical equations for free angular motion of the rigid body about fixed point (the SC's attitude motion) follow from (40) at constant inertia-mass parameters [19-24]:

$$\begin{cases} \dot{l} = L\left(\dfrac{1}{C} - \dfrac{\sin^2 l}{A} - \dfrac{\cos^2 l}{B}\right), \\ \dot{L} = -\dfrac{1}{2}\left(G^2 - L^2\right)\left(\dfrac{1}{A} - \dfrac{1}{B}\right)\sin 2l, \\ \dot{\varphi}_2 = G\left(\dfrac{\sin^2 l}{A} + \dfrac{\cos^2 l}{B}\right), \\ G = Const, \quad \varphi_3 = Const, \ I_3 = Const. \end{cases} \tag{41}$$

4 Chaotic motion cases at presence of harmonic perturbations

Initiation of complicated irregular (chaotic) modes is very important aspect of SC attitude dynamics' investigation. One of the main reasons of the motion chaotization is the splitting of the homo(hetero)clinic separatrices-orbits [6-11, 19, 25]. The separatrices-orbits splitting implies separation and multiple intersections of stable and unstable manifolds of saddle homo(hetero)clinic points. Therefore close to the homo(hetero)clinic orbit phase trajectories form the chaotic layer. Inside the chaotic layer phase trajectory can passes through different phase space regions and, therefore, the SC performs complicated chaotic evolutions with repeating modes (rotation-oscillation-rotation-…) and the complex tilting motion. Analytical detection of the chaotic layer existence we can provide on the base of the Melnikov method [25].

So, let us consider some cases of the SC motion at presence of harmonic perturbation.

4.1 Motion of the SC with synchronous modifications of the moments of inertia

4.1.1 Motion equations

Assume that the inertia moments have the following time-dependences:

$$A = A_0\left(1+\varepsilon\sin\Omega t\right),\; B = B_0\left(1+\varepsilon\sin\Omega t\right),$$
$$C = C_0\left(1-\varepsilon\sin\Omega t\right),$$

where ε is the small parameter $\left(\varepsilon \ll 1\right)$.

These time-dependences can be applied to modeling of the elastic properties of the SC's construction when the elastic vibrations lead to the compression of the body in one direction at the stretching along the other axes.

Then derivates of inertia moments take the form:

$$\dot{A} = \varepsilon A_0 \Omega\cos\Omega t,\; \dot{B} = \varepsilon B_0 \Omega\cos\Omega t,$$
$$\dot{C} = -\varepsilon C_0 \Omega\cos\Omega t.$$

It is needed to note that the many-sided analysis of the motion chaotization of the SC with harmonic inertia moments was conducted in the papers [7-10].

In this case we can rewrite equations of the SC motion (41):

$$\begin{cases} \dot{l} = \dfrac{L}{C_0\left(1-\varepsilon\sin\Omega t\right)} - \\ -L\left(\dfrac{\sin^2 l}{A_0\left(1+\varepsilon\sin\Omega t\right)} + \dfrac{\cos^2 l}{B_0\left(1+\varepsilon\sin\Omega t\right)}\right), \\ \dot{L} = -\dfrac{1}{2}\left(G^2-L^2\right)\left(\dfrac{1}{A_0}-\dfrac{1}{B_0}\right)\dfrac{\sin 2l}{1+\varepsilon\sin\Omega t} + \\ +\dfrac{\varepsilon L\Omega\cos\Omega t}{\alpha_L}\gamma_L, \\ \dot{\varphi}_2 = \dfrac{G}{1+\varepsilon\sin\Omega t}\bar{\alpha}_G, \\ \dot{G} = G\Omega\cos\Omega t\dfrac{\varepsilon}{1+\varepsilon\sin\Omega t}, \end{cases} \qquad (42)$$

where

$$\bar{\alpha}_L\left(l,t\right) = \frac{1}{C_0} - \frac{\sin^2 l}{A_0} - \frac{\cos^2 l}{B_0},$$

$$\bar{\alpha}_G\left(l,t\right) = \frac{\sin^2 l}{A_0} + \frac{\cos^2 l}{B_0},$$

$$\gamma_L = \frac{-1}{C_0\left(1-\varepsilon\sin\Omega t\right)^2} - \frac{\sin^2 l}{A_0\left(1+\varepsilon\sin\Omega t\right)^2} - \\ -\frac{\cos^2 l}{B_0\left(1+\varepsilon\sin\Omega t\right)^2}$$

Also it is possible to note that the considered SC represents a system with one and a half degrees of freedom, because the system (42) contains one independent equation (for $\dot{\varphi}_2$) which can be eliminated.

