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Stabilization of a Reentry Vehicle by a Partial Spin-up during Uncontrolled Descent

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Abstract—Stabilization of a reentry vehicle (RV) by a partial spin-up of it is considered for the case of uncontrolled descent into the atmosphere. In this case, the vehicle is a composite construction consisting of two rigid bodies, a return capsule and a stabilizing block, which is put in rotation. A model is developed for the spatial motion of the reentry vehicle considered as a system of coaxial rigid bodies rotating about a common axis of symmetry. The free motion is studied, and the stability of steady-state regimes is analyzed. The spatial motion of the system is considered for the case of a small asymmetry due to displacement of the axes of dynamic symmetry of the bodies with respect to the spin axis, and approximate solutions for the motion parameters of the free system are found.

1. PROBLEM FORMULATION

When an RV is undergoing uncontrolled descent, it is necessary for it to fall in a certain area of the earth's surface with a minimum dispersion of the falling points and to meet requirements associated with an overload level and the operating conditions of the parachute system. The success in hitting a given area with the required dispersion of the landing points is defined by the initial reentry conditions and by the features of the vehicle. To ensure the required angles of reentry into dense layers of the atmosphere, the RV must be oriented in a specific way and a brake impulse must be given out. The RV is stabilized by its spin-up about the longitudinal axis. After the RV has entered the atmosphere, the angular velocity of the spin should be cancelled in order to create operating conditions for the parachute system and to prevent resonant modes of motion [1, 2]. The cancellation of the RV angular velocity can be performed by applying a system of weights on unreeling cables, which are separated at the end of the process [3].

One method of stabilization without subsequent cancellation of the angular velocity of the entire vehicle is its partial spin-up, when some part of the RV is put into rotation and separated after the reentry. The return part of the RV is not put into rotation. In this case, the vehicle is a mechanical system of two rigid bodies with a common axis of rotation. One of the bodies is a stabilizing unit, and the other is the RV. The devices with partial spin-up can be used in remote systems for sensing the earth's surface to deliver the obtained photographic material to the earth. Body *1* is a stabilizing unit with a brake engine separated after the reentry, and body *2* is the proper reentry object (Fig. 1).

There are papers [4, 5] that study transient modes of motion and stability of stabilizable states of doubly spinning satellites and axisymmetric gyrostats, as well as some more general systems consisting of many coaxial flywheels.

Here we seek to study the free motion of coaxial bodies, including cases with asymmetry due to slight displacements of the longitudinal axis of symmetry of one body from the axis of rotation, and to analyze the stability of steady-state regimes.

2. EQUATIONS OF MOTION FOR A SYSTEM OF COAXIAL BODIES

Suppose the coordinate system $OXYZ$ moves translationally in an inertial space, and its origin coincides with the center of mass of a system of coaxial bodies. The coordinate systems $Ox'y'z'$ and $Oxyz$ are fixed to bodies *1* and *2*, respectively, and rotate with respect to

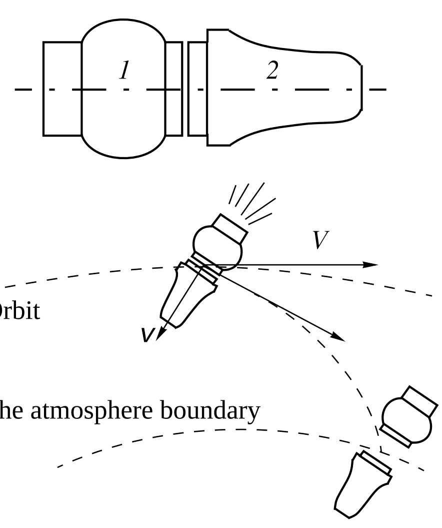

Fig. 1. Sketch of the RV and the reentry process.

the system *OXYZ*. The axes *Oz* and *Oz*' of the body-fixed systems coincide with a common axis of rotation of the bodies (Fig. 2). The position of the coaxial bodies relative to the system *OXYZ* will be characterized by the Eulerian angles: the precession angle ψ, the nutation angle θ, and the angles of proper rotation of bodies *1* and *2* (φ' and φ, respectively).

The vectors of angular velocities of bodies *1* and *2* $\boldsymbol{\omega}' = (p', q', r')$ and $\boldsymbol{\omega} = (p, q, r)$ are presented as projections onto the axes of the body-fixed coordinate systems (*Ox'y'z'* and *Oxyz*) and are expressed via the Eulerian angles as follows:

$$p = \dot\psi \sin\theta \sin\varphi + \dot\theta \cos\varphi,$$

$$p' = \dot\psi \sin\theta \sin\varphi' + \dot\theta \cos\varphi',$$

$$q = \dot\psi \sin\theta \cos\varphi - \dot\theta \sin\varphi,$$

$$q' = \dot\psi \sin\theta \cos\varphi' - \dot\theta \sin\varphi',$$

$$r = \dot\psi \cos\theta + \dot\varphi, \quad r' = \dot\psi \cos\theta + \dot\varphi'.$$

