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Influence of Disturbances on the Angular Motion of a Spacecraft in the Powered Section of Its Descent

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Abstract—The motion of a variable-mass spacecraft is considered in the powered section of a descending trajectory. Approximate analytical solutions are obtained for the angles of spatial orientation of the spacecraft, which allows one to analyze the nutation motion and to develop recommendations on the spacecraft's mass configuration, providing the smallest possible deviations of the longitudinal axis and thrust vector from specified directions. The errors of stabilization of the spacecraft's longitudinal axis are calculated by means of numerical integration of complete models and using the obtained analytical solutions, the results being in good agreement.

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1. FORMULATION OF THE PROBLEM

In order to impart to the braking thrust vector of a spacecraft, descending in the atmosphere, a specified direction, the vehicle is stabilized in space by spinning around the longitudinal axis. The retrorocket engine (RRE) operation lasts about 20 seconds, during which the inertial-mass characteristics of the spacecraft change because of fuel burning out. The initial angular disturbances result in occurrence, on a powered section, of nutation oscillations of the spacecraft's longitudinal axis with variable amplitude. The deviations of the longitudinal axis, and, hence, of a thrust vector cause spacecraft transition to a descending orbit that differs from calculated one, and, hence, to an increase of the area of scattering of landing points.

The problem is stated to obtain simple approximate analytical solutions for the angles of spatial orientation of spacecraft, which would allow one to analyze the motion and to develop recommendations on the vehicle's mass configuration ensuring the least deviations of a longitudinal axis from the specified direction, and, hence, the least scattering of landing points. The spatial motion of a vehicle around its center of mass determines the motion of its longitudinal axis too, and, hence, the direction of a braking thrust vector. The efficiency of gyroscopic stabilization is determined by the value of deviation of the final velocity of the spacecraft's center of mass from the nominal value on the powered section. As a rule, in problems of descent the braking impulse is supposed to be instantaneous, and its direction is considered to be constant [1]. However, under real conditions the direction of a braking thrust vector changes owing to nutation-precession motion during RRE operation.

It should be noted that the above-mentioned problem was considered earlier in a number of works, for example, in monograph [2]. However, in these works the solutions for kinematical parameters of spatial and trajectory motions of spacecraft on the powered section of descent trajectory were not presented in the explicit form. In the present paper, integration in quadratures of corresponding dynamic equations is performed, and the analytical solutions to indicated kinematical parameters are found.

It was stated in paper [3] that after terminating RRE operation the ratio of the transverse velocity magnitude to the total velocity magnitude should not exceed some specified value:

$$\Pi = \sqrt{V_{\xi k}^2 + V_{\eta k}^2}/|\mathbf{V}_k| \le \Pi_*, \qquad (1.1)$$

where $|\mathbf{V}_k| = \sqrt{V_{\xi k}^2 + V_{\eta k}^2 + V_{\zeta k}^2}$ is the value of a final velocity of the spacecraft's center of mass after RRE operation, axis ζ coincides with the specified direction of a braking impulse, and Π_* is the maximum allowable value of criterion Π.

Criterion Π, characterizing the angular error in applying the braking impulse, can be obtained either by numerical integration of corresponding equations of motion of the center of mass, for example, [3], simultaneously with the equations of motion with respect to the center of mass or with using the analytical solutions.

2. EQUATIONS OF MOTION OF A BODY WITH VARIABLE COMPOSITION

In describing the motion of a body with variable composition we make use of the "short-range effect" hypothesis [4, 5], according to which the particles are thrown away only from some part of the surface of a variable-mass body, and the particles not possessing relative velocity with respect to the body-fixed coordinate system are considered as belonging to the body. The particles possessing such a relative velocity do not belong to a body any longer and have no effect on its motion. We write the equations of motion in the *Oxyz* coordinate system, which is rigidly fixed to a spacecraft and has its origin at point *O* coinciding with the initial position of the center of mass. We note that in the process of fuel burning out in RRE the position of the spacecraft's center of mass relative to the spacecraft changes. We introduce the following coordinate systems: *OXYZ* is a movable and, in the general case, noninertial coordinate system, whose axes remain collinear with the axes of some inertial system; *Oxyz* is the spacecraft-fixed coordinate system, axis *Oz* is directed along the longitudinal axis of the vehicle, in the direction of which the braking thrust *P* is applied. We assume that the vehicle possesses axial dynamic symmetry which is not violated during mass changing, and the body's center of mass moves along the axis of symmetry *Oz*.