The Taylor series expansion (by ε) gives the following first approximation for the fourth equation (42):

$$\dot{G} = \varepsilon G\Omega\cos\Omega t \qquad (43)$$

Integration of (43) gives us the following dependence:

$$G = G_0\exp\left(\varepsilon\sin\Omega t\right). \qquad (44)$$

After ssubstitutions of the solution (44) and the Taylor series expansion with elimination of the terms of higher order of smallness $\left(\sim\varepsilon^i,\; i>1\right)$ we can obtain:

$$\begin{cases} \dot{l} = f_l\left(l,L\right) + \varepsilon g_l\left(L,l,t\right), \\ \dot{L} = f_L\left(l,L\right) + \varepsilon g_L\left(L,l,t\right), \end{cases} \qquad (45)$$

where

$$f_l\left(l,L\right) = L\bar{\alpha}_L, \qquad (46)$$

$$f_L\left(l,L\right) = -\frac{1}{2}\left(G_0^2-L^2\right)\left(\frac{1}{A_0}-\frac{1}{B_0}\right)\sin 2l, \qquad (47)$$

$$g_l\left(L,l,t\right) = L\left(\frac{2}{C_0}-\bar{\alpha}_L\right)\sin\Omega t, \qquad (48)$$

$$g_L\left(L,l,t\right) = \\ = -\frac{1}{2}\left(G_0^2-L^2\right)\left(\frac{1}{A_0}-\frac{1}{B_0}\right)\sin 2l\sin\Omega t - \\ -L\Omega\cos\Omega t\frac{\dfrac{2}{C_0}-\bar{\alpha}_L}{\bar{\alpha}_L} \qquad (49)$$

So, in this paper we consider analogues case of the SC motion chaotization [7-10] on the base of new form of perturbed dynamical equations (45).

4.1.2 The chaotization analysis

In considering case standard Melnikov function [25] (with multiplier ε) can be used for motion local chaotization detection:

$$M\left(t_0\right) = \varepsilon\int_{-\infty}^{\infty}\left[f_l g_L - f_L g_l\right]\left(\bar{L}\left(t\right),\bar{l}\left(t\right),t+t_0\right)dt \quad (50)$$

where $\bar{L}\left(t\right),\bar{l}\left(t\right)$ are the dependences which correspond to the heteroclinic phase-orbits.

These heteroclinic dependences for unperturbed $\left(\varepsilon=0\right)$ rigid body motion are the well-known classical solutions in phase space of body angular moment's components in the body coordinate frame $\left(G_X = A_0 p;\; G_Y = B_0 q;\; G_Z = C_0 r\right)$, for example [6-10]:

$$\begin{cases} G_X^* = (-1)^{[(k-1)/2]} G_0 \sqrt{\dfrac{a_3 - a_2}{a_3 - a_1}} \operatorname{sech}(n_2 t), \\ G_Y^* = (-1)^{k-1} G_0 \operatorname{tgh}(n_2 t), \\ G_Z^* = (-1)^{[k/2]} G_0 \sqrt{\dfrac{a_2 - a_1}{a_3 - a_1}} \operatorname{sech}(n_2 t), \end{cases} \tag{51}$$

$$k = 1,2,3,4,$$

$$n_2 = \sqrt{(a_2 - a_1)(a_3 - a_2)},$$

$$a_1 = \frac{1}{A_0}, \quad a_2 = \frac{1}{B_0}, \quad a_3 = \frac{1}{C_0}.$$

where [b] stands for the integer part of b.

From (30) auxiliary expressions follow:

$$\sin^2 \overline{l} = \frac{G_X^{*2}}{G_0^2 - G_Z^{*2}} \tag{52}$$

$$\cos^2 \overline{l} = \frac{G_Y^{*2}}{G_0^2 - G_Z^{*2}} \tag{53}$$

$$\sin 2\overline{l} = 2\sin\overline{l}\cos\overline{l} = 2\frac{G_X^* G_Y^*}{G_0^2 - G_Z^{*2}} \tag{54}$$

$$\overline{\alpha}_L(\overline{l}, \overline{L}) = a_3 - \frac{a_1 G_X^{*2} + a_2 G_Y^{*2}}{G_0^2 - G_Z^{*2}} \tag{55}$$

$$\overline{\alpha}_G(\overline{l}, \overline{L}) = \frac{a_1 G_X^{*2} + a_2 G_Y^{*2}}{G_0^2 - G_Z^{*2}} \tag{56}$$

With the help of (51)-(56) we rewrite expressions (46)-(49):

$$f_l\big|_{(\overline{l},\overline{L}),G_0} = G_Z^*\left(a_3 - \frac{a_1 G_X^{*2} + a_2 G_Y^{*2}}{G_0^2 - G_Z^{*2}}\right) \tag{57}$$

$$f_L\big|_{(\overline{l},\overline{L}),G_0} = -(a_1 - a_2) G_X^* G_Y^* \tag{58}$$

$$g_l\big|_{(\overline{l},\overline{L}),G_0} = G_Z^*\left(a_3 + \frac{a_1 G_X^{*2} + a_2 G_Y^{*2}}{G_0^2 - G_Z^{*2}}\right)\sin\Omega t \tag{59}$$

$$g_L\big|_{(\overline{l},\overline{L}),G_0} = -(a_1 - a_2) G_X^* G_Y^* \sin\Omega t - \\ -G_Z^*\Omega\cos\Omega t\left(\frac{a_3 + \dfrac{a_1 G_X^{*2} + a_2 G_Y^{*2}}{G_0^2 - G_Z^{*2}}}{a_3 - \dfrac{a_1 G_X^{*2} + a_2 G_Y^{*2}}{G_0^2 - G_Z^{*2}}}\right) \tag{60}$$