Let us introduce the angle and the velocity of relative twist:

$$\delta = \varphi' - \varphi, \quad \sigma = \dot\delta.$$

The vector components of the angular velocity $\boldsymbol{\omega}' = (p', q', r')$ expressed via the vector components p, q, r of the angular velocity of body *2* have the form

$$\begin{aligned} p' &= p\cos\delta + q\sin\delta, \\ q' &= q\cos\delta - p\sin\delta, \\ r' &= r + \sigma. \end{aligned} \quad (1)$$

To derive the equations of motion for the system of coaxial bodies let us use the center of mass of the system as a pole O and apply the angular momentum theorem [6]:

$$\frac{d\mathbf{K}_O}{dt} = \mathbf{M}_O^e. \quad (2)$$

Suppose this system includes dynamically symmetric bodies. Let us denote the principal moments of inertia of bodies *1* and *2* in the body-fixed coordinate systems *Ox'y'z'* and *Oxyz* by A_1, C_1 and A_2, C_2. These moments of inertia are not central, as the origins of the body-fixed systems *Ox'y'z'* and *Oxyz* coincide with the center of mass of the two-body system. This can be written for the above moments of inertia:

$$A_i = m_i l_i^2 + \bar{A}_i, \quad C_i = \bar{C}_i \quad (i = 1, 2),$$

where m_i, $\bar{A}_i$, and $\bar{C}_i$ are the mass and the natural principal moments of inertia for body i and l_i is the distance between the center of mass of the two-body system and that of body i.

The angular momentum of the system with respect to the center of mass is equal to the vectorial sum of the angular momenta of bodies *1* and *2* about the point O: $\mathbf{K}_O = \mathbf{K}_1 + \mathbf{K}_2$.

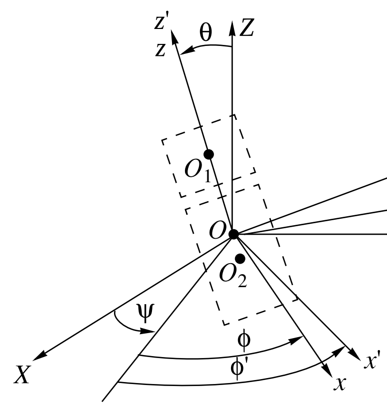

Fig. 2. Coaxial bodies and the applied coordinate systems.

Having calculated the derivative of the angular momentum of the system considered as a sum of the angular momenta of the bodies and having applied local derivatives in the body-fixed systems *Ox'y'z'* and *Oxyz*, we can rewrite Eq. (2) in the system *Oxyz* as

$$\hat{\boldsymbol{\delta}}\left[\frac{\tilde{d}\mathbf{K}_1}{dt} + \boldsymbol{\omega}' \times \mathbf{K}_1\right] + \left[\frac{\tilde{d}\mathbf{K}_2}{dt} + \boldsymbol{\omega} \times \mathbf{K}_2\right] = \mathbf{M}_O^e, \quad (3)$$

where $\boldsymbol{\omega}'$ and $\boldsymbol{\omega}$ are the angular velocities of rotation of the body-fixed coordinate systems with respect to the translationally moving system *OXYZ*; tilde denotes the local derivative in the corresponding moving coordinates; $\hat{\boldsymbol{\delta}}$ is the tensor of transition from the coordinates *Ox'y'z'* to the coordinates *Oxyz*; and $\mathbf{K}_1 = (A_1 p', A_1 q', C_1 r')$ and $\mathbf{K}_2 = (A_2 p, A_2 q, C_2 r)$.

Let us rewrite Eq. (3) in the matrix form:

$$\begin{bmatrix} \cos\delta & -\sin\delta & 0 \\ \sin\delta & \cos\delta & 0 \\ 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} A_1\dot{p}' \\ A_1\dot{q}' \\ C_1\dot{r}' \end{bmatrix} + \begin{bmatrix} p' \\ q' \\ r' \end{bmatrix} \times \begin{bmatrix} A_1 p' \\ A_1 q' \\ C_1 r' \end{bmatrix} \right)$$

$$+ \begin{bmatrix} A_2\dot{p} \\ A_2\dot{q} \\ C_2\dot{r} \end{bmatrix} + \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \begin{bmatrix} A_2 p \\ A_2 q \\ C_2 r \end{bmatrix} = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}, \quad (4)$$

where $M_x = M_{1,x} + M_{2,x}$; $M_y = M_{1,y} + M_{2,y}$; $M_z = M_{1,z} + M_{2,z}$ are the components of moments of external forces, which are the sums of corresponding projections of the moments applied to bodies *1* and *2*, respectively.