The dynamic equations of motion of a dynamically symmetric body of variable composition can be obtained from the dynamic equations of motion of a system of two coaxial bodies [3, 7] by letting the moments of inertia of one body to be zero:

$$\begin{aligned}(A - m\rho_C^2)\dot{p} + (C - A)qr &= M_x,\\ (A - m\rho_C^2)\dot{q} - (C - A)pr &= M_y,\\ C\dot{r} &= M_z,\end{aligned} \tag{2.1}$$

where $A = A(t)$ and $C = C(t)$ are the equatorial and longitudinal moments of inertia of a body, calculated in the body-fixed coordinate system *Oxyz*; $\rho_C = \rho_C(t)$ is the distance between the body's center of mass and the origin of the coordinate system *Oxyz*; M_x, M_y, and M_z are projections of the principal moment of external forces onto the body-fixed axes. Equations (2.1) coincide with well-known equations of motion of a solid body of variable mass [4–6] with invariable position of the body's center of mass $\rho_C = 0$.

Since the spacecraft size is small as compared to the radius of orbit, the moment from the gravitational force can be disregarded. We consider the process of symmetric burning out of fuel in RRE, when the throwing points away occurs strictly in the longitudinal axis direction, and the center of mass is only insignificantly displaced from its initial position: $\rho_C^2 \ll A/m$. There will be no moment of jet force relative to the center of mass in this case. By virtue of accepted assumptions, we re-write the dynamic equations (2.1) in the following form:

$$\dot{p} + b(t)qr = 0, \quad \dot{q} - b(t)pr = 0, \quad \dot{r} = 0, \tag{2.2}$$

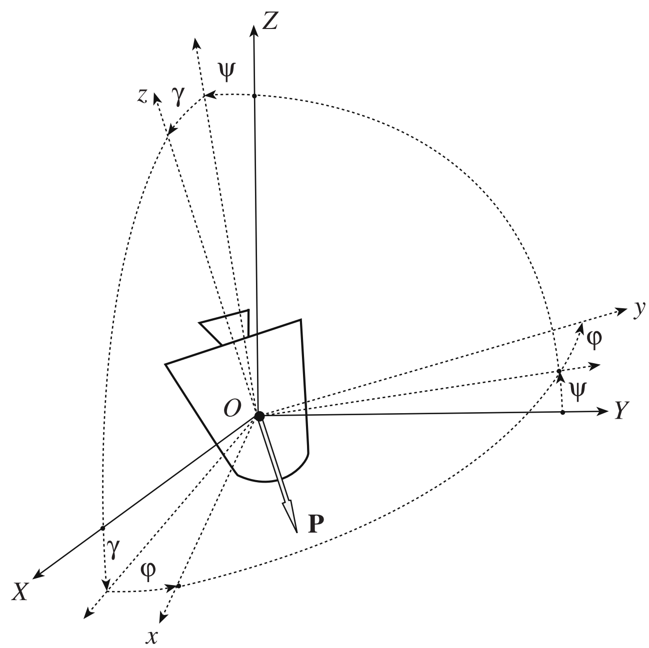

Fig. 1.

where

$$b(t) = C(t)/A(t). \tag{2.3}$$

We make use of the Euler-type angles $\psi \longrightarrow \gamma \longrightarrow \varphi$ as the angles determining the position of a body-fixed coordinate system *Oxyz* relative to the *OXYZ* system (see Fig. 1). The last turn by angle φ is made around the axis of the vehicle's dynamic symmetry, *Oz*. Such a choice will subsequently allow us to obtain the required approximate solutions, including those for the nutation angle.