Take into account (57)-(60) and (50) the following harmonic Melnikov function can be obtained [25]:

$$M(t_0) = \varepsilon\xi\cos(\Omega t), \quad \xi \neq 0, \tag{61}$$

where

$$\xi = \beta_1 I_1 + \beta_2 I_2 + \beta_3 I_3 + \beta_4 I_4 = \text{const},$$

$$\beta_1 = -2a_1(a_2 - a_1) < 0, \quad \beta_2 = -2\frac{(a_2 - a_1)^2}{a_1} < 0,$$

$$\beta_3 = -\Omega(a_1 + a_3) < 0, \quad \beta_4 = -\Omega\frac{a_2 - a_1}{a_1} < 0,$$

$$I_1 = \int_{-\infty}^{+\infty} G_X^* G_Y^* G_Z^* \sin(\Omega t)\,dt = \\ = \frac{G_0^3 n_2}{a_3 - a_1}\int_{-\infty}^{+\infty}\frac{\sinh(n_2 t)}{\cosh^3(n_2 t)}\sin(\Omega t)\,dt = \\ = \frac{G_0^3\Omega^2\pi}{2n_2^2(a_3 - a_1)\sinh\left(\dfrac{\Omega\pi}{2n_2}\right)} > 0$$

$$I_2 = \int_{-\infty}^{+\infty}\frac{G_X^* G_Y^{*3} G_Z^*}{G_0^2 - G_Z^{*2}}\sin(\Omega t)\,dt = \\ = \frac{G_0^3 n_2}{a_3 - a_1}\int_{-\infty}^{+\infty}\frac{\tanh^3(n_2 t)\sin(\Omega t)\,dt}{D + \sinh^2(n_2 t)} = \text{const} > 0$$

$$D = \frac{a_3 - a_2}{a_3 - a_1}$$

$$I_3 = \int_{-\infty}^{+\infty} G_Z^{*2}\cos(\Omega t)\,dt = \\ = \frac{G_0^2(a_2 - a_1)n_2}{a_3 - a_1}\int_{-\infty}^{+\infty}\frac{\cos(\Omega t)}{\cosh^2(n_2 t)}\,dt = \\ = \frac{G_0^2\Omega\pi(a_2 - a_1)}{n_2^2(a_3 - a_1)\sinh\left(\dfrac{\Omega\pi}{2n_2}\right)} > 0$$

$$I_4 = \int_{-\infty}^{+\infty}\frac{G_Y^{*2} G_Z^{*2}}{G_0^2 - G_Z^{*2}}\cos(\Omega t)\,dt = \\ = \frac{G_0^2(a_2 - a_1)}{a_3 - a_1}\int_{-\infty}^{+\infty}\frac{\tanh^2(n_2 t)\cos(\Omega t)\,dt}{D + \sinh^2(n_2 t)} = \text{const} > 0$$

This proves the fact of the multiple intersections of stable and unstable manifolds of saddle heteroclinic points and chaotic motion initiation.

For illustration of chaotic aspects of the motion we also present the Poincaré sections. As we can see, the Poincaré sections (fig. 2-5) include chaotic layers near the separatrix regions. These chaotic layers prove the fact of the multiple separatrices splitting and chaotic motion initiation. It is needed to note that the Poincaré sections are plotted on the base of condition $(\Omega t \bmod 2\pi) = 0$ in the phase space $\{l, L/G\}$ (the coordinate axes values are dimensionless) by numerical calculations.