Taking into consideration expressions (1) for the components of the angular velocities of the bodies, the last matrix relation can be reduced to the following system of scalar equations:

$$(A_1 + A_2)\dot{p} - (A_1 + A_2 - C_2)qr + C_1 q(r + \sigma) = M_x,$$
$$(A_1 + A_2)\dot{q} - (C_2 - A_1 - A_2)pr - C_1 p(r + \sigma) = M_y, \tag{5}$$
$$C_1(\dot{r} + \dot{\sigma}) + C_2\dot{r} = M_z.$$

To obtain the equation of relative motion of the bodies let us use the Lagrange equation of the second kind for the angle of relative twist δ. Let us write down the expression for the kinetic energy of the system and for the generalized force:

$$T = \frac{1}{2}\mathbf{K}_1 \cdot \boldsymbol{\omega}' + \frac{1}{2}\mathbf{K}_2 \cdot \boldsymbol{\omega}$$
$$= \frac{1}{2}[(A_1 + A_2)(p^2 + q^2) + C_2 r^2 + C_1(r^2 + 2r\sigma + \sigma^2)],$$
$$Q_\delta = M_{1z} + M_\delta,$$

where M_δ is the moment of the internal interaction of the bodies along the axis of rotation.

The quantities p, q, and r do not depend explicitly on the angle δ and velocity ($\sigma = \dot{\delta}$) of the relative twist; thus, the equation of relative motion will have the following form:

$$C_1(\dot{r} + \dot{\sigma}) = M_{1,z} + M_\delta. \tag{6}$$

Let us complete dynamic equations (5) and (6) with the kinematic equations for the Eulerian angles:

$$\dot{\theta} = p\cos\varphi - q\sin\varphi, \quad \dot{\varphi} = r - \cot\theta(p\sin\varphi + q\cos\varphi),$$
$$\dot{\psi} = \frac{1}{\sin\theta}(p\sin\varphi + q\cos\varphi), \quad \dot{\delta} = \sigma. \tag{7}$$

3. MOTION OF A FREE SYSTEM OF COAXIAL BODIES

Let the moments of external forces applied to the mechanical system be zero ($M_x = M_y = M_z = 0$), and let a constant moment of relative twist $M_\delta = M$ act between the coaxial bodies.

In this case, the set of equations (5) and (6) will be written in the form

$$\dot{p} = (A_1 + A_2)^{-1}([A_1 + A_2 - C_2]qr - C_1 q[r + \sigma]),$$
$$\dot{q} = (A_1 + A_2)^{-1} \times ([C_2 - A_1 - A_2]pr + C_1 p[r + \sigma]), \tag{8}$$
$$\dot{r} = \frac{-M}{C_2}, \quad \dot{\sigma} = \frac{M(C_1 + C_2)}{C_1 C_2}.$$

According to [6], let us choose a translationally moving coordinate system in such a way that the axis OZ is coincident with the invariably directed vector of the angular momentum. In this case, the expressions for the angular velocities and for the Eulerian angles have the form

$$p = \frac{K}{A_1 + A_2}\sin\theta_0\sin\varphi, \quad q = \frac{K}{A_1 + A_2}\sin\theta_0\cos\varphi,$$
$$r = \frac{-Mt}{C_2} + r_0, \quad \sigma = \frac{M(C_1 + C_2)}{C_1 C_2}t + \sigma_0,$$
$$\theta = \theta_0, \quad \varphi = \frac{-Mt^2}{2C_2} + \left(r_0 - \frac{K}{A_1 + A_2}\cos\theta_0\right)t + \varphi_0, \tag{9}$$
$$\psi = \frac{K}{A_1 + A_2}t + \psi_0, \quad \delta = \frac{M(C_1 + C_2)}{2C_1 C_2}t^2 + \sigma_0 t + \delta_0,$$

where K is the angular momentum of the system.

4. STABILITY OF STEADY-STATE ROTATIONS OF A FREE SYSTEM

Let us determine possible steady-state modes of the free system motion by equating to zero the derivatives of the angular velocities in Eqs. (8):

$$(A_1 + A_2)^{-1}([A_1 + A_2 - C_2]qr - C_1 q[r + \sigma]) = 0,$$
$$(A_1 + A_2)^{-1}([C_2 - A_1 - A_2]pr + C_1 p[r + \sigma]) = 0,$$
$$\dot{r} = 0, \quad \dot{\sigma} = 0.$$

Two steady-state modes exist:

$$(1)\ p = p_0, \quad q = q_0, \quad r = r_0,$$
$$\sigma = \frac{A_1 + A_2 - C_1 - C_2}{C_1}r_0,$$
$$(2)\ p = 0, \quad q = 0, \quad r = r_0, \quad \sigma = \sigma_0,$$

where p_0, q_0, r_0, and σ_0 are constants.