The kinematical equations for the introduced angles of spatial orientation have the form:

$$\begin{aligned}\dot{\gamma} &= p\sin\varphi + q\cos\varphi, \quad \dot{\psi} = (p\cos\varphi - q\sin\varphi)/\cos\gamma,\\ \dot{\varphi} &= r - \tan\gamma(p\cos\varphi - q\sin\varphi).\end{aligned} \tag{2.4}$$

We determine the nutation angle θ as an angle between axis *OZ* and the axis of vehicle's dynamic symmetry *Oz*. Then it follows from the spherical geometry, that

$$\cos\theta = \cos\psi\cos\gamma. \tag{2.5}$$

Note that for small values of the nutation angle (and, hence, of angles ψ and γ) formula (2.5) is re-written in the form:

$$\theta^2 \cong \psi^2 + \gamma^2. \tag{2.6}$$

3. APPROXIMATE SOLUTIONS

Let the mass, and longitudinal and transverse moments of inertia of a vehicle vary according to the linear law during the retrorocket engine operation, which is valid with sufficiently high accuracy for solid-propellant engines used as RRU, with fuel charges of star-shaped cross section and packet-grain charges, provided that they are burned out uniformly:

$$A(t) = A_0 - at, \quad C(t) = C_0 - ct, \tag{3.1}$$

where A_0 and C_0 are the initial values of corresponding moments of inertia; and $a, c > 0$.

We take advantage of the procedure of writing the equations of angular motion at small nutation angles in the complex form, which was used in a number of papers, for example, in [6]. We introduce the following complex variable:

$$\Theta = \psi + i\gamma, \tag{3.2}$$

whose real and imaginary parts represent the first two angles from the sequence of rotations (Fig.1), where, on the strength of (2.6), $|\Theta| = \theta$, i.e., the modulus of a complex variable characterizes the nutation angle value. For small nutation angles the real and imaginary parts of variable Θ describe the motion of the projection of the apex of spacecraft's longitudinal axis Oz along the motionless coordinate plane XOY. Omitting auxiliary derivations of the mentioned procedure, whose detailed description can be found in paper [6], one can easily reduce the first two equations of (2.2) to the following complex equation:

$$\ddot{\Theta} = ir_0 b(t)\dot{\Theta}. \tag{3.3}$$

From Eq. (3.3) one can obtain the following relationship for the complex angular velocity:

$$\dot{\Theta}(t) = \dot{\Theta}_0 \exp[iJ(t)], \tag{3.4}$$

where

$$J(t) = r_0\int_0^t b(t)dt = \frac{A_0 c r_0}{a^2} at/A_0 - \ln\left(1 - \frac{at}{A_0}\right)\left[\frac{C_0 r_0}{a} - \frac{A_0 c r_0}{a^2}\right]. \tag{3.5}$$

Separating the real and imaginary parts of solution (3.4), we write the expressions for angular velocities as:

$$\begin{aligned} \dot{\psi} &= \dot{\psi}_0\cos(J(t)) - \dot{\gamma}_0\sin(J(t)) \\ &= r_0 G\sin(F_0 - J(t)), \\ \dot{\gamma} &= \dot{\psi}_0\sin(J(t)) - \dot{\gamma}_0\cos(J(t)) \\ &= r_0 G\cos(F_0 - J(t)), \end{aligned} \tag{3.6}$$

where $G = \frac{1}{r_0}\sqrt{\dot{\psi}_0^2 + \dot{\gamma}_0^2}$, $\sin F_0 = \dot{\psi}_0/G$, and $\cos F_0 = \dot{\gamma}_0/G$.

To simplify further calculations, we expand the logarithm in integral (3.5) into the power-law series, whose interval of convergence is $t \in [0, A_0/a]$):

$$\begin{gathered} \frac{A_0 c r_0}{a^2} at/A_0 - \ln(1+\xi)\left[\frac{C_0 r_0}{a} - \frac{A_0 c r_0}{a^2}\right] \\ = \frac{c r_0}{a} t - \left[\frac{C_0 r_0}{a} - \frac{A_0 c r_0}{a^2}\right]\left(\xi - \frac{1}{2}\xi^2 + \frac{1}{3}\xi^3 - \ldots\right), \\ \xi = -at/A_0. \end{gathered} \tag{3.7}$$