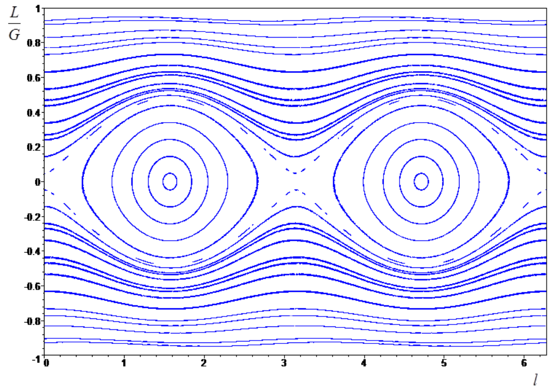

Figure 2 – Poincaré Section

$A_0 = 20, B_0 = 13, C_0 = 6,\ G_0 = 1, \Omega = 2, \varepsilon = 0.0$

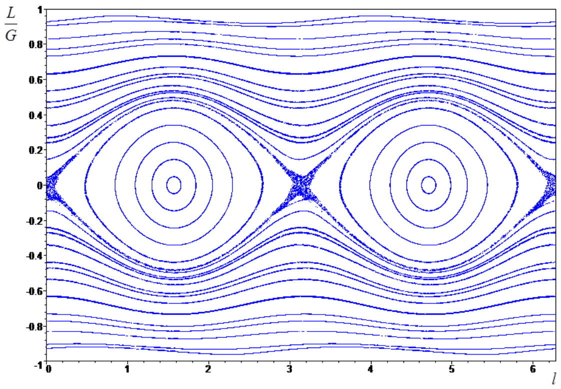

Figure 3 – Poincaré Section

$A_0 = 20, B_0 = 13, C_0 = 6,\ G_0 = 1, \Omega = 2, \varepsilon = 0.002$

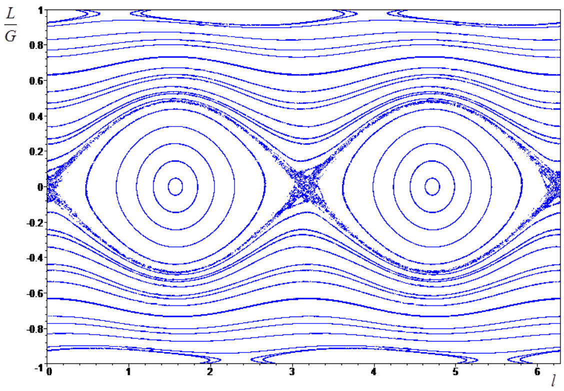

Figure 4 – Poincaré Section

$A_0 = 20, B_0 = 13, C_0 = 6,\ G_0 = 1, \Omega = 2, \varepsilon = 0.003$

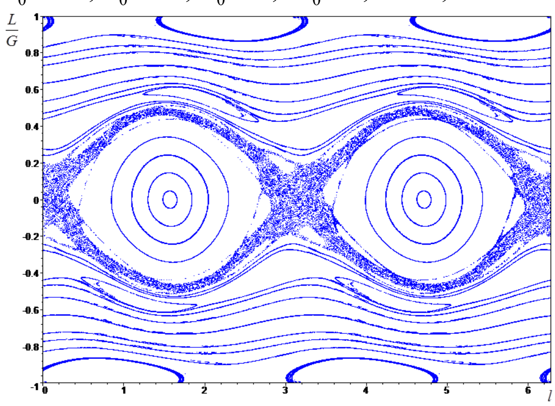

Figure 5 – Poincaré Section

$A_0 = 20, B_0 = 13, C_0 = 6,\ G_0 = 1, \Omega = 2, \varepsilon = 0.014$

4.2 Chaotization of attitude motion of a dual-spin spacecraft at presence of small harmonic internal perturbations

In this section we consider the attitude motion chaotization of one of the main types of SC-satellites with passive gyroscopic stabilization of the spatial (angular) orientation. This is a dual-spin spacecraft (DSSC), also called as gyrostat-satellite (GS). Many authors investigated complicated regimes of the DSSC attitude motion [2-20] in different formulations.

For example, papers [3-5] gave a description of motion of the dual-spin spacecraft at realization of a momentum transfer maneuver with rotor-body spinup. This maneuver is very important part of GS space mission. It can demonstrate motion evolutions with nontrivial change of attitude orientation and spacecraft longitudinal axis tumbling. These evolutions were explained with the help of direct analysis of motion equations, numerical experiments and on the base of probabilistic analysis of separatrices crossing [5].

In this paper we show possibility of the nontrivial motion modes realization as chaotic regimes initiation.

4.2.1 Motion equations

Let us examine the angular motion of the DSSC after realization of the spin-up maneuver [3, 4] in purposes to provide gyroscopic stabilization of the attitude DSSC position. The motion equations can be written as follows [19]:

$$\begin{aligned} &A\dot{p} + \left(C_2 - B\right)qr + q\Delta = 0, \\ &B\dot{q} + \left(A - C_2\right)pr - p\Delta = 0, \\ &C_2\dot{r} + \dot{\Delta} + \left(B - A\right)pq = 0, \\ &\dot{\Delta} = M_\Delta, \end{aligned} \qquad (62)$$

where $\left(p, q, r\right)^T$ are components of the main (carrier) body angular velocity (which represented in projections onto axes of the $Ox_2y_2z_2$ frame); $\Delta = C_1\left(r + \sigma\right) -$ the longitudinal angular moment of the rotor along Oz_1; $\sigma -$ the rotor angular velocity relative to the carrier body; $\mathbf{I}_2 = diag\left(A_2, B_2, C_2\right)$ is the triaxial inertia tensors of the carrier body in the connected frame $Ox_2y_2z_2$; $\mathbf{I}_1 = diag\left(A_1, A_1, C_1\right)$ is the inertia tensors of the dynamically symmetrical rotor in the connected frame $Ox_1y_1z_1$; $A = A_1 + A_2$, $B = A_1 + B_2$, $C = C_1 + C_2$ are the main inertia moments of the coaxial bodies system in the frame $Ox_2y_2z_2$ (including rotor); $M_\Delta -$ is the internal torque of the coaxial bodies interaction; $C_1\sigma = h_{z_1}$ – the rotor relative angular moment in the carrier body frame $Ox_2y_2z_2$. Assume that $A_2 > B_2 > C_2 > A_1 > C_1$.