Let us analyze the stability of the first mode. Let us introduce small perturbations of the angular velocities Δp, Δq, Δr, and $\Delta\sigma$ and write the equations of the perturbed motion:

$$\frac{d\Delta p}{dt} = (A_1 + A_2)^{-1} \times ([A_1 + A_2 - C_2]\Delta r - C_1[\Delta r + \Delta\sigma])(q_0 + \Delta q),$$
$$\frac{d\Delta q}{dt} = (A_1 + A_2)^{-1} \times ([C_2 - A_1 - A_2]\Delta r + C_1[\Delta r + \Delta\sigma])(p_0 + \Delta p), \tag{10}$$
$$\frac{d\Delta r}{dt} = 0, \quad \frac{d\Delta\sigma}{dt} = 0.$$

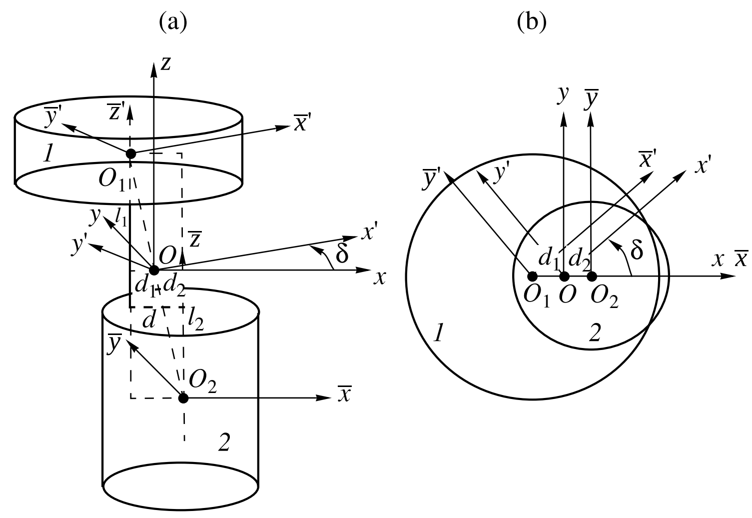

Fig. 3. Coaxial bodies in the case of small asymmetry.

A general solution to system (10) has the following form:

$$\Delta p = D_1\cos\chi t + D_2\sin\chi t - p_0,$$
$$\Delta q = D_2\cos\chi t - D_1\sin\chi t - q_0, \quad (11)$$
$$\Delta\sigma = (\Delta\sigma)_0, \quad \Delta r = (\Delta r)_0,$$

where $\chi = (A_1 + A_2)^{-1}([A_1 + A_2 - C_2]\Delta r - C_1[\Delta r + \Delta\sigma])$; $D_1 = (\Delta p)_0 + p_0$, $D_2 = (\Delta q)_0 + q_0$, and $(\Delta p)_0$, $(\Delta q)_0$, $(\Delta\sigma)_0$, and $(\Delta r)_0$ are the initial small perturbations.

It can be seen from solutions (11) that the first steady-state mode is stable in the linear approximation in the space of angular velocities. It can be demonstrated that the vectors of the angular momentum of the system and the angular velocity of the body *2* are codirectional: $\mathbf{K} = (A_1 + A_2)\boldsymbol{\omega}$.

The second mode allows a stability study by means of the Lyapunov function: $L = (\Delta p)^2 + (\Delta q)^2 + (\Delta r)^2 + (\Delta\sigma)^2$. The derivative of this function is identically equal to zero by virtue of the equations of the corresponding perturbed system; hence, this mode is stable and characterizes the rotation of the body about the principal axis of the ellipsoid of inertia coincident with the common axis of rotation of the bodies.

5. MOTION OF A FREE SYSTEM OF COAXIAL BODIES IN THE CASE OF SMALL ASYMMETRY

Let us consider the motion of a system of coaxial bodies in the case of small asymmetry associated with a displacement d of the common axis of rotation from the axis of dynamical symmetry of body *2* (Fig. 3). Here, the axis of dynamical symmetry of body *1* coincides with the axis of rotation and is parallel to the axis of dynamical symmetry of body *2*. In this case, the system's center of mass O will belong to the interval O_1O_2 between the centers of mass of the bodies and will not change its position with respect to body *2*.

Let us bring into consideration the following systems of coordinates. The system $Oxyz$ is the basic moving coordinate system with the axes firmly connected with body *2*. The axis Ox lies in the equatorial plane, which is perpendicular to the axis of rotation, and coincides with the projection of the interval O_1O_2 onto this plane. The axis Oy lies in the above-mentioned plane. The system $O_2\bar{x}\bar{y}\bar{z}$ is the principal central system of coordinates connected with body *2* with the axes parallel to those of the system $Oxyz$. The system $O_1\bar{x}'\bar{y}'\bar{z}'$ is the principal central coordinate system connected with body *1*. The system $Ox'y'z'$ is the system of coordinates with the origin at the center of mass O and with the axes parallel to those of the system $O_1\bar{x}'\bar{y}'\bar{z}'$. Let us take the angle between the equatorial axes Ox and Ox' as the angle of relative twist. Let m_1 and m_2 be the masses of bodies *1* and *2*, respectively, and l be the distance between the bodies' centers of mass O_1 and O_2.