For the considered class of spacecraft the quantity ξ during RRE operation does not exceed a value of 0.2. Therefore, rejecting in the expansion the terms containing ξ of the third and higher powers and letting $F_0 = 0$, we obtain the following approximate equations for the angles of spatial orientation:

$$\dot{\gamma} = r_0 G\cos(\lambda t + \mu t^2), \quad \dot{\psi} = r_0 G\sin(\lambda t + \mu t^2), \tag{3.8}$$

where

$$\lambda = -r_0\frac{C_0}{A_0}, \quad \mu = \frac{r_0}{2A_0}\left(c - \frac{aC_0}{A_0}\right). \tag{3.9}$$

For a complex angular velocity the following equation is valid:

$$\dot{\Theta} = \dot{\psi} + i\dot{\gamma} = ir_0 G\overline{\exp}[i(\lambda t + \mu t^2)], \tag{3.10}$$

from which it follows that further integration in complex form seems to be inexpedient, since it results in a formalized form of solution in special functions of complex variable. Proceeding from this circumstance, we carry out further transformations on the basis of separated equations (3.8).

In virtue of spacecraft design features, μ and λ parameters can take either identical or opposite signs, and combination of signs, accordingly, depends on validity or invalidity of the following condition:

$$\Lambda = (cA_0 - aC_0) < 0. \tag{3.11}$$

In the general case, when any combination of signs of μ and λ is possible, the integrals of equations (3.8), with regard to expressions (3.9), are written in the following form:

$$\begin{aligned} \gamma(t) &= r_0 G\int\cos(|\lambda|t \pm |\mu|t^2)dt, \\ \psi(t) &= -|r_0|G\int\sin(|\lambda|t \pm |\mu|t^2)dt, \end{aligned} \tag{3.12}$$

where the upper sign is taken for the case, when condition (3.11) is met, and the lower sign, on the contrary, in the case, when it is not met.

After integrating (3.12) the solutions are written in the Fresnel's integrals:

$$\gamma(t) = r_0 G\sqrt{\frac{\pi}{2|\mu|}}\Big[\cos\Big(\Big|\frac{\lambda^2}{4\mu}\Big|\Big)C\Big(\sqrt{\frac{2}{\pi|\mu|}}\Big(|\mu|t \pm \frac{|\lambda|}{2}\Big)\Big)$$
$$+ \sin\Big(\Big|\frac{\lambda^2}{4\mu}\Big|\Big)S\Big(\sqrt{\frac{2}{\pi|\mu|}}\Big(|\mu|t \pm \frac{|\lambda|}{2}\Big)\Big)\Big] + \Gamma,$$
$$\psi(t) \qquad (3.13)$$
$$= \mp|r_0|G\sqrt{\frac{\pi}{2|\mu|}}\Big[\cos\Big(\Big|\frac{\lambda^2}{4\mu}\Big|\Big)S\Big(\sqrt{\frac{2}{\pi|\mu|}}\Big(|\mu|t \pm \frac{|\lambda|}{2}\Big)\Big)$$
$$- \sin\Big(\Big|\frac{\lambda^2}{4\mu}\Big|\Big)C\Big(\sqrt{\frac{2}{\pi|\mu|}}\Big(|\mu|t \pm \frac{|\lambda|}{2}\Big)\Big)\Big] + \Psi,$$

where $C(x) = \int_0^x \cos\Big(\frac{\pi}{2}x^2\Big)dx$, and $S(x) = \int_0^x \sin\Big(\frac{\pi}{2}x^2\Big)dx$, are Fresnel's integrals, and Γ and Ψ are constants of integration.

Now we make use of the following series expansion of the Fresnel's integrals [8]:

$$C(x) = \frac{1}{2} + \frac{1}{\pi x}\sin\Big(\frac{\pi}{2}x^2\Big) + O\Big(\frac{1}{x^2}\Big),$$
$$S(x) = \frac{1}{2} - \frac{1}{\pi x}\cos\Big(\frac{\pi}{2}x^2\Big) + O\Big(\frac{1}{x^2}\Big). \qquad (3.14)$$