We also note, that axes $\{x_i, y_i, z_i\}$, coincide with the principal axes of the coaxial bodies $i = 1,2$.

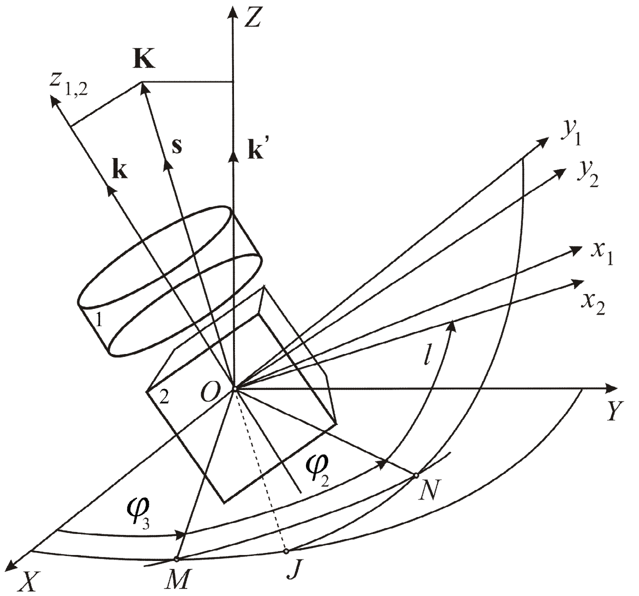

Figure 6 – The DSSC coaxial bodies system and the coordinate frames

The Serret-Andoyer variables can be expressed with the help of the coaxial system angular moment **K** (fig. 6):

$$L = \frac{\partial \mathrm{T}}{\partial \dot{l}} = \mathbf{K}\cdot\mathbf{k};$$

$$G = \frac{\partial \mathrm{T}}{\partial \dot{\varphi}_2} = \mathbf{K}\cdot\mathbf{s} = |\mathbf{K}| = K;$$

$$I_3 = \frac{\partial \mathrm{T}}{\partial \dot{\varphi}_3} = \mathbf{K}\cdot\mathbf{k}'; \quad L \le G$$

$$\begin{aligned} K_{x_2} &= Ap = \sqrt{G^2 - L^2}\sin l; \\ K_{y_2} &= Bq = \sqrt{G^2 - L^2}\cos l; \\ K_{z_2} &= C_2 r + \Delta = L. \end{aligned} \tag{63}$$

In the Serret-Andoyer variables the system Hamiltonian takes the form:

$$\begin{aligned} &H = H_0 + \varepsilon H_1; \\ &H_0 = T = \frac{G^2 - L^2}{2}\left[\frac{\sin^2 l}{A_1 + A_2} + \frac{\cos^2 l}{A_1 + B_2}\right] + \\ &+ \frac{1}{2}\left[\frac{\Delta^2}{C_1} + \frac{(L-\Delta)^2}{C_2}\right], \\ &H_1 = H_1(l, \varphi_2, \varphi_3, \delta, L, G, I_3, \Delta), \end{aligned} \tag{64}$$

where T – system kinetic energy; ε – small non-dimensional parameter; H_1 is the general form of the perturbed part of the Hamiltonian.

As it follows from the Hamiltonian (64), G, I_3 and φ_3 are constants in unperturbed case. Then corresponding dynamical system has one degree of freedom $\{l, L\}$. In the case when the perturbed part of the Hamiltonian (H_1) also depend only on l and L; then we obtain one-degree of freedom perturbed system:

$$\begin{aligned} &\dot{L} = f_L(l,L) + \varepsilon g_L(t); \\ &\dot{l} = f_l(l,L) + \varepsilon g_l(t) \\ &f_L(l,L) = -\frac{\partial H_0}{\partial l} = \alpha\left(G^2 - L^2\right)\sin l\cos l \\ &f_l(l,L) = \frac{\partial H_0}{\partial L} = \\ &= L\left[\frac{1}{C_2} - \frac{\sin^2 l}{(A_1 + A_2)} - \frac{\cos^2 l}{(A_1 + B_2)}\right] - \frac{\Delta}{C_2} \\ &g_L = -\frac{\partial H_1}{\partial l}; \\ &g_l = \frac{\partial H_1}{\partial L} \end{aligned} \tag{65}$$

where $\alpha = (A_1 + B_2)^{-1} - (A_1 + A_2)^{-1}$; g_L, g_l – are the general form of perturbations.

Let us consider the system motion in the case when the carrier coaxial body with triaxial inertia tensor has non-nil longitudinal angular moment (angular velocity) along Oz_2-direction, and the second coaxial dynamically symmetrical body has not longitudinal angular moment along Oz_1-direction $(\Delta = 0)$. For example, it was realized in the framework of Galileo space mission[1] [26].