The projections of the angular velocity of body *2* in the systems $Oxyz$ and $O_2\bar{x}\bar{y}\bar{z}$ are the same and equal to p, q, and r. Similarly, the projections of the angular velocity of body *1* in the systems $Ox'y'z'$ and $O_1\bar{x}'\bar{y}'\bar{z}'$ are identical and equal to p', q', and r' because the corresponding axes are parallel. The distances between the centers of mass of bodies *1* and *2*, and the center of mass of the system are correspondingly equal (Fig. 3) to $l_1 = lm_2/(m_1 + m_2)$ and $l_2 = lm_1/(m_1 + m_2)$ in the direction along the axis of rotation and $d_1 = dm_2/(m_1 + m_2)$ and $d_2 = dm_1/(m_1 + m_2)$ in the direction perpendicular to it.

A small displacement d (Fig. 3) results in variations of the moments of inertia of body *2* in the system $Oxyz$:

$$I_{xx} = \bar{A}_2 + m_2 l_2^2, \quad I_{yy} = \bar{A}_2 + m_2 l_2^2 + m_2 d_2^2,$$

$$I_{zz} = \bar{C}_2 + m_2 d_2^2,$$

$$I_{xy} = I_{yz} = 0, \quad I_{xz} = -m_2(-l_2)d_2 = m_2 l_2 d_2,$$

where $\bar{A}_2$, $\bar{C}_2$ are the equatorial and longitudinal moments of inertia of body *2* as calculated in the central system $O_2\bar{x}\bar{y}\bar{z}$.

Let us write down the angular momentum theorem in the coordinate system *Oxyz* in form (3). To do this, let us find the projections of the angular momenta of bodies *1* and *2* onto the axes of the systems *Ox'y'z'* and *Oxyz*, respectively. The angular momentum of body *1* is calculated as the sum of the angular momenta of motion of the center of mass of body *1*, $\mathbf{K}_1^e$, and the angular momentum of body *1* in the case of its rotation about the center of mass, $\mathbf{K}_1^r$: $\mathbf{K}_1 = \mathbf{K}_1^e + \mathbf{K}_1^r$.

In projections onto the axes of the coordinate system *Ox'y'z'* we obtain

$$\mathbf{K}_1 = m_1[\delta]^{-1}\left(\begin{bmatrix}-d_1\\0\\l_1\end{bmatrix}\times\begin{bmatrix}p\\q\\r\end{bmatrix}\times\begin{bmatrix}-d_1\\0\\l_1\end{bmatrix}\right)+\begin{bmatrix}\bar{A}_1 p'\\\bar{A}_1 q'\\\bar{C}_1 r'\end{bmatrix}$$
$$= m_1[\delta]^{-1}\begin{bmatrix}l_1^2 p + d_1 l_1 r\\q(l_1^2+d_1^2)\\d_1^2 r + d_1 l_1 p\end{bmatrix}+\begin{bmatrix}\bar{A}_1 p'\\\bar{A}_1 q'\\\bar{C}_1 r'\end{bmatrix}, \quad (12)$$

where $\bar{A}_1$ and $\bar{C}_1$ are the principal equatorial and longitudinal central moments of inertia of body *1* and $[\delta] = \begin{bmatrix}\cos\delta & -\sin\delta & 0\\\sin\delta & \cos\delta & 0\\0 & 0 & 1\end{bmatrix}$ is the matrix of transition from the coordinate system *Ox'y'z'* to the system *Oxyz*.

Let us write down the angular momentum of body *2* in the system *Oxyz*:

$$\mathbf{K}_2 = \hat{\mathbf{I}}\cdot\boldsymbol{\omega} = \begin{bmatrix}(\bar{A}_2 + m_2 l_2^2)p + m_2 l_2 d_2 r\\(\bar{A}_2 + m_2 l_2^2 + m_2 d_2^2)q\\(\bar{C}_2 + m_2 d_2^2)r + m_2 l_2 d_2 p\end{bmatrix}. \quad (13)$$

Taking (12) and (13) into account, vector equation (3) can be written in the coordinate system *Oxyz* in the form