Replacing in solution (3.13) Fresnel's integrals by their representations (3.14), rejecting here the quantities of the order of $O(\pi\mu(\mu t + \lambda/2)^{-2})$, we write down:

$$\gamma = r_0 G\sqrt{\frac{\pi}{2|\mu|}}$$
$$\times\Big[\frac{1}{2}(\cos y_0 + \sin y_0) + \frac{1}{\pi x}\sin\Big(\frac{\pi x^2}{2} - y_0\Big)\Big] + \Gamma,$$
$$\psi = \mp|r_0|G\sqrt{\frac{\pi}{2|\mu|}}$$
$$\times\Big[\frac{1}{2}(\cos y_0 - \sin y_0) - \frac{1}{\pi x}\cos\Big(\frac{\pi x^2}{2} - y_0\Big)\Big] + \Psi, \qquad (3.15)$$
$$y_0 = \Big|\frac{\lambda^2}{4\mu}\Big| = \Big|\frac{r_0 C_0^2}{2(cA_0 - aC_0)}\Big|,$$
$$x = \sqrt{\frac{2}{\pi|\mu|}}\Big(|\mu|t \pm \frac{|\lambda|}{2}\Big).$$

Note that in the case, when $\Lambda = 0$, quantity $\mu = 0$, and equations (3.12) have the exact solutions:

$$\gamma = \frac{A_0}{C_0}r_0\sin(\lambda t) + \Gamma, \quad \psi = \frac{A_0}{C_0}r_0\cos(\lambda t) + \Psi,$$

for which the constant amplitude and frequency of oscillations are characteristic.

Substituting these solutions into expression for the nutation angle value (2.6) and making averaging, we write:

$$\langle\theta^2\rangle - \text{const} = \frac{r_0^2G^2}{2|\mu|\pi x^2} = \frac{r_0^2G^2}{4\mu^2(t \pm |\lambda/(2\mu)|)^2}. \qquad (3.16)$$

As is seen from expression (3.16), where the upper sign and lower sign correspond to validity or invalidity of condition (3.11), the average value of nutation angle will monotonously decrease beginning from a zero time instant in the case, when the quantity $(t \pm |\lambda/(2\mu)|)^2$ monotonously increases. Thus, the monotonous decrease of average values of the nutation angle will occur in the case of coincidence of the signs of μ and λ quantities, when condition (3.11) is met. In the case of non-coincidence of signs of μ and λ the average values of the nutation angle will increase beginning from a zero time instant until the instant $T_* = |\lambda/(2\mu)|$, at which, as our approximate calculations have shown, the nutation angle value grows without limit.

From solutions (3.15) one can easily find the average values for angles ψ and γ, and the envelopes for the nutation angle:

$$\langle\gamma\rangle = \frac{r_0G}{2}\sqrt{\frac{\pi}{2|\mu|}}(\cos y_0 + \sin y_0) + \Gamma,$$
$$\langle\psi\rangle = \frac{\mp|r_0|G}{2}\sqrt{\frac{\pi}{2|\mu|}}(\cos y_0 - \sin y_0) + \Psi, \qquad (3.17)$$
$$\theta_{\substack{\max\\ \min}} = \langle\theta\rangle \pm A_\theta, \quad A_\theta = (\langle\gamma\rangle^2 + \langle\psi\rangle^2)^{1/2}.$$

Figure 2 presents the results for nutation angle (2.5) obtained by means of numerical integration of relations (3.8), as well as its averaged (3.16) value (curves *1*, *2*) and envelopes $\theta_{\substack{\max\\ \min}}$ (3.17). Curve *1* (Fig. 2) corresponds to coincidence of the signs of λ and μ quantities, and curve *2* to their distinction. The initial data and parameters of a system for two calculations are presented in table.

Thus, the value of the nutation angle will decrease at fulfillment of condition (3.11), which corresponds to such nutation-precession motion of spacecraft, when the braking impulse accuracy increases spontaneously, and, hence, the area of landing points scattering decreases. The growth of the nutation angle results in "spraying" of the braking impulse and in increasing errors of transition to the calculated orbit of descent.