Assume that small $(\mu << 1)$ harmonic internal torque between spinning and despinning sections takes place:

$$M_\Delta = \mu\cos\nu t \tag{66}$$

The torque (66) describes, for example, a disturbing signal of the control system of internal

[1] Galileo was the fifth spacecraft to visit Jupiter, launched on October 19, 1989 [26]. This spacecraft was built as the coaxial DSSC with rotating and nonrotating sections. Scientific instruments were mounted on the big rotating section of the spacecraft (the DSSC carrier body), together with the main antenna, power supply, the propulsion module and most of Galileo's computers and control electronics. The nonrotating section included the camera system, the spectrometers and the photo-polarimeter radiometer.

spinup engine (the rotor angular velocity stabilization system) at presence of latency of angular velocity sensor.

From the last equation (62) at presence of the small torque (66) the analytical solution follows $(\Delta(0)=0)$:

$$\Delta(t)=(\mu/\nu)\sin\nu t \tag{67}$$

After substitution of solution (67) into (64) we can write:

$$H=\frac{G^2-L^2}{2}\left[\frac{\sin^2 l}{A_1+A_2}+\frac{\cos^2 l}{A_1+B_2}\right]+$$
$$+\frac{1}{2}\left[\frac{1}{C_1}\left(\frac{\mu}{\nu}\sin\nu t\right)^2+\frac{\left(L-\frac{\mu}{\nu}\sin\nu t\right)^2}{C_2}\right],$$

and with elimination of terms proportional to μ^2 we obtain the perturbed Hamiltonian form:

$$H=\frac{G^2-L^2}{2}\left[\frac{\sin^2 l}{A_1+A_2}+\frac{\cos^2 l}{A_1+B_2}\right]+$$
$$+\frac{1}{2}\frac{L^2}{C_2}-\frac{L\frac{\mu}{\nu}\sin\nu t}{C_2}+O\left(\mu^2\right) \tag{68}$$

Then from (68) the separated shape of the Hamiltonian follows:

$$H=H_0+\mu H_1,$$
$$H_0=\frac{G^2-L^2}{2}\left[\frac{\sin^2 l}{A_1+A_2}+\frac{\cos^2 l}{A_1+B_2}\right]+\frac{1}{2}\frac{L^2}{C_2}, \tag{69}$$
$$H_1=-\frac{L}{C_2\nu}\sin\nu t.$$

However, in order to use of the small dimensionless parameter we can proceed to the following expressions:

$$H=H_0+\varepsilon H_1,$$
$$H_0=\frac{G^2-L^2}{2}\left[\frac{\sin^2 l}{A_1+A_2}+\frac{\cos^2 l}{A_1+B_2}\right]+\frac{1}{2}\frac{L^2}{C_2}, \tag{70}$$
$$H_1=-L\nu\sin\nu t,\ \varepsilon=\frac{\mu}{C_2\nu^2},$$

where ε is the small non-dimentional parameter.

In this case we get the system (65) with the perturbations:

$$g_L(t)=0;\quad g_l(t)=-\nu\sin\nu t \tag{71}.$$

4.2.2 Solutions for heteroclinic separatrix-orbits

In order to apply the Melnikov method we need to have the heteroclinic exact explicit solutions. Solutions (51) are not suitable for considering case, because despite the Δ-vanishing ($\Delta(0)=0$) coaxial bodies rotate relative to each other ($\sigma\neq 0$). So we still have the mechanical system of coaxial bodies with the internal degree of freedom, not the mono-rigid-body. Therefore, the mono-rigid-body solutions (51) cannot be used, and we should apply coaxial-bodies-solutions (for example the solutions [19]).

However, the problem of obtaining of the homo(hetero)clinic solutions is very challenging and exciting. In this work we consider an alternative (in contrast with [16, 19]) method of heteroclinic solutions obtaining – the method of direct substitutions [20].

So, let us briefly describe the methodology and obtain the heteroclinic solutions, which are appropriate for the considering case of the DSSC motion.

Starting from the symmetry of the equations (62) and noting the similar symmetry of the hyperbolic functions and their derivatives, we can perform the following direct substitutions into (62):

$$\overline{p}(t)=\frac{p_0}{\operatorname{ch}\lambda t},\quad \overline{q}(t)=b\operatorname{th}\lambda t,$$
$$\overline{r}(t)=\frac{r_0}{\operatorname{ch}\lambda t},\quad \Delta\equiv 0, \tag{72}$$

and then after reducing we obtain the algebraic equations system for the unknown b and λ. After some transformations we obtain the expressions for "variables" b, λ as dependences on the "arbitrary" value p_0:

$$b^2=\frac{(C_2-A)A}{(C_2-B)B}p_0^2,\quad \lambda^2=\frac{(B-A)(C_2-A)}{C_2B}p_0^2. \tag{73}$$

Knowing that the asymptotes of the hyperbolic polhodes (heteroclinic polhodes) passes through the intermediate axis of the angular moment's ellipsoid (point $(p_0,0,r_0)^T$) under condition $2TB=K^2$ [2], it is possible to finally write the heteroclinic solution's parameters:

$$\forall r_0\ (r(0)=r_0):\ b^2=\frac{(C_2-A)A}{(C_2-B)B}p_0^2,$$
$$\lambda^2=\frac{(B-A)(C_2-A)}{C_2B}p_0^2, \tag{74}$$
$$p_0^2=\frac{C_2(C_2-B)}{A(B-A)}r_0^2.$$

Then functions (72) with parameters (74) are the required heteroclinic solutions for the considering case. We need to note that form of these solutions is new regardless of its connection with previous results [16, 19].