$$\begin{bmatrix}(\bar{A}_2 + m_2 l_2^2)\dot{p} + m_2 l_2 d_2 \dot{r}\\(\bar{A}_2 + m_2 l_2^2 + m_2 d_2^2)\dot{q}\\(\bar{C}_2 + m_2 d_2^2)\dot{r} + m_2 l_2 d_2 \dot{p}\end{bmatrix}+\begin{bmatrix}p\\q\\r\end{bmatrix}\times\begin{bmatrix}(\bar{A}_2 + m_2 l_2^2)p + m_2 l_2 d_2 r\\(\bar{A}_2 + m_2 l_2^2 + m_2 d_2^2)q\\(\bar{C}_2 + m_2 d_2^2)r + m_2 l_2 d_2 p\end{bmatrix}$$
$$+[\delta]\left\{\frac{d}{dt}\left(m_1[\delta]^{-1}\begin{bmatrix}l_1^2 p + d_1 l_1 r\\q(l_1^2+d_1^2)\\d_1^2 r + d_1 l_1 p\end{bmatrix}+\begin{bmatrix}\bar{A}_1 p'\\\bar{A}_1 q'\\\bar{C}_1 r'\end{bmatrix}\right)+\begin{bmatrix}p'\\q'\\r'\end{bmatrix}\right.$$
$$\left.\times\left(m_1[\delta]^{-1}\begin{bmatrix}l_1^2 p + d_1 l_1 r\\q(l_1^2+d_1^2)\\d_1^2 r + d_1 l_1 p\end{bmatrix}+\begin{bmatrix}\bar{A}_1 p'\\\bar{A}_1 q'\\\bar{C}_1 r'\end{bmatrix}\right)\right\}=\begin{bmatrix}M_x\\M_y\\M_z\end{bmatrix}. \quad (14)$$

Applying relations (1), Eqs. (14) can be rewritten in the following form:

$$(A_1 + A_2)\dot{p} + qr(C_1 - A_1 + C_2 - A_2) + C_1 q\sigma$$
$$= M_x - (m_1 l_1 d_1 + m_2 l_2 d_2)(pq + \dot{r}),$$
$$(A_1 + A_2 + m_1 d_1^2 + m_2 d_2^2)\dot{q} +$$
$$+ pr(A_1 - C_1 + A_2 - C_2 - m_1 d_1^2 - m_2 d_2^2) - C_1 p\sigma$$
$$= M_y - (m_1 l_1 d_1 + m_2 l_2 d_2)(r^2 - p^2), \quad (15)$$
$$(C_1 + C_2 + m_1 d_1^2 + m_2 d_2^2)\dot{r} + C_1\dot{\sigma}$$
$$= M_z - (m_1 l_1 d_1 + m_2 l_2 d_2)(\dot{p} - qr) - (m_1 d_1^2 + m_2 d_2^2)pq,$$

where $A_i = \bar{A}_i + m_i l_i^2$, $C_i = \bar{C}_i$ $(i = 1, 2)$.

In this case, the equation of relative motion of the bodies turns out to be similar to Eq. (6). System (15), together with Eq. (6), presents dynamical equations of motion of coaxial bodies in the case of the above-indicated asymmetry.

Let us take a dimensionless variable characterizing the displacement of the axes of dynamic symmetry of the bodies from the axis of rotation as a small parameter ε:

$$\varepsilon = \frac{m_1 d_1 l_1 + m_2 d_2 l_2}{A_1 + A_2} = \frac{m_1 m_2 dl}{(m_1 + m_2)(A_1 + A_2)}. \quad (16)$$

Suppose that no moments of external forces and internal interaction are acting; then, to an accuracy of the order of ε, the dynamic equations can be written as

$$\dot{p} + aqr + bq\sigma = -\varepsilon(pq + \dot{r}),$$
$$\dot{q} - apr - bp\sigma = -\varepsilon(r^2 - p^2), \quad (17)$$
$$(C_1 + C_2)\dot{r} + C_1\dot{\sigma} = -\varepsilon(A_1 + A_2)(\dot{p} - qr),$$

$$\dot{\sigma} = -\dot{r},$$

where $a = \dfrac{C_1 - A_1 + C_2 - A_2}{A_1 + A_2}$ and $b = \dfrac{C_1}{A_1 + A_2}$ are the dimensionless parameters.

Let us change from the equatorial angular velocities to the variables of the "amplitude–phase" type, G and F, with the aid of the following substitution:

$$p = G(t)\cos F(t), \quad q = G(t)\sin F(t). \tag{18}$$

To an accuracy of the order of ε, dynamic equations (17) can be rewritten as follows:

$$\begin{gathered} \dot{G} = -\varepsilon r^2 \cos F, \\ \dot{F} = -(ar + b\sigma) - \frac{\varepsilon[G^2 - r^2]\sin F}{G}, \\ \dot{r} = \frac{\varepsilon(A_1 + A_2)}{C_2} G[r + (ar + b\sigma)]\cos F, \\ \dot{\sigma} = -\dot{r}. \end{gathered} \tag{19}$$

The solutions to the generating system of equations ($\varepsilon = 0$) have the form

$$\begin{gathered} \bar{G} = \frac{K\sin\theta_0}{A_1 + A_2}, \quad \bar{F} = \omega t + \varphi_0, \\ \bar{r} = r_0, \quad \bar{\sigma} = \sigma_0, \end{gathered} \tag{20}$$

where $\omega = r_0 - \dfrac{K}{A_1 + A_2}\cos\theta_0 = r_0 - \dfrac{K_z}{A_1 + A_2} = r_0 - \dfrac{C_1\sigma_0 + (C_1 + C_2)r_0}{A_1 + A_2} = -ar_0 - b\sigma_0$.