4. MOTION OF THE CENTER OF MASS AND CALCULATION OF ERRORS OF APPLYING THE BRAKING IMPULSE

The solutions for the angles of spatial orientation (3.13), (3.15), and (3.16), obtained above, allow one to get analytical estimates of the efficiency of gyroscopic

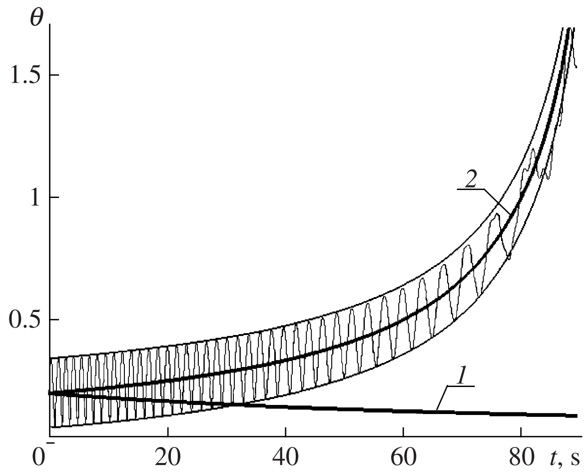

Fig. 2.

stabilization of the longitudinal axis, expressed by integrated criterion (1.1).

The linear law of mass variation

$$m(t) = m_0(1 - \nu t), \quad \nu = (m_0 - m_k)/(m_0 T) \quad (4.1)$$

corresponds to the constant magnitude of jet thrust force.

Now we write the equations of motion of the spacecraft's center of mass in projections onto the axes of the $O_1\xi\eta\zeta$ system (Fig. 3) with regard to spatial motion of the spacecraft in the $OXYZ$ system at small angles:

$$m(t)\dot{V}_\xi = -P\gamma, \quad m(t)\dot{V}_\eta = P\psi, \qquad (4.2)$$
$$m(t)\dot{V}_\zeta = -P,$$

where $V_\xi = \dot{\xi}$, $V_\eta = \dot{\eta}$, and $V_\zeta = \dot{\zeta}$ are the components of the velocity of the center of mass. Note, that in Eqs. (4.2) we have neglected the so-called gravitational losses [10], which, if necessary, can be calculated by separate integration and added as an additive quantity [3]. Thus, Eqs. (4.2) describe the motion of the spacecraft's center of mass on the powered section only under the action of a constant jet thrust **P**.

For calculating the stabilization errors $\Pi(t)$ (1.1) the equations of motion (4.2), (2.2), and (2.4) are numerically integrated [3]. As an alternative to numerical integration, quantity $\Pi(t)$ can be determined analytically by substituting into equations of motion (4.2) the averaged dependences for spatial angles $\bar{\gamma}$ and $\bar{\psi}$:

$$\bar{\gamma}(t) = \frac{1}{2}[\gamma_{max}(t) + \gamma_{min}(t)],$$
$$\bar{\psi}(t) = \frac{1}{2}[\psi_{max}(t) + \psi_{min}(t)], \qquad (4.3)$$

which follow from solutions (3.15) when condition (3.11) is met. With regard to (4.3) the solutions of equations (4.2) have the form:

$$V_i(t) = D_i \ln|1 - \nu t|, \quad (i = \overline{\xi, \eta, \zeta}) \qquad (4.4)$$

$$D_{\xi,\eta} = \frac{PGr_0}{4m_0\nu}\sqrt{\frac{2\pi}{|\mu|}}(\cos y_0 \pm \sin y_0) + \frac{P\Gamma}{m_0\nu},$$

where

$$D_\zeta = \frac{P}{m_0\nu}.$$

Then the value of a final stabilization error (1.1) is determined by the formula:

$$\Pi = \frac{\sqrt{D_\xi^2 + D_\eta^2}}{\sqrt{D_\xi^2 + D_\eta^2 + D_\zeta^2}}. \qquad (4.5)$$

For the comparative analysis and estimation of the efficiency of obtained analytical formulas, the error $\Pi(t)$ has been calculated in three ways (Fig. 4). The thick oscillatory line corresponds to the results obtained by means of simultaneous numerical integration of differential equations (4.2), (2.2), and (2.4); the thin oscillatory line represents the results obtained by numerical integration of analytical expressions (3.12) and (4.2), while the horizontal line corresponds to the final error Π calculated by formula (4.5).