4.2.3 The chaotization analysis

In purposes to analyze the possibility of the attitude motion chaotization we can use classical Melnikov method [26]. The Melnikov function (with multiplier ε) in considered case for the perturbed system (65) with perturbations (71) has the form:

$$\begin{aligned} M(t_0) &= \varepsilon \int_{-\infty}^{\infty} \left[f_L g_l - f_l g_L \right]\left(\bar{L}(t), \bar{l}(t), t+t_0\right) dt = \\ &= -\varepsilon \int_{-\infty}^{+\infty} f_L\left(\bar{l}(t), \bar{L}(t)\right) g_l\left(t+t_0\right) dt, \end{aligned} \qquad (75)$$

Taking into account (63) and (72) integral (75) is rewritten as:

$$\begin{aligned} M(t_0) &= \varepsilon\nu \int_{-\infty}^{+\infty} \alpha\left(\bar{G}^2 - \bar{L}^2\right) \sin\bar{l} \cos\bar{l} \sin\left(\nu\left(t+t_0\right)\right) dt = \\ &= \varepsilon\nu\alpha \int_{-\infty}^{+\infty} A\bar{p}(t) B\bar{q}(t) \sin\left(\nu\left(t+t_0\right)\right) dt = \\ &= \varepsilon\nu\alpha b p_0 AB \int_{-\infty}^{+\infty} \frac{\operatorname{th}\lambda t}{\operatorname{ch}\lambda t} \cdot \sin\left(\nu\left(t+t_0\right)\right) dt = \\ &= \alpha b p_0 AB\left[\cos\nu t_0 J_1 + \cos\nu t_0 J_2\right] = R\cos\nu t_0, \end{aligned}$$

where

$$J_1 = \int_{-\infty}^{+\infty} \frac{\operatorname{th}\lambda t}{\operatorname{ch}\lambda t} \cdot \sin\left(\nu t\right) dt = \mathrm{const} = J,$$

$$J_2 = \int_{-\infty}^{+\infty} \frac{\operatorname{th}\lambda t}{\operatorname{ch}\lambda t} \cdot \cos\left(\nu t\right) dt = 0, \quad R = \varepsilon\nu\alpha b p_0 ABJ.$$

We can analytically reduce integral J_1 to the following expression

$$J = \int_{-\infty}^{+\infty} \frac{\operatorname{th}\lambda t}{\operatorname{ch}\lambda t} \cdot \sin\left(\nu t\right) dt = \int_{-\infty}^{+\infty} \frac{\operatorname{sh}\lambda t}{\operatorname{ch}^2 \lambda t} \cdot \sin\left(\nu t\right) dt \qquad (76)$$

With the help of integration "by parts" we obtain [27]:

$$J = \frac{\nu}{\lambda} \int_{-\infty}^{+\infty} \frac{\cos\left(\nu t\right)}{\operatorname{ch}\lambda t} dt - \left. \frac{\sin\left(\nu t\right)}{\lambda \operatorname{ch}\lambda t} \right|_{-\infty}^{+\infty} = \frac{\nu\pi}{\lambda^2} \operatorname{sech} \frac{\nu\pi}{2\lambda}. \qquad (77)$$

Then the Melnikov function takes the final analytical harmonic form:

$$M(t_0) = \frac{\varepsilon\nu}{\lambda^2} \alpha b p_0 AB\nu\pi \operatorname{sech} \frac{\nu\pi}{2\lambda} \cos\nu t_0 \qquad (78)$$

Also taking into account (74) it is possible to write the following Melnikov functions' expression containing the small disturbance amplitude (μ):

$$M(t_0) = \mu\pi \sqrt{\frac{AB}{\left(A - C_2\right)\left(B - C_2\right)}} \operatorname{sech} \frac{\nu\pi}{2\lambda} \cos\nu t_0 \qquad (79)$$

Thus, the Melnikov function has infinite number of simple roots. This proves the fact of the multiple intersections of stable and unstable manifolds of saddle heteroclinic points. Therefore chaotic layer takes place near the separatrix region. The chaotic layer has been illustrated (fig. 7-10) with the help of Poincaré sections $\left(\nu t \bmod 2\pi\right) = 0$ in the phase space $\{l, L/G\}$ (the coordinate axes values are dimensionless).

It is needed to note, that the analytical form of the Melnikov function (79) and considered form of the heteroclinic solutions (72) (with parameters (74)) are the main new results in comparison with previous works [19, 16, 11].