Applying Poincaré's theorem [7], we seek the solutions to perturbed system (19) in the form of the following expansions, confining ourselves to two terms of the asymptotic series:

$$\begin{gathered} G(t) = \bar{G} + \varepsilon g(t), \quad F(t) = \bar{F} + \varepsilon f(t), \\ \sigma(t) = \bar{\sigma} + \varepsilon \Sigma(t), \quad r(t) = \bar{r} + \varepsilon R(t), \\ \delta(t) = \bar{\delta} + \varepsilon \Delta(t), \quad \dot{\Delta} = \Sigma, \end{gathered} \tag{21}$$

where $\bar{\delta} = \sigma_0 t + \delta_0$ and $g(t)$, $f(t)$, $R(t)$, $\Sigma(t)$, and $\Delta(t)$ are the functions to be found.

Substituting expressions (21) into perturbed system (19) and equating the terms of the order of ε, we obtain the system of equations for perturbations:

$$\begin{gathered} \dot{g} = -r_0^2\cos(\omega t + \varphi_0), \\ \dot{f} = -(aR + b\Sigma) - \frac{\bar{G}^2 - r_0^2}{\bar{G}}\sin(\omega t + \varphi_0), \\ \dot{R} = \frac{A_1 + A_2}{C_2}\bar{G}[r_0 - \omega]\cos(\omega t + \varphi_0), \\ \dot{\Sigma} = -\frac{A_1 + A_2}{C_2}\bar{G}[r_0 - \omega]\cos(\omega t + \varphi_0). \end{gathered} \tag{22}$$

Following [7], we obtain the solution to system (22) for zero initial values of small perturbations:

$$\begin{gathered} g(t) = -\frac{r_0^2}{\omega}[\sin(\omega t + \varphi_0) - \sin\varphi_0], \\ f(t) = \gamma t + \beta[\cos(\omega t + \varphi_0) - \cos\varphi_0], \\ R(t) = \alpha(\sin(\omega t + \varphi_0) - \sin\varphi_0), \\ \Sigma(t) = -\alpha(\sin(\omega t + \varphi_0) - \sin\varphi_0), \end{gathered} \tag{23}$$

where

$$\alpha = \frac{A_1 + A_2}{C_2\omega}\bar{G}[r_0 - \omega],$$

$$\beta = -\frac{1}{\omega}\left\{\frac{A_1 + A_2}{C_2\omega}\bar{G}[r_0 - \omega](b - a) - \frac{\bar{G}^2 - r_0^2}{\bar{G}}\right\},$$

where

$$\gamma = \alpha\sin\varphi_0(a - b).$$

The time dependences for the amplitude–phase variables and for the angular velocities r and σ of the system of coaxial bodies with a small asymmetry follow from Eqs. (23) and (21).

To an accuracy of the order of ε, we can write the dependences for the equatorial angular velocities:

$$p(t) = \bar{G}\cos\bar{F} + \varepsilon P(t), \; q(t) = \bar{G}\sin\bar{F} + \varepsilon Q(t), \tag{24}$$

where

$$\begin{gathered} P(t) = g\cos\bar{F} \\ -\bar{G}(\beta[\cos(\omega t + \varphi_0) - \cos\varphi_0] + \gamma t)\sin\bar{F}, \\ Q(t) = g\sin\bar{F} \\ +\bar{G}(\beta[\cos(\omega t + \varphi_0) - \cos\varphi_0] + \gamma t)\cos\bar{F}. \end{gathered}$$

Let us pass on to determining the dependences for the Eulerian angles and seek them in the form of the following expansions in the small parameter ε:

$$\begin{gathered} \psi = \bar{\psi} + \varepsilon\Psi(t), \quad \theta = \bar{\theta} + \varepsilon\Theta(t), \\ \varphi = \bar{\varphi} + \varepsilon\Phi(t). \end{gathered} \tag{25}$$

The generating solutions ($\bar{\psi}$, $\bar{\theta}$, $\bar{\varphi}$) are defined by dependences (9) with allowance made for zero moment of the internal interaction of the bodies ($M = 0$). Substituting expansion (25) and the obtained dependences for the angular velocities into Euler's equations (7) and

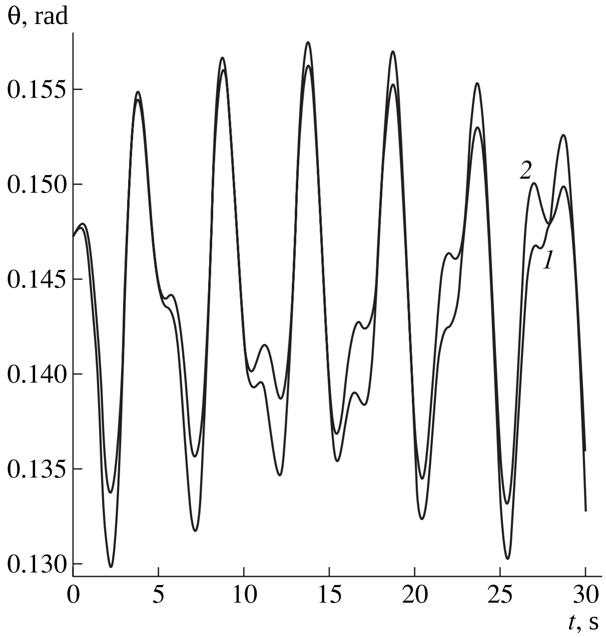

Fig. 4. Comparison of nutation angles calculated by (*1*) numerical integration and (*2*) the approximate analytical relation.