The calculations were carried out for the parameters indicated in the table (the line of coincidence of the signs of λ and μ implies fulfillment of (3.11)), and the initial values of the velocity of the center of mass had zero values, since the inertial coordinate system $O_1\xi\eta\zeta$ moved by itself with orbital velocity for the instant of the powered section beginning. It is seen from the figure that all ways of calculating the error Π give the same result near the final instant of RRE operation time.

Table

Spacecraft parameters and initial conditions of motion	r_0, rad/s	G, rad/s	C_0, kg m^2	A_0, kg m^2	c, kg m^2/s	a, kg m^2/s	γ_0, rad	λ, rad/s	μ, rad/s^2	Λ, (kg m^2)^2/s
Signs of λ and μ quantities coincide	10	0.1	10	20	0.1	0.5	0.1	–5	–0.04	–3
Signs of λ and μ quantities are opposite	10	0.1	10	20	0.4	0.6	0.1	–5	0.03	2

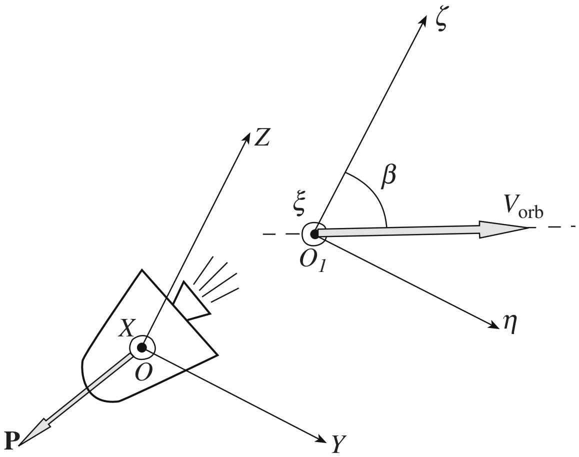

Fig. 3.

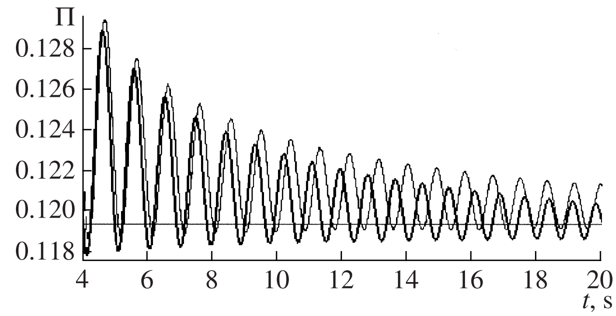

Fig. 4.

In conclusion, it should be noted that condition (3.11) should be considered as a criterion for choosing the "optimum" combinations of inertial-mass parameters of a spacecraft. For example, condition (3.11) can be written in the form:

$$\frac{c}{C_0} < \frac{a}{A_0}, \tag{4.6}$$

where, according to laws (3.1), quantities a and c can be represented as follows:

$$a = \frac{A_0 - A_k}{T} = \frac{\Delta_A}{T}, \quad c = \frac{C_0 - C_k}{T} = \frac{\Delta_C}{T}.$$

With regard to the last formulas relation (4.6) is reduced to the form:

$$\frac{\Delta_C}{C_0} < \frac{\Delta_A}{A_0}, \tag{4.7}$$

where quantities Δ_A and Δ_C represent the final changes of transverse and longitudinal moments of inertia of the spacecraft, respectively. Relation (4.7) indicates that, in order to decrease the amplitude of nutation oscillations and to increase stabilization efficiency, it is necessary to provide for such internal configuration of the spacecraft, at which the relative change of the longitudinal moment of inertia is less than the relative change of the transverse moment of inertia. This can be achieved, for example, by disposing solid-propellant charges as close to the spacecraft's longitudinal axis, as possible. From the viewpoint of increasing the efficiency of gyroscopic stabilization of a spacecraft on the powered section, the arrangement of a package of solid-propellant charges in RRE's combustion chamber in the form of a "rod" is more expedient, than its arrangement in the form of a "washer." Other variants of the internal RRE configuration that provide for fulfillment of condition (4.7) are also possible. The suggested techniques of estimating the efficiency of gyroscopic stabilization are quite applicable both for analyzing the quality of dynamic processes in spacecraft motion on a powered section and for synthesizing the internal configuration of a spacecraft.

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