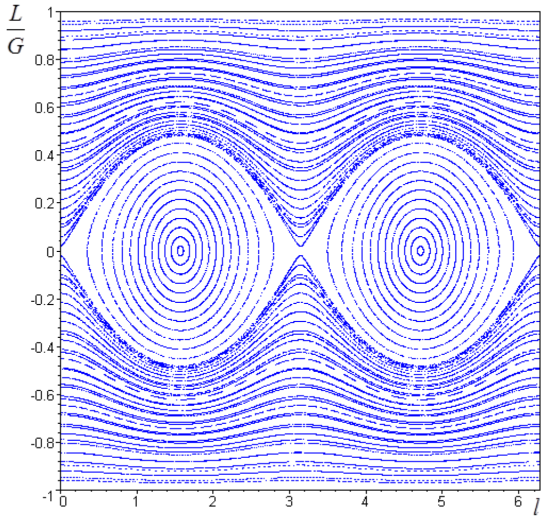

Figure 7 – Poincaré Section:
$A_2 = 15,\ B_2 = 8,\ C_2 = 6,\ A_1 = 5,\ C_1 = 4,$
$G = 10,\ \varepsilon = 0.0,\ \nu = 1$

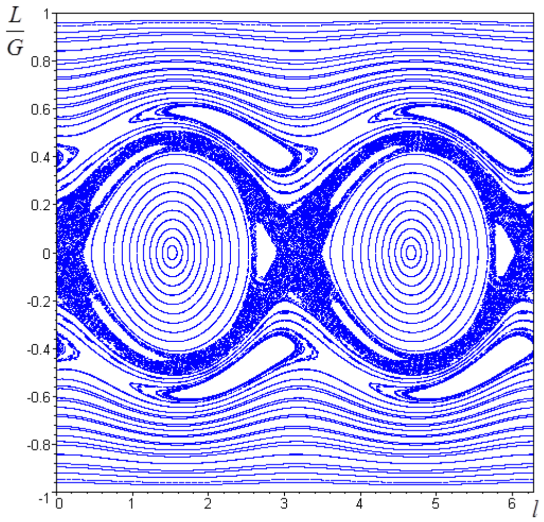

Figure 8 – Poincaré Section:
$A_2 = 15,\ B_2 = 8,\ C_2 = 6,\ A_1 = 5,\ C_1 = 4,$
$G = 10,\ \varepsilon = 0.1,\ \nu = 1$

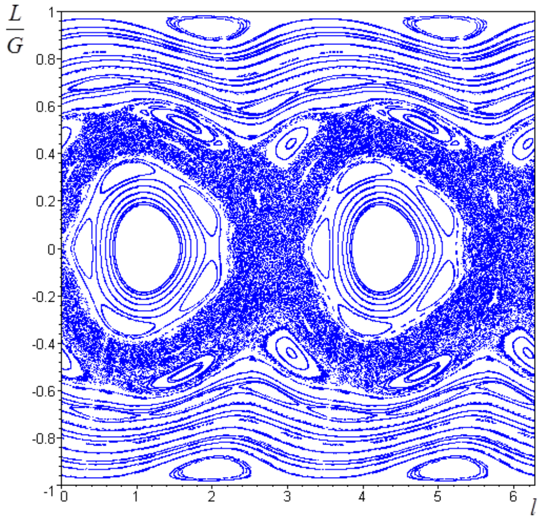

Figure 9 – Poincaré Section:
$A_2 = 15,\ B_2 = 8,\ C_2 = 6,\ A_1 = 5,\ C_1 = 4,$
$G = 10,\ \varepsilon = 0.4,\ \nu = 2$

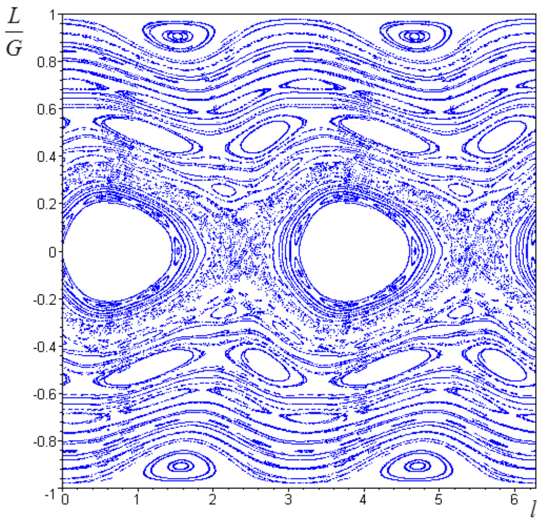

Figure 10 – Poincaré Section
$A_2 = 15,\ B_2 = 8,\ C_2 = 6,\ A_1 = 5,\ C_1 = 4,$
$G = 10,\ \varepsilon = 0.8,\ \nu = 2$

5 Conclusion

Attitude dynamics of the SC with variable structure was examined. Equations of the motion of the SC were obtained on the base of Hamiltonian formalism in the Serret-Andoyer variables. These equations can be used for analysis and synthesis of conditions of the SC attitude motion on active legs of orbital trajectories.

Analytical and numerical modeling of the SC motion was realized. Existence of the SC chaotic modes of motion was demonstrated with the help of Melnikov method and Poincaré sections.

Chaotic aspects of the DSSC's attitude dynamics were analytically studied at presence of small internal harmonic torque between coaxial bodies.

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