equating the terms of the order of ε, we obtain the kinematical equations of the first approximation:

$$\dot{\Psi} = -\frac{K\cos\bar{\theta}}{(A_1 + A_2)\sin\theta}\Theta + \frac{1}{\sin\theta}[P\sin\bar{\varphi} + Q\cos\bar{\varphi}],$$

$$\dot{\Theta} = -\frac{K\sin\bar{\theta}}{A_1 + A_2}\Phi + P\cos\bar{\varphi} - Q\sin\bar{\varphi}, \qquad (26)$$

$$\dot{\Phi} = \frac{K}{(A_1 + A_2)\sin\bar{\theta}}\Theta - \cot\bar{\theta}[P\sin\bar{\varphi} + Q\cos\bar{\varphi}].$$

Now, we integrate separately the two last equations of the inhomogeneous linear system of equations (26). Let us apply the Cauchy formula [8]:

$$\mathbf{y}(t) = \mathbf{M}(t)\mathbf{M}^{-1}(t_0)\mathbf{y}_0 + \mathbf{M}(t)\int_{t_0}^{t}\mathbf{M}^{-1}(s)\mathbf{f}(s)ds, \quad (27)$$

where $\mathbf{y}(t)$ is the general solution to the inhomogeneous linear system of differential equations $L(\mathbf{y}) = \mathbf{f}(t)$, L is the linear differential operator, t_0 is the initial value of the independent variable, $\mathbf{y}_0$ is the vector of initial values of the required functions, $\mathbf{M}(t)$ is the fundamental matrix of the corresponding homogeneous system, and $\mathbf{f}(t)$ is the vector of perturbing functions.

Finally, let us write the solutions for small perturbations of the nutation angles and the angles of proper rotation at zero initial values:

$$\begin{bmatrix}\Theta(t)\\ \Phi(t)\end{bmatrix} = \mathbf{M}(t)\int_0^t \mathbf{M}^{-1}(s)\begin{bmatrix}f^{\Theta}(s)\\ f^{\Phi}(s)\end{bmatrix}ds, \qquad (28)$$

where

$$f^{\Theta} = P\cos\bar{\varphi} - Q\sin\bar{\varphi},$$

and

$$f^{\Phi} = -\cot\bar{\theta}[P\sin\bar{\varphi} + Q\cos\bar{\varphi}]$$

are the known perturbing functions and

$$\mathbf{M}(t) = \begin{bmatrix} -\sin\bar{\theta}\sin\left|\frac{K}{A_1 + A_2}\right|t & \sin\bar{\theta}\cos\left|\frac{K}{A_1 + A_2}\right|t \\ \cos\left|\frac{K}{A_1 + A_2}\right|t & \sin\left|\frac{K}{A_1 + A_2}\right|t \end{bmatrix}$$

is the fundamental matrix of solutions to the corresponding homogeneous system.

If we substitute the time dependence of the nutation angle perturbation from (28) into the first equation in (26), the perturbation of the precession angle $\mathbf{\Psi}(t)$ will be determined by integration:

$$\Psi(t) = \frac{1}{\sin\theta_0}$$

$$\times\int_0^t\left(-\frac{K\cos\theta_0}{(A_1 + A_2)}\Theta(t) + P(t)\sin\bar{\varphi} + Q(t)\cos\bar{\varphi}\right)dt.$$

Let us give the nutation angle $\theta(t)$ (Fig. 4) calculated by the approximate analytical dependences (28) and (25) and by numerical integration with the following initial conditions of motion and inertia–mass parameters of the system:

$m_1 = 15$ kg, $m_2 = 30$ kg,

$l = 0.4$ m, $d = 0.01$ m,

$A_1 = 2$ kg m^2, $A_2 = 1.5$ kg m^2,

$C_1 = 1.2$ kg m^2, $C_2 = 1.3$ kg m^2,

$p_0 = 0.3$ rad/s, $q_0 = 0.2$ rad/s,

$r_0 = 1.1$ rad/s, $\sigma_0 = 5$ rad/s.

The small parameter is $\varepsilon = 0.01$ for the above numerical values.

The obtained results permit an analysis of the motion of reentry vehicles with partial spin-up to be made. They also allow one to choose the initial conditions of motion and inertia–mass parameters for this type of spacecraft.

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